Problem Set 7: Due Dec 4th in Class
Econ 14.04

Note: This problem set is due on December 4th. Be sure to read the undergrad Varian text book Chapters 26 – 28 and 31 before the test.

1. Consider the simultaneous move game below:

	Left	*Center*	*Right*
Top	$-1, 3$	$3, -1$	$5, 0$
Middle	$3, -1$	$-1, 3$	$5, 0$
Bottom	$0, 5$	$0, 5$	$1000, 4$

   (a) Show that Player 2s action of "right" is dominated by a mixed strategy of Left and Center.

   (b) Show that there is no nash equilibrium in dominant strategies.

   (c) Determine the NE in mixed strategies as formally as possible. (Hint: remember that in order to mix the expected value of both choices must be equal. Start with the row player - determine the y that will make choosing Top and Middle equal as a function of the mixing probability of the column player. Do the same thing for the column player. The point where both of these equations are satisfied will be the mixed strategy nash equilibrium).

2. Will and Davy are playing the following game of "Liar's poker" With the following form:

   - A deck of cards consisting of 4 kings and 4 aces is shuffled and put in front of Will. Will looks at the top card and makes a (potentially untruthful) announcment of "King" or "Ace".
     - If he announces "King" the game ends with no exchange of money.
     - If he announces "Ace", Davy gets to take an action:
   - Davy has the option of "Folding" or "Calling".
     - If he folds, Davy pays Will $.50.
     - If he Calls and Will is holding the Ace, Davy pays Will $1.00.
     - If he Calls and Will is holding the King, Will pays Davy $1.00.

   (a) Draw the extensive form and normal form of this game.

   (b) Show that there is no pure strategy Nash Equilibirum.

   (c) Given that Will has a King, how often should he bluff?

   (d) Given that Will announces a King, how often should Davy call him?

3. Consider an industry with 3 firms, each having marginal costs equal to 0. The inverse demand curve facing this industry is:

$$P(q_1, q_2, q_3) = 60 - (q_1 + q_2 + q_3)$$

   (a) If each firm behaves as a Cournot competitor (quantity competition), what is firm 1's best response function?

   (b) Calculate the Cournot equilibrium.

   (c) Firms 2 and 3 decide to merge and form a single firm (MC still 0). Calculate the new industry equilibrium. Is firm 1 better or worse off as a result? Are the combined profits from firm 2 and 3 greater or less than before? Would it be a profitable idea for all three firms to organize into a cartel?

   (d) Suppose firm 1 can commit to a certain level of output in advance. If the choice of firm 1 is q_1,what would be the optimal choices of firms 2 and 3?

4. Consider the following game. An incumbent makes a decision to advertise at a cost K or not. This action is observed by a challenger who has the option of entering the market at cost F or staying out. If the challenger stays out of the market, the incumbent firm is a monopolist. If the challenger enters the market, the two parties compete a la Cournot.

(a) Draw the extensive form of this game. (5 points)

(b) Advertisement increases demand at any given price. The inverse demand curves when incumbent advertises and when not are (correspondingly):

$$P_{Ad}(q_1, q_2) = 60 - Q$$
$$P_{NoAd}(q_1, q_2) = 48 - Q$$

Assume that all parties have zero marginal costs.

1. Calculate the profits of each firm when the challenger enters depending on whether the incumbent advertises or not. (9 points)
2. Suppose that $F = 350$. Should the incumbent advertise? (3 points)
3. Suppose that $F = 100$. For what levels of K should the incumbent advertise? (3 points)

5. *Consider the following game. There are N players, each with an initial endowment of k. Each agent has the opportunity to invest $x^i \in [0, k]$ in the spraying of mosquitos, effects of which are enjoyed evenly by all individuals. The remaining amount $y^i = k - x^i$ is privately consumed. If $X = x^1 + x^2 + \cdots + x^n$ is invested in bug spraying, the total benefits from the public good that is generated is $G = \alpha X$, with each individual receiving $g^i = \frac{G}{N}$. Assume that all agents have a linear utility function $U^i(g^i, y^i) = g^i + y^i$.

(a) Solve for NE of this game as a function of α.

(b) What is socially optimal level of investment as a function of α.

(c) Suppose that the local, peoples' government can observe individual contributions and distribute the outcome of investment as it wishes, not necessarily equally. Can you think of a scheme that would obtain the socially optimal level of investment?

(d) **What if the government can observe the contributions, but does not know how much original resources each individual has? Can you think of a scheme that would obtain the socially optimal level of investment?