Problem Set 5

14.003/14.03, Fall 2010

1 Short Questions [30 points]

To receive full credit, provide a complete defense of your answer.

1. [5 points] True/False and Explain: Jason has a von Neumann-Morgenstern utility function $u(W) = 1 - e^{-\frac{1}{2}W}$. When he becomes richer, he is willing to pay strictly less for full insurance against a shock that would make him lose \$150.00 with probability p. [*Hint: set up the equation that determines the price he is willing to pay and see which terms cancel out*].

2. [5 points] True/False and Explain: Ashley is risk averse and her income and wealth are entirely unrelated to NBA results, except for occasional betting. She would be willing to bet on the Celtics winning the World Championship if it paid her 3 dollars for every dollar bet. Then she must believe that the probability of that happening is at least 1/3. [*Allow the bet amount to be any positive real number. If she bets b, she always pays b and in case she wins, she gets back $3b$. Think about the condition that needs to hold for Ashley to choose not to bet.*]

3. [10 points] A sports fan has an expected utility function of the form $u(w) = \ln w$. She has subjective probability p that the Patriots will win their next football game and probability $1-p$ that they will not win. She chooses to bet \$$x$ on the Patriots, so that if the Patriots win, she wins \$$x$ and if they lose, she loses \$$x$.

   You know the fan's initial wealth W_0. How can you determine her subjective odds $\frac{p}{1-p}$ by observing the size of her bet x?

4. [10 points] In 1953, Maurice Allais proposed the following thought experiment. You must make a choice between Gamble A and Gamble B (you can interpret these dollar amounts as final wealth levels):

   Gamble **A**. \$1,000,000 for sure

   Gamble **B**. \$1,000,000 with probability 0.89

   \$5,000,000 with probability 0.10

   \$0 with probability 0.01

   Which would you choose?

   Next, you must make a choice between Gamble C and Gamble D:

   Gamble **C:.**\$1,000,000 with probability 0.11

$0 with probability 0.89

Gamble **D:** $\$5,000,000$ with probability 0.10

$0 with probability 0.90

Which would you choose?

Most people choose Gambles A and D. Explain why, no matter what utility function a person has over final wealth, this pattern of choices is inconsistent with expected utility maximization.

2 State Dependent Utility [15 points]

Jeff works in construction, a job that has a reasonably high risk of injury. If Jeff stays healthy and continues to work, his utility of wealth is:

$$U = 30W^{.75}$$

If he gets severely injured on the job and can no longer work, his utility of wealth will fall to:

$$U = 5W^{.75}$$

due to the fact that, if disabled, he cannot enjoy as much the things he would have been able to purchase with this wealth. Assume Jeff's current wealth is $\$20,000$

1. Calculate the loss of wealth (in the healthy state) that would be equivalent in utility impact to suffering a permanent disability.
2. If Jeff faces a probability of .03 of becoming permanently disabled, what would be an actuarially fair price for an insurance policy paying the monetary value you calculated in part (1)?
3. Calculate whether or not Jeff would purchase such a policy. Explain.

3 Insurance Theory [25 points]

Suppose there is a 5% chance that Max gets in a car accident. If there is no accident, he gets to spend his full income of 9 dollars. If there is an accident, he has to spend 5 dollars on car repairs so he only gets to spend 4 dollars for his other needs. He has a von Neumann-Morgenstern utility function $U(w) = \sqrt{w}$.

1. What is his expected utility with no insurance?

2. Suppose now he can buy insurance, at a cost of 5 cents for one dollar of coverage (he pays 5 cents for insurance, which pays out one dollar if there is an accident). How much coverage will he buy? How much will he spend on insurance? How does his utility compare to the case with no insurance?

3. Suppose now that he can buy insurance, but it costs 6 cents for one dollar of coverage. How much coverage does he buy now? Is his utility higher or lower compared to part 2?

4. Now suppose that he can buy insurance at a cost of 5 cents for one dollar of coverage, buy he also has to pay 0.02 dollars in underwriting fees (fixed cost) in addition to whatever he pays for coverage. What is the expression for the total cost of insurance? How much coverage does he buy now and how does it compare to parts 2 and 3? How does his utility compare to part 2? Should he buy insurance? If the fixed cost became 0.05, should he still buy insurance?

5. Now think about the problem from the point of view of the insurance company. Write an expression for the profits of the insurance company [from selling one dollar of coverage] as a function of π, the insurance premium (the cost of one dollar of coverage). What has to be true about π for profits to be zero?

6. Suppose there is only one insurance company monopolizing the auto industry (and only one consumer of insurance). The company cannot charge a fixed cost, but can set the premium π however it wants. Write down an expression for the insurance company's profits as a function of π. [*Hint: First figure out how much coverage the consumer buys as a function of* π, *then substitute this into the expression for the monopolists total profits and maximize profits with respect to* π].

7. Now suppose the insurance company can charge a fixed cost F as well. What are the optimal π and F that maximize profits? How does this relate to the 'certainty equivalent' income for the consumer?

4 Market Unravelling and Mandates [30 points]

The government of Rothschildia (a small, densely populated island nation off of the coast of Cambridge) is considering implementing a *voluntary* national health insurance plan. Everyone in Rothschildia is risk averse with VNM utility function $U(W) = W^{0.5}$. Each citizen has a wealth of 1 Stiglitz (the local currency) but should he or she become ill, s/he must spend her entire wealth of 1 Stiglitz on health care. (Since the cure is immediate and complete, the only disutility of illness is this 1 Stiglitz cost.) The only respect in which Rothschildians differ from one another is that each has a different *ex ante* probability of becoming ill, p_i. Illness probability is distributed *uniformly* among citizens on the $[0, 1]$ interval, meaning that *on average* one-half of the population becomes sick in a year, but that each person has a different probability, p_i, ranging from zero to one where all values of p_i are equally likely. If $p_i = 0$, person i is certain not to become ill, if $p_i = 1$, person i is certain to become ill, if $p_i = .5$, person i has a 50% probability of becoming ill, etc. Moreover, each citizen *knows* his own individual illness probability but the government only knows about the distribution of probabilities. Finally, for this problem, you should assume that each citizen lives only 1 year, but that a new generation is born every year (each also knowing his or her p_i). The government, of course, persists from year to year.

You should also bear in mind the following facts about uniform probability distributions:

1. If a variable is distributed uniformly on the $[0, 1]$ interval, the mean (i.e., expected) value of the variable greater than or equal to a given cutoff, $L \in [0, 1]$ is $\frac{L+1}{2}$. Similarly, the expected value below a given cutoff, $U \in [0, 1]$, is $\frac{U}{2}$. Hence, the expected value of observations $\geq \frac{1}{2}$ is $\frac{3}{4}$, and the expected value of observations $\leq \frac{1}{2}$ is $\frac{1}{4}$.

2. If you want to calculate the expected value of a *function of a random variable,* you must integrate that function over the probability distribution of the random variable which is specified by its 'probability density function' (PDF).

   The PDF of a $U[0, 1]$ (where U stands for uniform) variable is $f(x) = 1$. So, for example, if $y = x^2$ and x is distributed $U[0, 1]$, and you want to know the expectation of y, you would integrate the function y over the PDF of x :

$$E[y] = E\left[x^2\right] = \int_0^1 \left(x^2 \cdot 1\right) dx = \frac{1}{3}$$

The Problem

A. A policy maker for the Board of Health sits down to design the new national health plan. She reasons that since half of all Rothschildians get sick each year, the government should offer an actuarially fair full insurance policy that charges a premium of $\frac{1}{2}$ Stiglitz and pays a benefit of 1 Stiglitz to any enrollee who gets sick (of course, the enrollee pays the premium regardless of whether or not she becomes ill). Given this premium, calculate who chooses to enroll in the plan, and:

- What is the expected illness probability of the most healthy and least healthy person to enroll in the plan?
- What is the average health of those who enroll in the plan?
- Does the plan break even, make money, or lose money in year 1, and by how much per person on average?

B. In year 2, a different policy maker at the board of health (recall, the first has passed away) notes that something went wrong in the first year: the plan made/lost money (depending on your answer above). He reasons, "Clearly we set the premium too high/low in the first year. What we'll do is set the new premium to reflect our average cost from last year. This should straighten things out."

- What is the new premium?
- What is the expected illness probability of the most healthy and least healthy person to enroll in the plan?
- What is the average health of those who enroll in the plan?
- Does the plan break even, make money, or lose money in year 2, and by how much (per person average)?

C. In year 3, a third policy maker observes that something is again amiss. The plan made/lost money *again* last year, although the intention was to break even. This policy maker suggests that the board fix the problem by setting the new premium at the average cost for year 2.

- What is the new premium?
- What is the expected illness probability of the most healthy and least healthy person to enroll in the plan?
- What is the average health of those who enroll in the plan?
- Does the plan break even, make money, or lose money in year 3, and by how much (per person average)?

D. In year 4, a fourth policy maker notes that something went wrong again. This policy maker has taken 14.03, however, and says, "Alas, I see the error of our ways! Every time we change the premium, a different pool of citizens enroll in the plan. I wonder if there is a premium we could set so that the pool of citizens who enrolls at that price costs us on average exactly that price. That way, we'd break even and provide insurance to all those who want to buy it." After a few strokes of the pen, she shouts, "Eureka! There is."

- What is that premium? *Hint: you can either solve this problem analytically (i.e., on paper with a simple equation) or with a spreadsheet by repeating the steps you used for A, B, and C, until you get a convergent solution.*

E. In year 5, an economist (also schooled in 14.03) from the Rothschildian Board of Social Welfare visits the Board of Health and says, "I see that you've worked out the national health plan premium so that it no longer makes/loses money every year. That's a step forward. However, I'm concerned that your break-even program is not actually maximizing average social welfare. What I'd like you to do is calculate average well-being (utility) under three different policy options: 1) No health plan; 2) Your current break-even plan; 3) A third mandatory plan for all citizens that also breaks even. Then report back to me."

- Please perform these calculations for the three plans.
- Which health plan do you recommend based upon your calculations?

F. Explain substantively why your preferred policy option yields higher average social utility than the other two health insurance plans.

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