14.03 Fall 2010

Problem Set 2

Professor: David Autor

No late problem sets are accepted

1 Indirect utility function and expenditure function (18 points)

Let $U = \frac{1}{2}\ln x + \frac{1}{3}\ln y$ be the utility function, where x and y are two goods. Denote p_x and p_y as respectively the prices of the two goods x and y, and m as the income of the consumer.

1. Derive the Marshallian demand functions $d_x(p_x, p_y, m)$, $d_y(p_x, p_y, m)$, and the indirect utility function $V(p_x, p_y, m)$.
2. Apply Roy's Identity to find the Marshallian demand functions.
3. Derive the Hicksian demand functions $h_x(p_x, p_y, U_0)$, $h_y(p_x, p_y, U_0)$, and the expenditure function $E(p_x, p_y, U_0)$.
4. Apply Shephard's lemma to find the Hicksian demand functions.
5. Find $V(p_x, p_y, E(p_x, p_y, U_0))$ and $E(p_x, p_y, V(p_x, p_y, m))$ and explain.

2 Coffee Subsidy Program (28 points + 6 bonus)

Concerned that too many undergraduates are falling asleep during lecture, MIT is considering a program to subsidize coffee for students that would offer a 50% coffee subsidy

up to \$100 (i.e. a maximum subsidy of \$50). We want to help MIT design an experiment to estimate the effects of this policy on coffee consumption, and to compute the deadweight loss of this policy.

You run a randomized experiment that lasts for a semester. You randomly assign students to one of two groups. You give students in the treatment group a 50% coffee subsidy (as described above). For the students in the control group, you just monitor their coffee consumption during the semester.

You observe that in the control group (i.e. the group that does not get a subsidy) one half of the recipients spend \$55 on coffee and the other half spend \$120 on coffee. Assume that the treatment group is identical to the control group, so half of the students in the treatment group would spend \$55 on coffee and half would spend \$120, if assigned to the control group. The price of coffee is equal to \$1/cup. [this means that the subsidized group is paying 50 cents/cup up to 100 cups and 1\$/cup for any extra cups].

1. Draw a diagram that shows the budget set, indifference curve and optimal consumption of coffee for the group that would consume \$55 on coffee under the control group, both under the subsidy treatment and under no treatment. Draw the same diagram for the group that would consume \$120 of coffee. Explain and show diagramatically how the MIT policy constrains student choice relative to an equivalent cash transfer (i.e., a Carte Blanche policy). Indicate for which group coffee consumption at the optimum is distorted (relative to the equivalent cash transfer). [note: we say that a student's choice is distorted if her MRS under the policy is differ from the MRS she would obtain if given Carte Blanche.].

2. Do you expect coffee consumption for the group that spent \$55 on coffee to be higher, lower under the coffee subsidy policy, or is this indeterminate? How about for the group that spent \$120 on coffee? You can assume that coffee is a normal good. [Hint: think about the income and substitution effects of the subsidy]

3. You find that for the group receiving a subsidy, the consumption of coffee is 100 cups for half of the sample and 120 cups for the other half. What is an upper and lower bound of the deadweight loss for the students that bunch at the kink point (i.e. consume 100 cups of cofee). [explain]

   Based on prior research, you believe that the utility function of students over coffee and other goods takes the following form: $U_i(y, c) = y + 0.5\alpha_i \ln c$, where y

is the amount of money spent on all other goods, c is the number of cups of coffee consumed and α_i can take one of two values depending on how much students like coffee (i.e. depending on whether we are looking at the group that consumes \$55 or \$120 in coffee with no subsidy).

4. Assume the price of coffee is equal to p and the price of good y is 1. Derive the Hicksian demand functions and the expenditure function. What is the compensated elasticity of demand for coffee? Recall that the formula for the price elasticity of demand for a good is $\frac{\partial \ln Q}{\partial \ln P} = \frac{\partial Q}{\partial P}\frac{P}{Q}$ and represents the percent change in quantity demanded for a one percent change in price. [Note that the Hicksian demand function for coffee does not depend on the level of utility, and is equal to the Marshallian demand function at price p]

5. Derive the value of α for each of the groups, the group that spends \$55 on coffee (when in the control group), and the group that spends \$120 on coffee (when in the control group) when the price of coffee p is equal to 1.

6. Given the demand functions derived above, compute the quantity of coffee each group would consume under the subsidy treatment. How does the consumption of coffee change for each group under the subsidy? Why? [explain using the income and substitution effects]

7. Draw the compensated demand curve for the students whose consumption is distorted and for the students whose consumption is undistorted by the subsidy. For the distorted students, show on the demand curve, the point where they consume when facing the subsidy, and the point where they would consume if they faced the true price ratio (holding utility constant). Identify the DWL of the program graphically. Explain using a diagram how the DWL loss of the coffee subsidy program would change if the elasticity of the compensated demand curve was $-\frac{1}{2}$. [Hint: The elasticity is related to the steepness of the compensated demand curve.]

8. Using the graph from the previous part, approximate the value of the DWL by a triangle and calculate its area. [note: you should be able to get a numerical value]

9. [Extra credit] How would you compute the exact DWL using the expenditure function derived above, where the DWL is the difference between what MIT spent on the subsidy and the minimum amount of cash MIT would have had to spend to

raise student welfare by an equivalent amount. [Hint: Think about the total expenditure under the coffee subsidy vs. the minimum expenditure that gives the same utility].

10. Ignore all the utility assumptions in part (3). You find that the grades of the group receiving a subsidy have increased relative to the control group. The following semester MIT decides that the program is too expensive and removes the coffee subsidy. However, coffee consumption for the students that were in the experimental group stays higher than for the control group and they continue to receive better grades. Would this lead you to question the carte blanche principle?

3 Are Tortillas a Giffen Good? (15 points)

Researchers are trying to design an experiment to determine whether tortillas are a Giffen good in Mexico.

1. The authors have monthly data on the total quantity of tortillas consumed in 20 cities across Mexico and the average monthly price for tortillas in each city. They find that the total quantity of tortillas consumed and the average price are positively correlated. Can they conclude then that tortillas are a Giffen good in Mexico? Why or why not?

2. They decide instead to run a randomized experiment to determine whether tortillas are a Giffen good. Their hypothesis is that as the price of tortillas increases, households will reduce their consumption of the"fancy" good, which is beef, and increase their consumption of tortillas. The researchers wish to pick a sample of households for their experiment. Their goal is to maximize the chance that they would observe Giffen behavior over the consumption of tortillas for the sample chosen. For each of the following choices, which one of the demographic groups should the researchers prefer and why?

   (a) Poor v. middle-class households

   (b) Households that consume meat regularly, consume meat occasionally, or never consume meat

(c) In the town where the households are located, tortillas are a more expensive or a less expensive source of calories than meat (measured in terms of calories per peso).

Once they have selected the sample according to the criteria above, they send a survey to measure baseline consumption of food and other demographic characteristics for the households. Then they randomly assign households to either a treatment or a control group. Households in the treatment group are given printed vouchers entitling them to a price reduction of 2 pesos off the price of each kilogram of tortillas. Households receive 10 vouchers every month for five months. A kilogram of tortillas currently costs 10 pesos. You measure the consumption of tortillas for each household in the treatment and the control groups during the period in which they are receiving the subsidy.

3. Using the data collected, the researchers want to estimate the causal effect of receiving the subsidy on the consumption of tortillas for the group that received a subsidy $T^* = E[Y_1 - Y_0|X = 1]$ [we call this the treatment effect on the treated]. They compare the consumption of tortillas during the treatment period between the group that receives the subsidy and the control group. Write out the estimate with the causal notation used in class. State the assumptions required to obtain a valid estimate of T^*. Are these assumptions likely to be satisfied?

4. The researchers are worried that the treatment and control group resulting after the randomization might not have been perfectly balanced. Suggest an alternative estimate of the causal effect that uses the data they have collected in the pre-period (and the treatment period) and might provide a better estimate for the causal effect of interest. Explain why this estimate might be preferred to the estimate in part (3).

5. The researchers learn that some recipients of tortilla coupons have sold these coupons for cash to residents of another town rather than use them to buy and consume tortillas. Assume for now that tortillas are an inferior good among consumers. [Recall that for an inferior good**:** $\partial x/\partial I < 0$]. If some consumers did sell their stamps for cash, how would you expect this behavior to bias your test of whether or not rice is a Giffen good? Explain carefully.

6. Assume the researchers find that when the price of tortillas decreases (through subsidies) the quantity of tortillas consumed also decreases, i.e. that tortillas are a Giffen good for the households they studied (ignore the possible bias in the question above). There are numerous programs that aim to improve nutrition by giving food subsidies to very poor households. Can we be confident that these programs will improve nutrition? If they do not, can we be confident that they will improve consumer welfare?

4 Revealed Preference (15 points)

You are given the following partial information about John's purchases. He consumes only two goods:

	Year 1			Year 2	
	Quantity	Price		Quantity	Price
Good 1	100	100	Good 1	120	100
Good 2	100	100	Good 2	?	80

Over what range of quantities of good 2 consumed in year 2 would you conclude:

1. That his behavior is inconsistent (i.e contradicts the weak axiom of revealed preference)
2. That the consumer's consumption bundle in year 1 is revealed preferred to that in year 2?
3. That the consumer's bundle in year 2 is revealed preferred to that in year 1?
4. That there is insufficient information to justify (4.1), (4.2) or (4.3)?

5 Short questions (24 points total + 4 bonus)

You must explain your answers to get ANY credit.

1. A consumer with convex, well-behaved indifference curves who is indifferent between the bundles (1,4) and (9,2) will like the bundle (5,3) better than either of the first two bundles.

2. (a) Which (if any) of the 5 consumer theory axioms are violated for Alice's preferences? There are only two things Alice likes: golf and soccer. However, she mostly likes golf, so when comparing two consumption bundles she always picks the one with the most golf; only if they contain equal amounts of golf does she prefer the one with more soccer.

   (b) BONUS - Try drawing Alice's indifference curves.

3. True, False or unknown - explain for credit.

   Dr. Cyclone has recently discovered a way to clone consumers, and he practices his technique on Silicon dormitory at MIT. Each original resident of Silicon dorm got a clone with exactly the same income and preferences (presumably, he expands the dormitory as well!) All (and only) residents of Silicon eat at the dining hall for dinner each night. We conclude that at every price, the price elasticity of demand for tuna casserole at Silicon dining hall will double after the cloning.

   Questions on the readings:

4. In Waldfogel's study, grandparents are most likely to give cash whereas boyfriends/girlfriends are most likely to give in-kind gifts. Does this suggest that grandparents better understand the principle of deadweight loss than do boyfriends/girlfriends? If not, what is the likely explanation.

5. Imagine that there is a well-functioning underground market for food stamps in Boston, so that food stamps can be resold for $0.90 on the dollar. In Los Angeles, however, enforcement is very effective and food stamps tend to sell at only $0.50 on the dollar. In which city would you expect the deadweight losses from food stamp provision to be greater, and why?

6. Based on the evidence in the Jensen and Miller paper, you conclude that rice is a Giffen good in rural China in the year 2003. Between 2003 and 2004, incomes rise substantially in rural China but the prices of rice and all other goods remain the same. Would you expect rice consumption to rise or fall? Explain rigorously.

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