14.01 Principles of Microeconomics, Fall 2007
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Lecture 29

Strategic Games

Outline

1. Chap 12, 13:   *Collusion – Prisoners' Dilemma*

2. Chap 12, 13:   *Repeated Games*

3. Chap 12, 13:   *Threat, Credibility, Commitment*

4. Chap 14:   *Maximin Strategy*

1   Collusion – Prisoners' Dilemma

Last time we talked about the prisoners' dilemma. The conclusion is that they will betray the other.

Now apply it to the cases of Cournot and Bertrand models.

In the Cournot model, the demand is

$$P = 30 - Q_1 - Q_2.$$

The equilibrium will be

$$Q_1 = Q_2 = 10,$$

with

$$\pi_1 = \pi_2 = 100.$$

However, to maximize their total profits, they should choose a total quantity Q so that

$$\frac{d}{dQ}(Q(30 - Q)) = 0,$$

which follows that

$$Q = 15.$$

If they share profit equally,

$$Q_1 = Q_2 = 7.5,$$
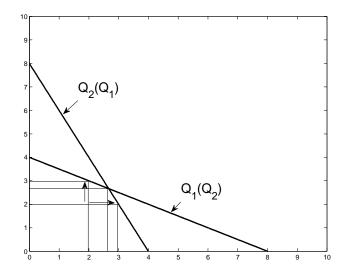
and

$$\pi_1 = \pi_2 = 112.5.$$

Figure 1: Reaction Curves in Cournot Model.

Obviously, the latter case will make both of them better off. But given the opponent produces 7.5, each of them can increase the profit by producing more (see Figure 1).

In Bertrand model, demand functions for firm 1 and firm 2 are

$$Q_1 = 12 - 2P_1 + P_2,$$

and

$$Q_2 = 12 - 2P_2 + P_1.$$

Equilibrium is

$$P_1 = P_2 = 4,$$

with

$$\pi_1 = \pi_2 = 32.$$

However, firms can choose P_1 and P_2 together to maximize the total revenue

$$\pi = P_1(12 - 2P_1 + P_2) + P_2(12 - 2P_2 + P_1).$$

By first order condition, we obtain

$$12 - 4P_1 + 2P_2 = 0,$$

and

$$12 - 4P_2 + 2P_1 = 0.$$

Thus

$$P_1 = P_2 = 6,$$

with

$$\pi_1 = \pi_2 = 36.$$

But in this case, each firm has incentive to lower its price given the other firm's price (see Figure 2).
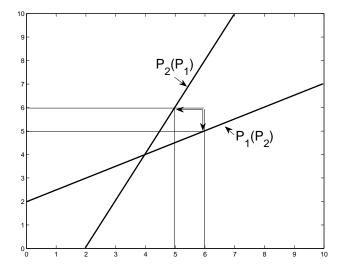

Figure 2: Reaction Curves in Bertrand Model.

2   Repeated Games

Back to the prisoners' problem. If suspect A and B will cooperate for infinite periods, and they are both patient, they care about future payoffs. Because if one of them betrays this time, the opponent will lose the trust and betray in the future; the payoff changes from -1 to -3 for each time. Therefore, both A and B would like to keep silence. But if they are impatient, and only consider today's payoff, they will still betray. Now move on to the case that A and B will cooperate for finite number times which is fairly large. We deduce from the last time they cooperate; the answer is that they will betray for the last time, so will they for other opportunities. Therefore, the collusion between A and B succeed only if they will be cooperative forever and are patient.
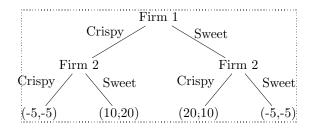
3   Threat, Credibility, Commitment

Back to the crispy-sweet question.

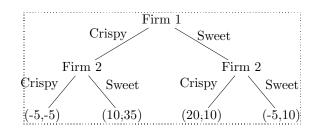|		Firm 2	
		Crispy	Sweet
Firm 1	Crispy	-5,-5	10,20
	Sweet	20,10	-5,-5

Table 1: Payoffs of Firm 1 and 2.

In order to get the largest 20 by producing sweet, firm 2 tries to make firm 1 believe that firm 1 should choose crispy by claiming that it always produces sweet no matter what firm 1 produces. However, firm 1 can ignore firm 2's announcement because once firm 1 choose sweet, firm 2 will produce crispy.

Suppose that firm 2 will advertise and so change the payoffs.

		Firm 2	
		Crispy	Sweet
Firm 1	Crispy	-5,-5	10,35
	Sweet	20,10	-5,10

Table 2: Payoffs of Firm 1 and 2.

In this case, firm 2 feels indifferent between choosing crispy or sweet when firm 1 produces sweet, and chooses sweet when firm 1 produces crispy. So it is credible if firm 2 claims that it always chooses sweet, and then firm 1 had better choose crispy. This example tells us that firm 2 had to do something to make the announcement credible.

4 Maximin Strategy

See Table 3. Firm B has dominant strategy: advertise.

Therefore, the equilibrium should be both A and B advertise.

However, if firm B does not choose the rational option, the minimum payoff of A is 5 if A advertises, and 8 if A does not advertise.

The maximin strategy is the strategy that renders the highest minimum payoff.

When A cannot tell whether B is rational or not, A might use maximin strategy. In this case, the maximin strategy of A is:

		Firm B	
		Advertise	Not Advertise
Firm A	Advertise	10,5	5,0
	Not Advertise	8,8	15,2

Table 3: Payoffs of Firm A and B.