

12.864 Inference from Data and Models 23 February 2004
Problem Set No. 2 Due: 1 March 2004

1. You have a single vector $\mathbf{v}_1 = [1, 1, 1, 1]^T$, but need three more vectors in order to have a complete spanning set. Starting with \mathbf{v}_1 , construct a complete orthonormal basis set.

2. For the tracer balance model shown on P. 9 of the notes, there are three reservoirs with tracer concentrations $C_i, 1 \leq i \leq 3$, where $C_1 = 1.1045, C_2 = 1.2385, C_3 = 1.2900$. Reservoir 0 is observed to have concentration, $C_0 = 1.2$. The problem is to make some inference about the values of the exchange coefficient ratios, $x_i = J_i/J_0$, given also that mass is conserved ($J_0 = J_1 + J_2 + J_3$).

(a) Find the solution of minimum norm, $H = \mathbf{x}^T \mathbf{x}$ in two ways, one of which uses Lagrange multipliers.

(b) Find the change—by direct calculation—in H when C_0 is changed from 1.2 to 1.18. Compare it to the value inferred from the Lagrange multipliers of part (a).

(c) What would your conclusion be if $C_0 = 1.3$?

(d) Repeat (a), (b) except find the solution minimizing $H = (x_2 - x_1)^2 + 4(x_3 - x_2)^2$.

3. In problem 2(a), the situation is the same, except that it is thought that mass balance is probably violated at an accuracy of about $\pm 10\%$, so that the mass balance equation is changed to $J_0 = J_1 + J_2 + J_3 + n$ where n represents the error. Can you find a new solution? Can you find more than one?