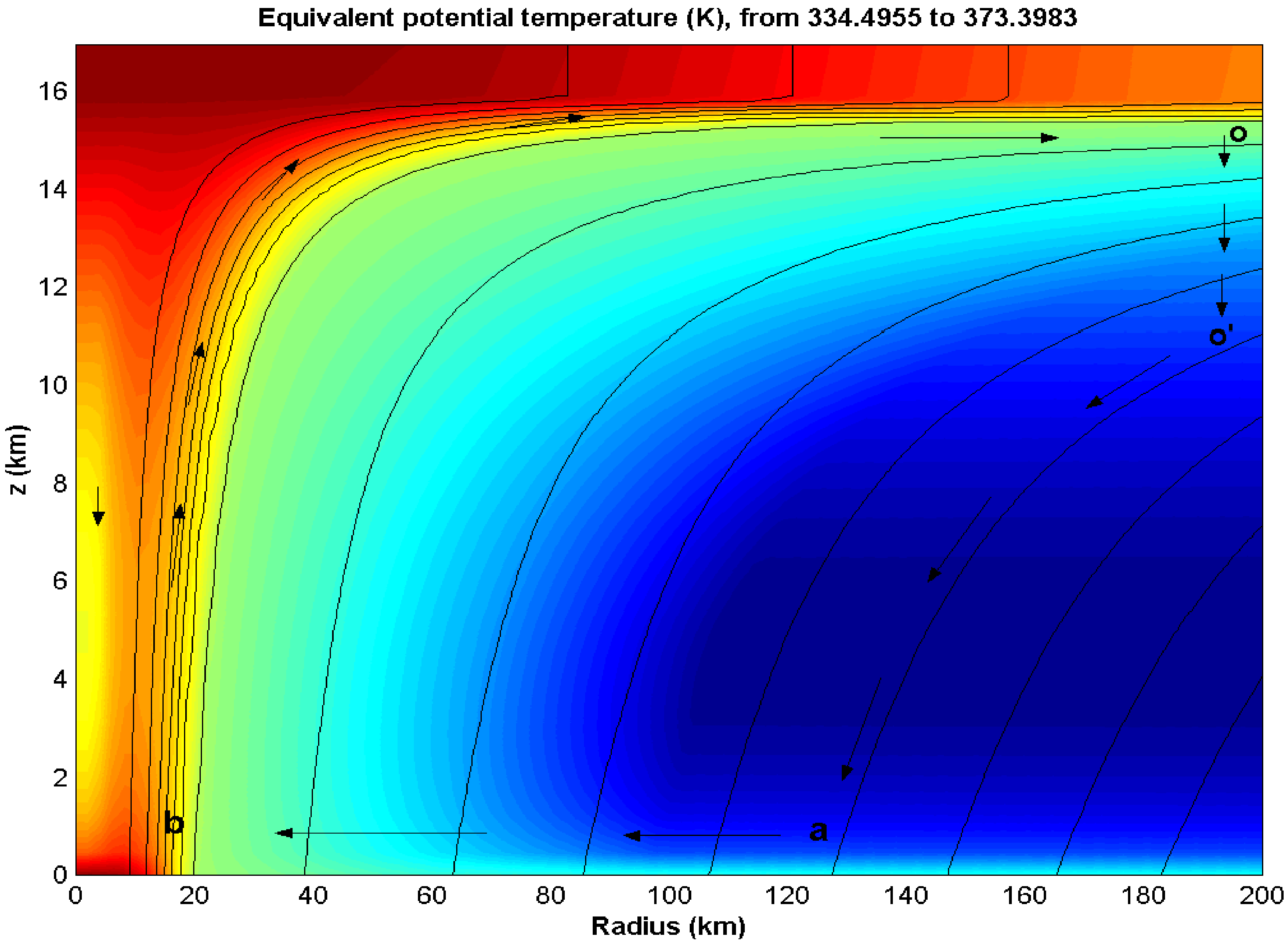
Tropical Cyclones: Steady State Physics

Energy Production



Carnot Theorem: Maximum efficiency results from a particular energy cycle:

- Isothermal expansion
- Adiabatic expansion
- Isothermal compression
- Adiabatic compression

Note: Last leg is not adiabatic in hurricane: Air cools radiatively. But since environmental temperature profile is moist adiabatic, the amount of radiative cooling is the same as if air were saturated and descending moist adiabatically.

Maximum rate of working:

$$W = \frac{T_s - T_o}{T_s} \dot{Q}$$

Total rate of heat input to hurricane:

$$\dot{Q} = 2\pi \int_0^{r_0} \rho \left[C_k \, |\, \mathbf{V} \,| \left(k_0^* - k \right) + C_D \, |\, \mathbf{V} \,|^3 \right] r dr$$

Surface enthalpy flux

Dissipative heating

In steady state, Work is used to balance frictional dissipation:

$$W = 2\pi \int_0^{r_0} \rho \left[C_D \, |\, \mathbf{V} \,|^3 \right] r dr$$

Plug into Carnot equation:

$$\int_0^{r_0} \rho\left[C_D \,|\, \mathbf{V} \,|^3\right] r dr = \frac{T_s - T_o}{T_o} \int_0^{r_0} \rho\left[C_k \,|\, \mathbf{V} \,|\left(k_0^* - k\right)\right] r dr$$

If integrals dominated by values of integrands near radius of maximum winds,

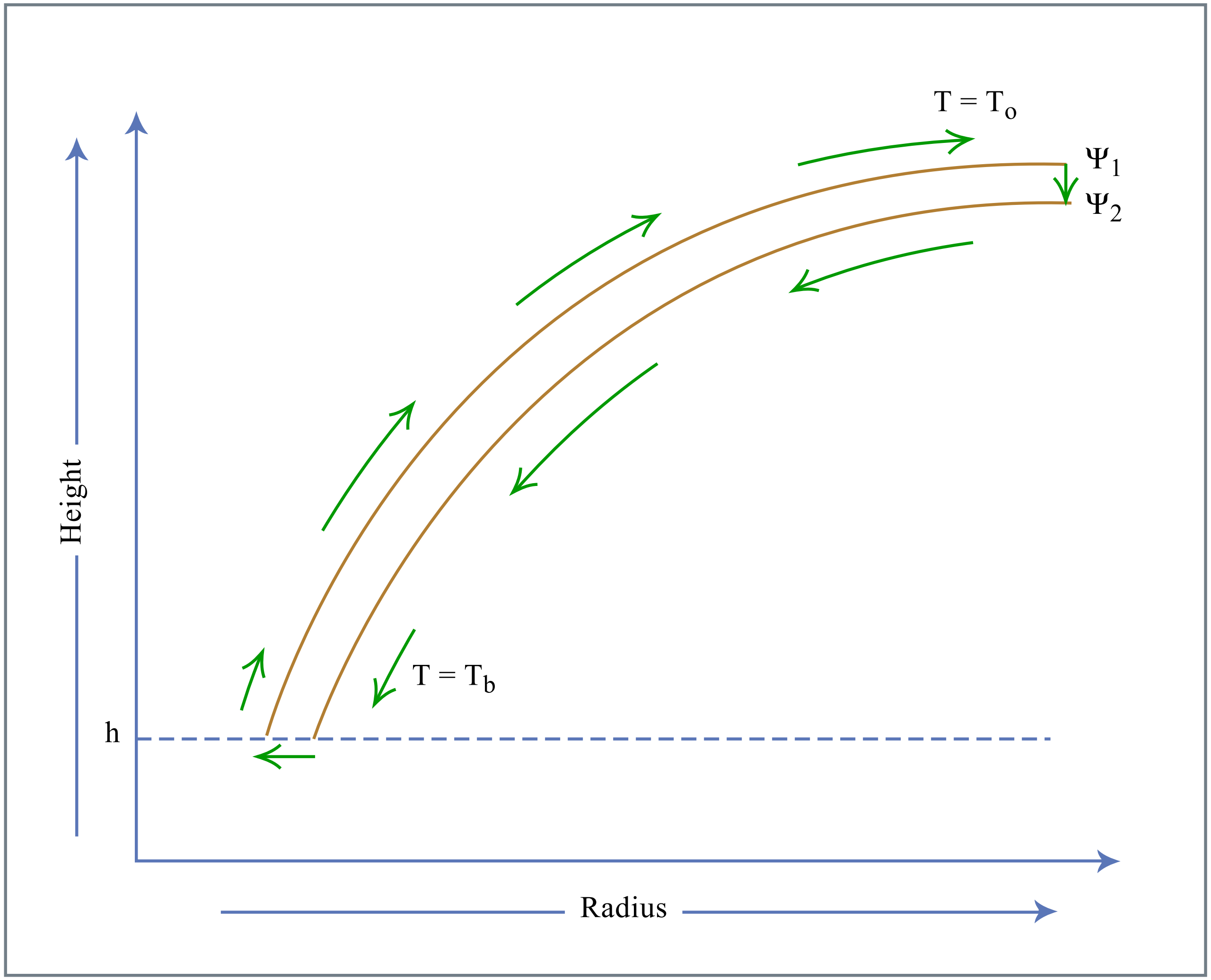
$$\rightarrow \quad |V_{\max}|^2 \cong \frac{C_k}{C_D} \frac{T_s - T_o}{T_o}\left(k_0{}^* - k\right)$$

Not a closed expression since k at radius of maximum winds is unknown

Problems with Energy Bound:

- Implicit assumption that all irreversible entropy production is by dissipation of kinetic energy. But outside of eyewall, cumuli moisten environment....accounting for almost all entropy production there
- Approximation of integrals dominated by high wind region is crude

Local energy balance in eyewall region:



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Definition of streamfunction, ψ:

$$\rho r w = \frac{\partial \psi}{\partial r}, \qquad \rho r u = -\frac{\partial \psi}{\partial z}$$

Flow parallel to surfaces of constant ψ, satisfies mass continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(\rho r u) + \frac{1}{r}\frac{\partial}{\partial z}(\rho r w) = 0$$

Variables conserved (or else constant along streamlines) above PBL, where flow is considered reversible, adiabatic and axisymmetric:

Energy: $$E = c_p T + L_v q + gz + \frac{1}{2}|\mathbf{V}|^2$$

Entropy: $$s^* = c_p \ln T - R_d \ln p + \frac{L_v q^*}{T}$$

Angular Momentum: $$M = rV + \frac{1}{2} f r^2$$

First definition of *s*:*

$$Tds^* = c_p dT + L_v dq^* - \alpha dp \qquad (1)$$

Steady flow:

$$\alpha dp = \alpha \frac{\partial p}{\partial r} dr + \alpha \frac{\partial p}{\partial z} dz$$

Substitute from momentum equations:

$$\alpha dp = -dz\left[g + \mathbf{V} \bullet \nabla w\right] + dr\left[\frac{V^2}{r} + fV - \mathbf{V} \bullet \nabla u\right] \qquad (2)$$

Identity:

$$\left(\mathbf{V}\bullet\nabla w\right)dz+\left(\mathbf{V}\bullet\nabla u\right)dr=\frac{1}{2}d\left(u^2+w^2\right)+\frac{1}{\rho r}\varsigma d\psi, \quad (3)$$

where $\varsigma \equiv \dfrac{\partial u}{\partial z}-\dfrac{\partial w}{\partial r}$ azimuthal vorticity

Substituting (3) into (2) and the result into (1) gives:

$$Tds^* = dE - VdV - \left(\frac{V^2}{r}+fV\right)dr+\frac{1}{\rho r}\varsigma d\psi \quad (4)$$

One more identity:

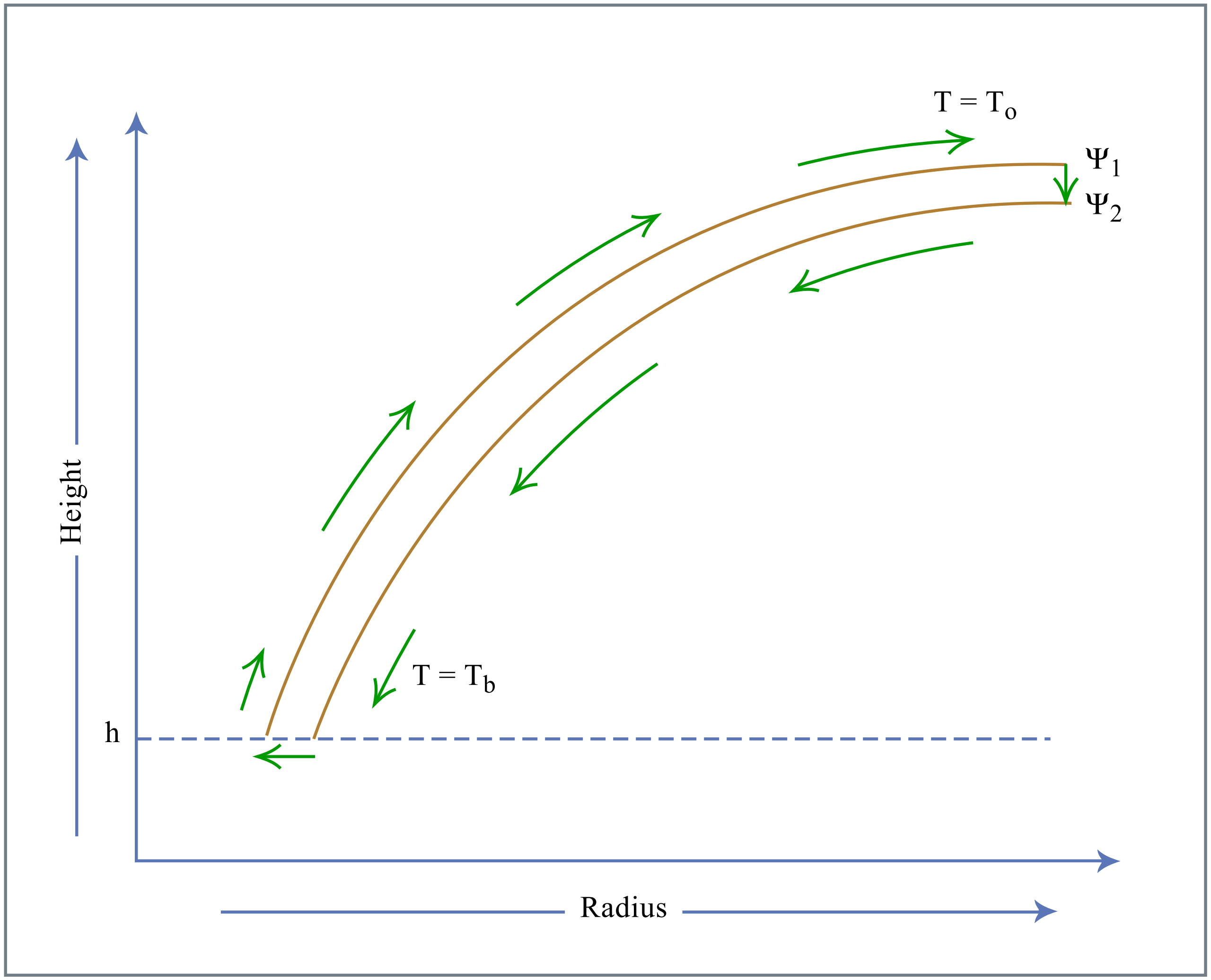
$$VdV + \left(\frac{V^2}{r} + fV \right) dr = \left(\frac{M}{r^2} - \frac{1}{2} f \right) dM$$

Substitute into (4):

$$Tds^* + \frac{M}{r^2} dM - \frac{1}{\rho r} \xi d\psi = d\left[E + \frac{1}{2} fM \right] \qquad (5)$$

Note that third term on left is very small: Ignore

Integrate (5) around closed circuit:



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Right side vanishes; contribution to left only from end points

$$\left(T_b - T_o\right) ds^* + \left(\frac{1}{r_b^2} - \frac{1}{r_o^2}\right) M dM = 0$$

$$\rightarrow \frac{1}{r_b^2} = \frac{1}{r_o^2} - \left(T_b - T_o\right) \frac{ds^*}{M dM} \qquad (6)$$

Mature storm: $r_o >> r_b$:

$$\rightarrow \frac{M}{r_b^2} \cong -\left(T_b - T_o\right) \frac{ds^*}{dM} \qquad (7)$$

In inner core, $V >> fr$

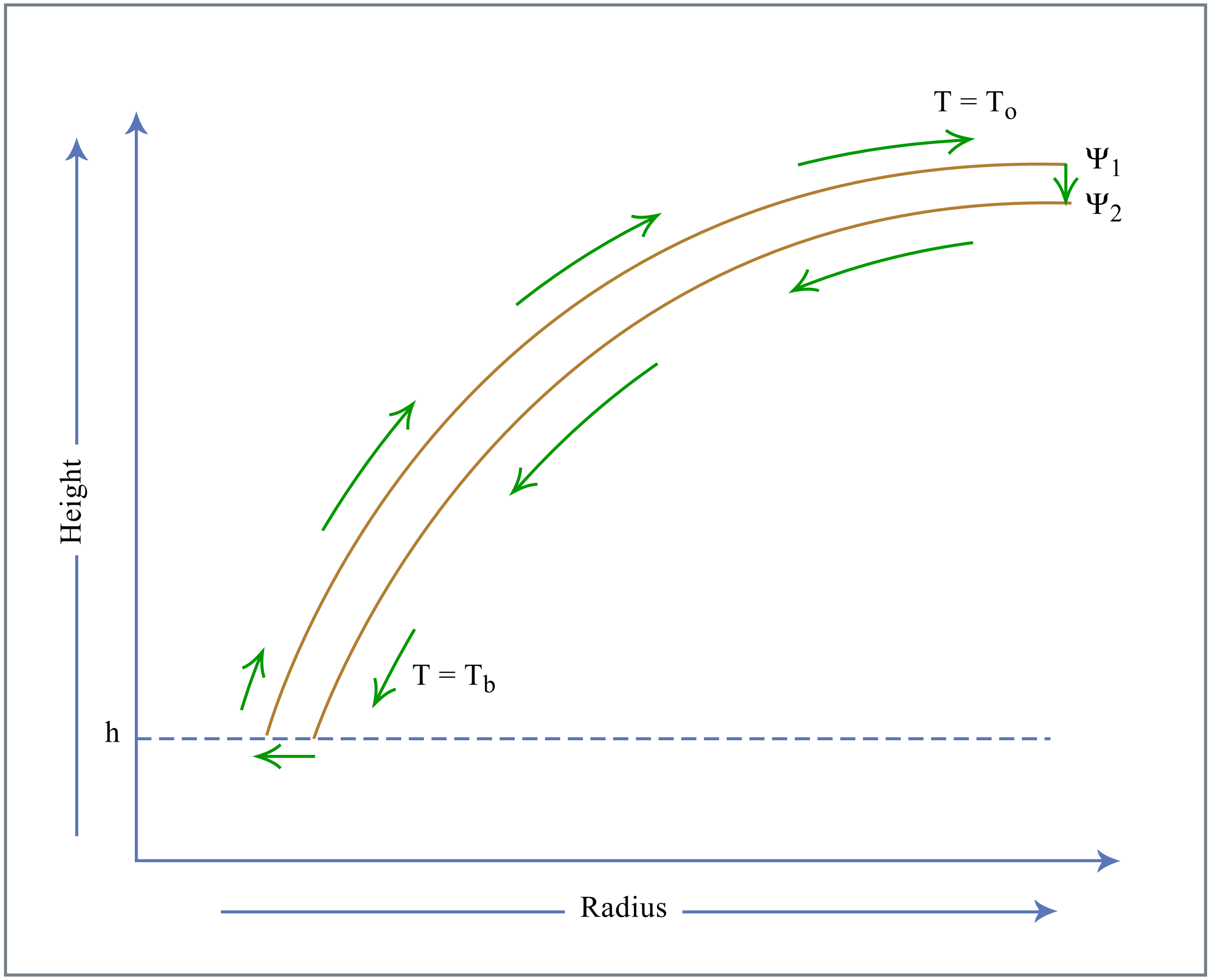
$$\rightarrow M \cong rV$$

$$V_b \cong -r_b \left(T_b - T_o\right) \frac{ds^*}{dM} \qquad (8)$$

Convective criticality: $s^* = s_b$

$$\rightarrow V_b \cong -r_b \left(T_b - T_o\right) \frac{ds_b}{dM} \qquad (9)$$

ds_b/dM determined by boundary layer processes:



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Put (7) in differential form:

$$\left(T_b - T_o\right)\frac{ds}{dt} + \frac{M}{r^2}\frac{dM}{dt} = 0, \qquad (10)$$

Integrate entropy equation through depth of boundary layer:

$$h\frac{d\overline{s}}{dt} = \frac{1}{T_s}\left[C_k\,|\mathbf{V}|\left(k_s{}^* - k\right) + C_D\,|\mathbf{V}|^3 + F_b\right], \qquad (11)$$

where F_b is the enthalpy flux through PBL top. Integrate angular momentum equation through depth of boundary layer:

$$h\frac{d\overline{M}}{dt} = -C_D r\,|\mathbf{V}|\,V \qquad (12)$$

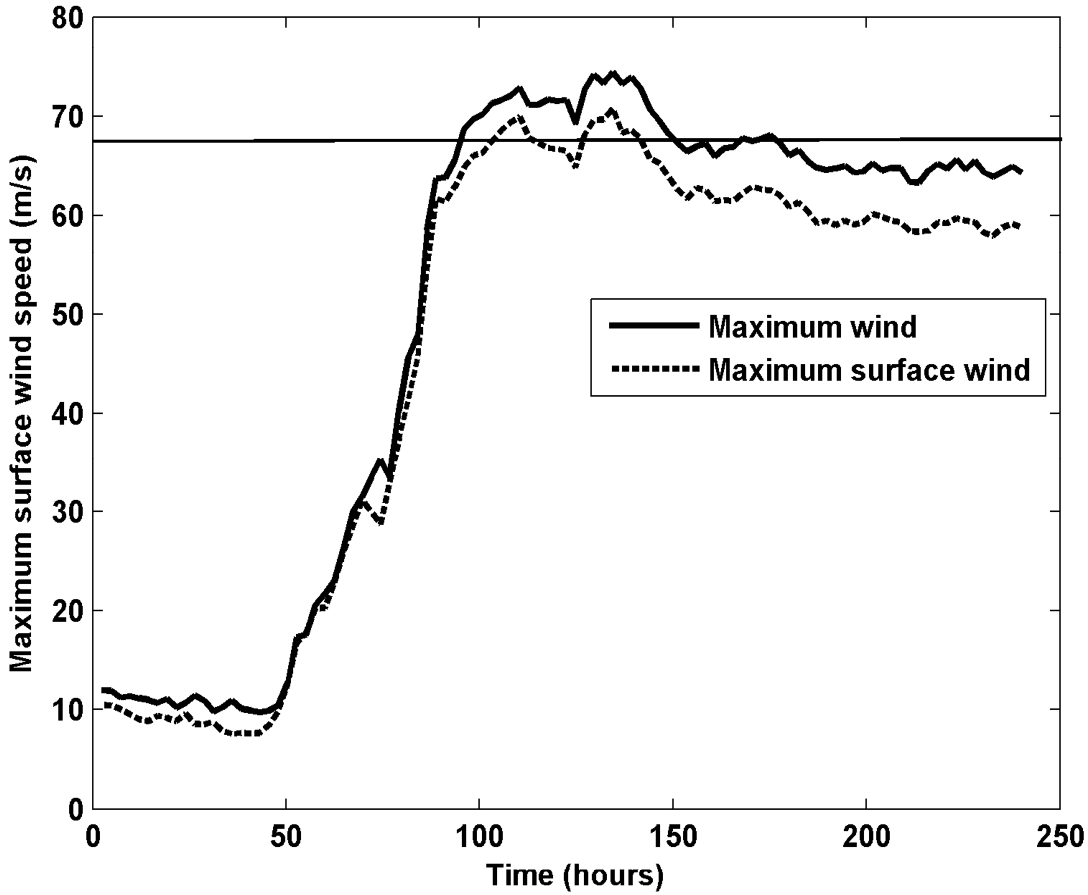
Substitute (11) and (12) into (10) and set F_b to 0:

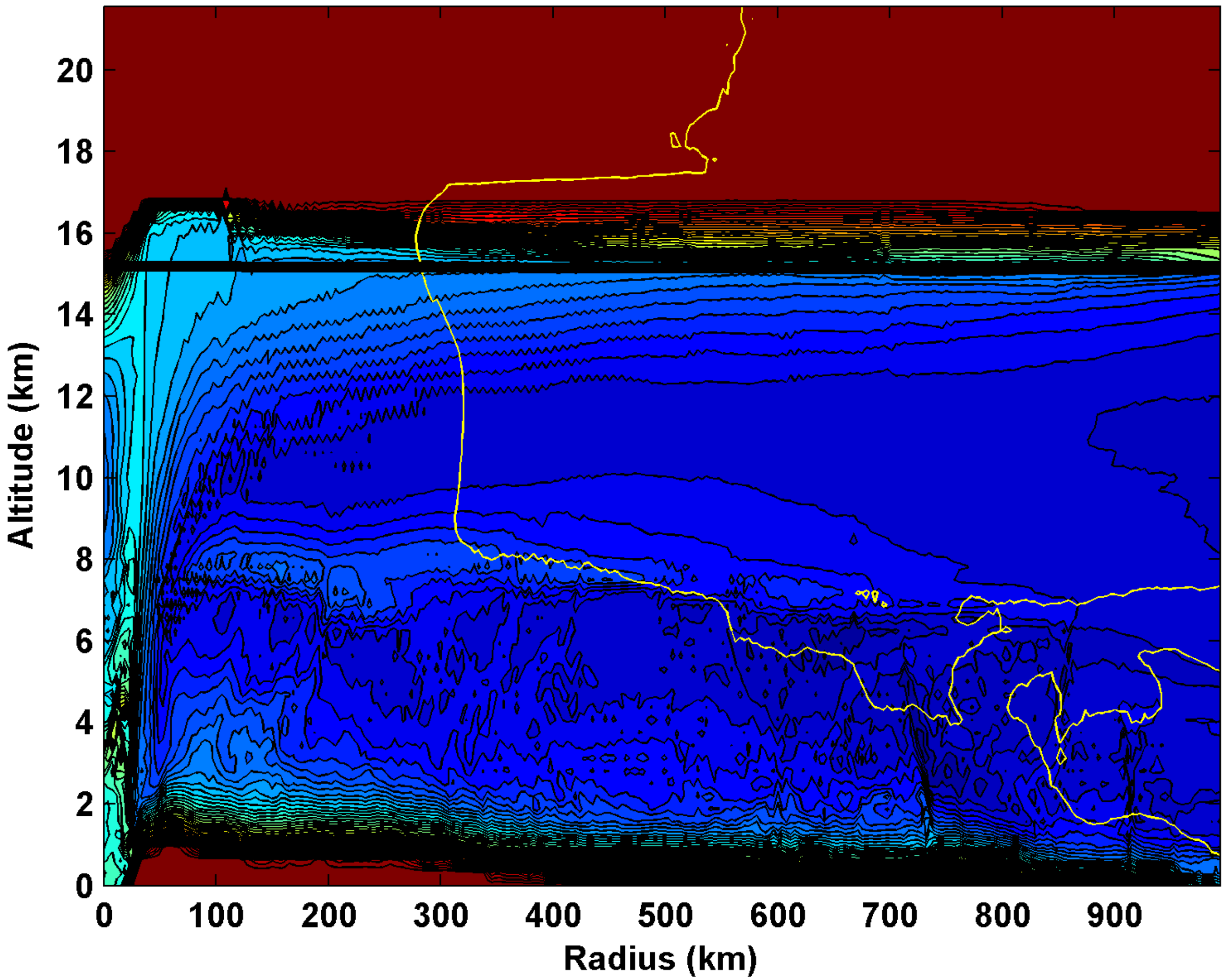
$$\rightarrow \quad |V|^2 = \frac{C_k}{C_D} \frac{T_s - T_o}{T_o} \left(k_0^{\,*} - k \right) \qquad (13)$$

Same answer as from Carnot cycle. This is still not a closed expression, since we have not determined the boundary layer enthalpy, *k*

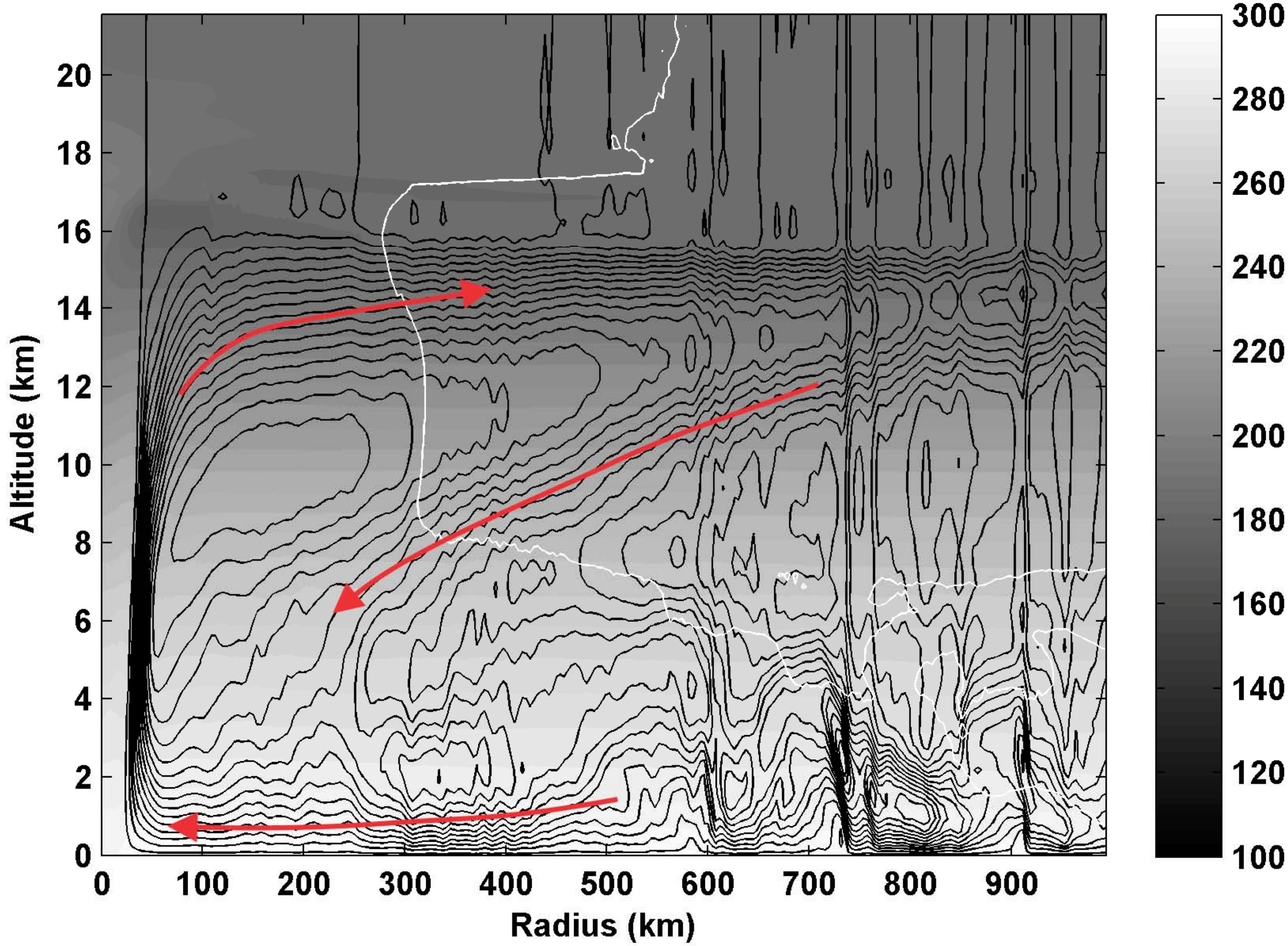
What Determines Outflow Temperature?

Simulations with Cloud-Permitting, Axisymmetric Model

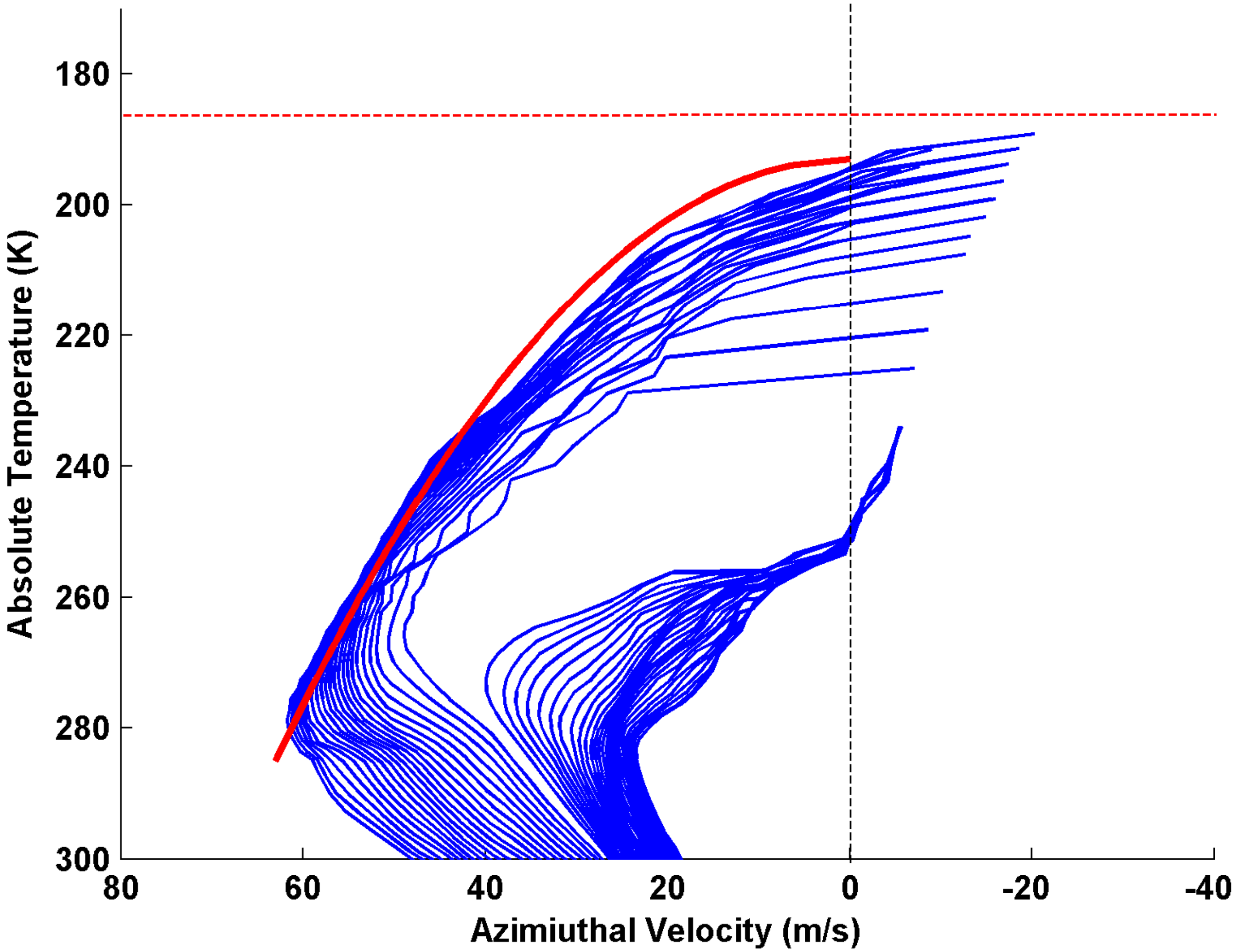




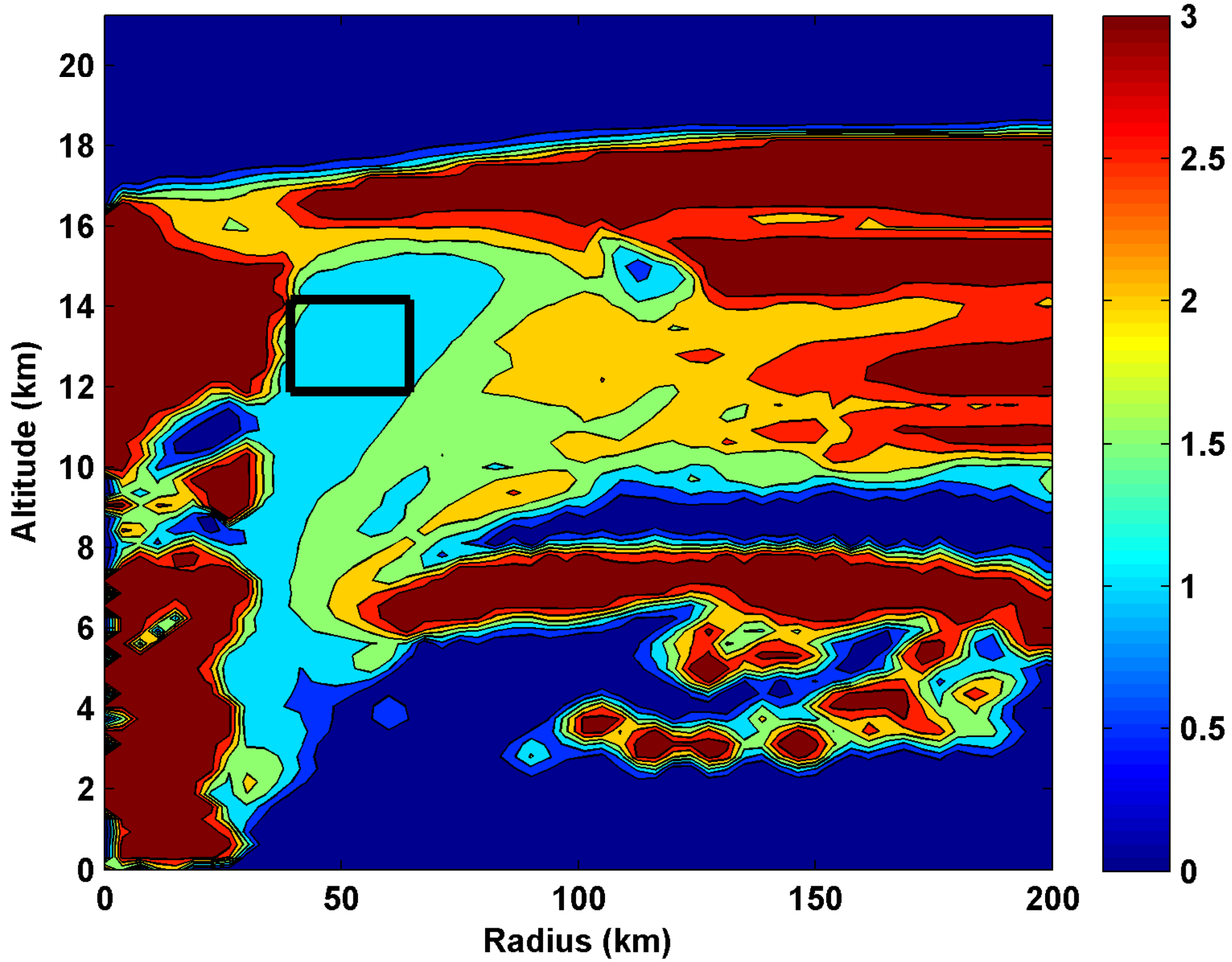
Saturation entropy (contoured) and V=0 line (yellow)



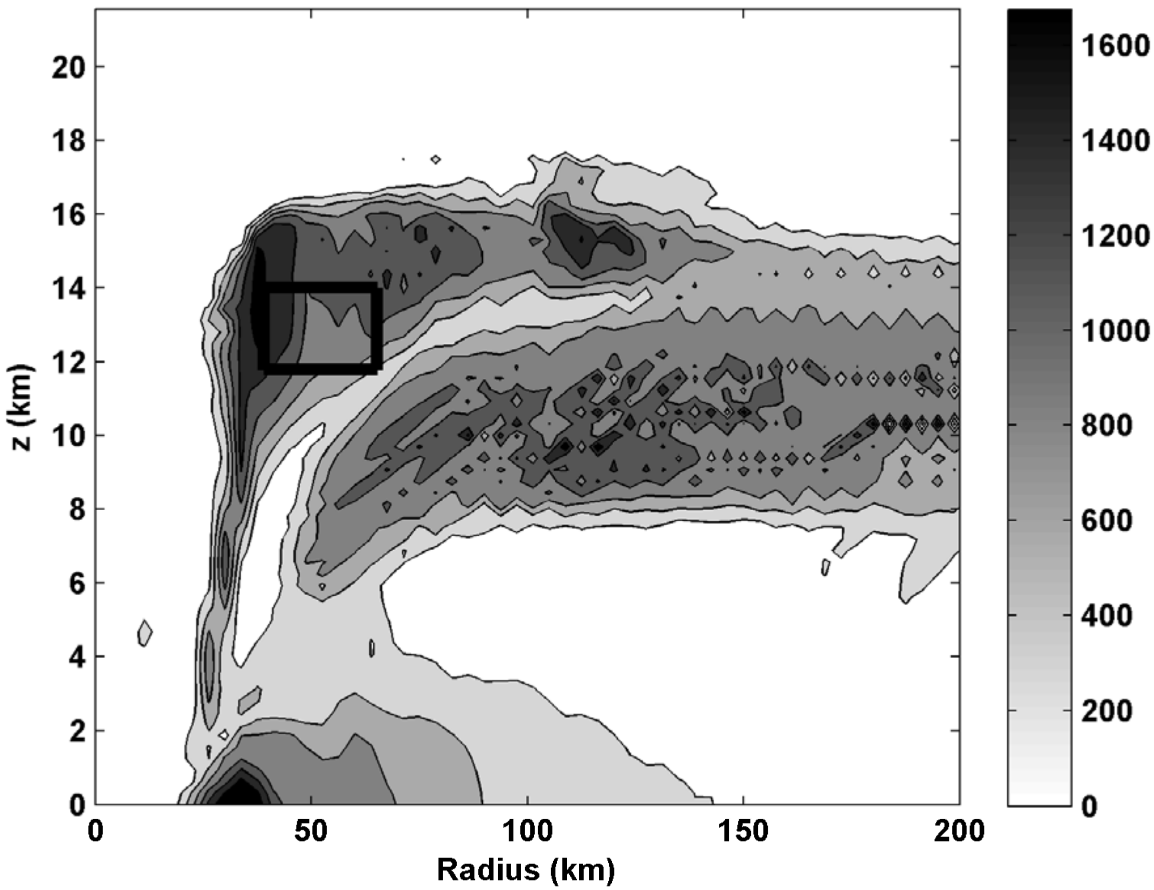
Streamfunction (black contours), absolute temperature (shading) and V=0 contour(white)



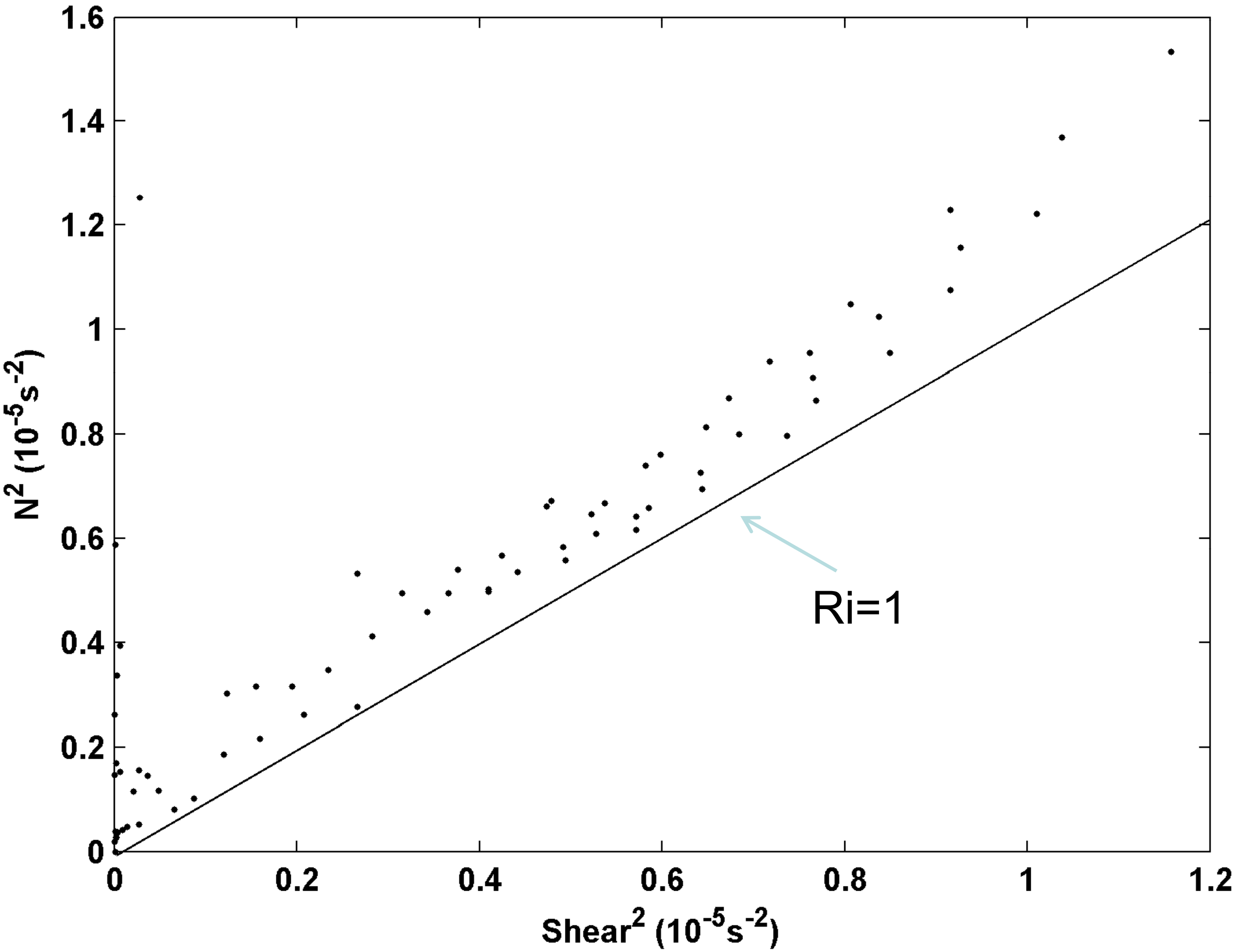
Angular momentum surfaces plotted in the V-T plane. Red curve shows shape of balanced M surface originating at radius of maximum winds. Dashed red line is ambient tropopause temperature.



Richardson Number (capped at 3). Box shows area used for scatter plot.



Vertical Diffusivity (m^2s^{-1})



1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
N^2 ($10^{-5}s^{-2}$)
Ri=1
0
0.2
0.4
0.6
0.8
1
1.2
$Shear^2$ ($10^{-5}s^{-2}$)

Implications for Outflow Temperature

$$Ri \cong \frac{r^2 \Gamma_m \dfrac{ds^*}{dM}}{\dfrac{\partial M}{\partial z}}.$$

$$\rightarrow \frac{\partial M}{\partial z} \cong \frac{r_t^2 \Gamma_m \dfrac{ds^*}{dM}}{Ri_c}.$$

$$\frac{\partial s^*}{\partial z} = \frac{ds^*}{dM} \frac{\partial M}{\partial z},$$

$$\rightarrow \frac{\partial s^*}{\partial z} \cong \frac{r_t^2 \Gamma_m \left(\frac{ds^*}{dM} \right)^2}{Ri_c}. \quad (14)$$

But the vertical gradient of saturation entropy is related to the vertical gradient of temperature:

$$\frac{\partial T}{\partial s^*} = \left(\frac{\partial T}{\partial s^*} \right)_p + \frac{\left(\frac{\partial T}{\partial p} \right)_{s^*}}{\frac{\partial s^*}{\partial p}}, \quad (15)$$

Use definition of s* and C.-C.:

$$\left(\frac{\partial T}{\partial s^*} \right)_p = \frac{\frac{T}{c_p}}{\left[1 + \frac{L_v^2 q^*}{R_v c_p T^2} \right]}, \quad (16)$$

Substitute (16) into (15) and use hydrostatic equation:

$$\frac{\partial T}{\partial s^*} = \frac{\dfrac{T}{c_p}}{\left[1+\dfrac{L_v^2 q^*}{R_v c_p T^2}\right]} - \frac{\Gamma_m}{\dfrac{\partial s^*}{\partial z}}. \qquad (17)$$

If $V_b^2 << c_p \dfrac{(T_b - T_o)^2}{T_b}\left(1+\dfrac{L_v^2 q^*}{R_v c_p T^2}\right)\left[Ri_c \dfrac{r_b^2}{r_t^2}\right]$ we can neglect first term on left of (17)

Substitute (14) into (17):

$$\frac{\partial T_o}{\partial s^*} \cong -\frac{Ri_c}{r_t^2}\left(\frac{dM}{ds^*}\right)^2. \qquad (18)$$

Gives dependence of Outflow T on s*

Using $$\frac{\partial T}{\partial s*} = \frac{\partial T}{\partial M}\frac{dM}{ds*},$$

We can re-write (18) as $$\frac{\partial T_o}{\partial M} \cong -\frac{Ri_c}{r_t^2}\left(\frac{dM}{ds*}\right). \qquad (19)$$

We can also re-write (6) as $$M_b = r_b^2\left(\frac{1}{2}f - \left(T_b - T_o\right)\frac{ds*}{dM}\right) \qquad (20)$$

Boundary layer entropy $$h\frac{ds_b}{dt} = C_k\,|\mathbf{V}|\left(s_0^* - s_b\right) + C_d\,\frac{|\mathbf{V}|^3}{T_b} \qquad (21)$$

Boundary layer angular momentum $$h\frac{dM}{dt} = -r\,|\mathbf{V}|V \qquad (22)$$

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