Introduction to “waves” and time dependent motion (12.808)

One aspect of observational physical oceanography not treated in the course thus far is time-dependent motion. This can be considered to be of two types: *forced* [such as the annual cycle of mixed layer deepening/shoaling, and tidal forcing] and *free* [natural modes of oceanic response]. Since how the ocean responds is not unrelated to how it wants to respond, we will consider the natural modes first. Natural time-dependent modes most obvious are waves. Many types of these are familiar to everyone: the ocean surface rises and falls due to natural fluctuations that are not locally generated, but arise from distant forcing. Long swell is a good example. When the “surf is up” is when waves generated by distant storms arrive at local beaches. They have traveled thousands of km. from their point of origin. Other types of surface waves are a tsunami, a disturbance caused by sudden shifts in the earth’s bathymetry [e.g. earthquakes], or a storm surge, caused by rapid, large-scale shifts in surface pressure [hurricanes approaching the coast]. We will begin with these long, low frequency waves. An essential characteristic of these motions is that they are ***hydrostatic***: vertical acceleration in these waves are a small fraction of gravity. While limiting, there is still a rich pantheon of wave motion contained within this class of wave. It includes the following:

- Long gravity waves
- Mixed gravity/inertial waves
- Kelvin & edge waves
- Vorticity waves [i.e. Rossby waves]

The latter contains waves that exist over flat bottoms and those which rely on variations in bottom topography (topographic Rossby waves). Under each of the above categories, there are waves that have their maximum vertical displacement at the ocean surface & are insensitive to the ocean stratification [surface waves] and those which have their maxima in vertical motion within the water column with little vertical motion at the free surface [internal waves]. We will try to treat both of these. We will see that some waves exist in the absence of “rotation”, others depend upon rotation, and yet others depend on variation of the rotation rate with latitude. We will make this introduction as simple as possible by considering most of the mathematics for a homogeneous density ocean [no stratification], but will show that each of the internal wave counterparts has exactly the same dynamics but with a apparent gravity that is much reduced. It is this *reduced* *gravity* approximation that will allow us to skip lots of math already covered by the surface wave discussion.

Long [hydrostatic] surface waves

The equations of motion we have derived for a shallow layer of constant density are, ignoring external forces and friction

$$\frac{du}{dt} - fv = -g\frac{\partial h}{\partial x},$$
$$\frac{dv}{dt} + fu = -g\frac{\partial h}{\partial y},$$
$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}[u(H+h)] + \frac{\partial}{\partial y}[v(H+h)] = 0, where$$
$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}$$

We will be linearizing these (valid for small perturbations) about a state of no motion over a flat bottom (or nearly flat). This will simplify the above to the following:

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial h}{\partial x},$$
$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial h}{\partial y},$$
$$\frac{\partial h}{\partial t} + H[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}] = 0.$$

We will also be making use of the potential vorticity equation in a linearized form, so let's write that down as well (it can be derived from the above three).

$$\frac{d}{dt}[\frac{f+\zeta^z}{H+h}] = 0 \approx \frac{1}{H}[\frac{\partial \zeta^z}{\partial t} - \frac{f}{H}\frac{\partial h}{\partial t} + \beta v]$$

We will ***ignore*** the variation of the Coriolis parameter at this time compared to other, larger terms, making this equation reduce to a simple relation between surface elevation and vorticity for a fluid where potential vorticity is constant:

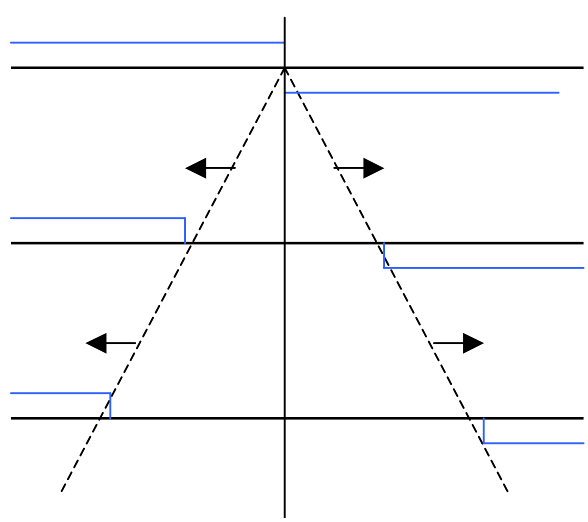
$$\zeta^z = \frac{f}{H}h + \zeta_0^{\ z} = \frac{f}{H}[h + h_0], where$$
$$h_0 = h_0(x, y)$$

Gill makes the point in his book that historically, the shallow water wave equations in a rotating system were "solved" many decades before Rossby noticed that a non-zero constant representing some initial vorticity or surface elevation was an important factor. We will see what this does in what follows. Next, take the x-derivative of the u-momentum equation and the y-derivative of the v-momentum equation to get

$$-c^2\nabla_H^2 h + \frac{\partial^2 h}{\partial t^2} + f^2 h = -fH\zeta_0^{\ z} = -f^2 h_0(x, y),\ or$$
$$-c^2\nabla_H^2 h + \frac{\partial^2 h}{\partial t^2} = 0,\ for\ f = 0,\ where$$
$$c \equiv [gH]^{1/2}$$

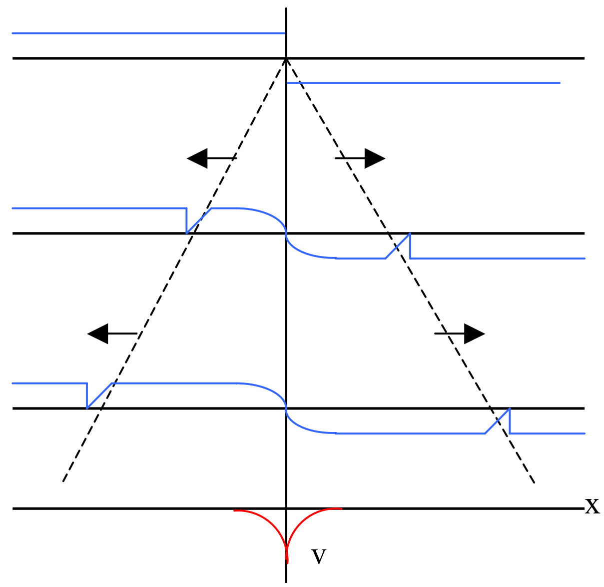
We have made use of the continuity equation in deriving the above "wave" equation for the surface elevation. The quantity, c, has units of velocity and, for the case of no rotation, is the phase speed at which an initial disturbance will propagate.

Consider the ***non-rotating*** case at the right. The ocean surface to the left of the origin (taken to be the x-axis) is initially high and the surface to the right of the origin is low. As time develops, the interface will flatten as fluid moves from left to right and the regions of high and low water will become separated as each side propagates as a wave to the right and left (at speed c).



Eventually, there will be a flat interface in the field of view but the fluid will still be moving from left to right as the "far field" adjusts to the initial disturbance. This is an example of waves in a non-rotating system.

The case for a ***rotating system*** is very different, as illustrated in the next diagram (at right). Here we have the same initial condition, but as the interface adjusts, a steady, geostrophic flow develops at the initial discontinuity. A "wave" signal propagates away as before (with a slightly different phase speed as we will see shortly). But behind it there is no u-component of velocity as in the previous case and the surface does not become flat. Instead, a v-component of flow (out of the page for the northern hemisphere) is left behind. This steady flow is a solution to the steady part of the "wave" equation for a rotating system, which must satisfy the following:



$$-c^2\nabla_H^2 h + f^2 h = -fH\zeta_0^{\,z} = -f^2 h_0(x,y),\ or$$

$$if\ \ h_0(x,y) = -h_0\,\mathrm{sgn}(x),\ then$$

$$h = h_0[-1+e^{-x/a}],\ \ for\ \ x>0$$

$$h = h_0[1-e^{x/a}],\ \ for\ \ x<0$$

$$u = 0$$

$$v = -(gh_0/|f|a)e^{-|x|/a},\ where$$

$$a \equiv \sqrt{gH}/|f| = c/|f|$$

The quantity a , has units of distance and is called the Rossby Radius of Deformation, after Carl Rossby who first solved the problem of "geostrophic adjustment". In order to put some real numbers into the above, consider an ocean of depth 4000m. In this case the phase speed c = sqrt(gH) = 200 m/s, and with a value of $f = 10^{-4}\ s^{-1}$, we get a = 2000 km. The waves that spread out from the origin have a permanent shape in the non-rotating case: they do not change shape. This is because all wavelengths move at the same speed.

This condition is call "***non-dispersive***". With rotation, this changes. To better see this, we need to consider how individual waves of a given frequency and scale propagate. It is customary to consider a sinusoidal wave as a test case, recognizing that we can always sum up many individual waves to solve a given problem. Consider a wave of the following structure:

$$u, v, h \approx (\hat{u}, \hat{v}, \hat{h})e^{i(kx+ly-\sigma t)} = (\hat{u}, \hat{v}, \hat{h})[\cos(kx+ly-\sigma t) + i\sin(kx+ly-\sigma t)]$$

The quantity σ is the frequency of the wave and the x-wavenumber $k = 2\pi/\lambda_x$ is inversely related to the wavelength in the x-direction. If, for example, waves are only propagating in the x-direction, as in our examples above, then the y-wavenumber, l, would be zero (an infinite wavelength in the y-direction: or constant in y). If we insert this test wave into the equations, we get the relationships of the wave variables (shown above with carats) to one another. If we substitute this form into the wave equation (ignoring the constant term depending on $h_0(x,y)$ as we are now talking above the waves) we get the following:

$$\sigma^2 = f^2 + c^2\kappa_H^2, \; where$$
$$\kappa_H \equiv (k, l), \& \; \kappa_H^2 = (k^2 + l^2)$$

The quantity κ_H is the horizontal wavenumber in the direction of wave propagation. This equation relating frequency to wavenumber is called a dispersion relation and is important for the study of wavemotion. We see that the *lowest* frequency of surface gravity waves is the *inertial* frequency and that inertial waves are a special case in which the horizontal wavenumber is zero (infinite wavelength). We can also see that for waves with frequencies that are much larger than f,

$$\sigma^2 \approx c^2\kappa_H^2, \; or$$
$$\sigma \approx \pm c\kappa_H, \; for \; \sigma^2 >> f^2$$

We define the **wave phase speed as the ratio of the frequency to wavenumber** and we see that for this limit, the wave speed is identical to c. In this limit, all wavelengths travel at the same speed, which we have estimated above to be 200 m/s for water of 4000m depth. This is pretty fast: a wave could cross an ocean of width 4000 km (like the N. Atlantic) in a little more than 3 hours! This is why tsunami warnings are important to get out quickly: if an earthquake near the Aleutian Island arc causes a rise in sea level locally, this will arrive at the Hawaiin Islands in about 2 hours. Of course, once the wave gets into shallow water, it will start to slow down since the phase speed depends on the square root of the local depth, but at this point another important thing happens: the wave amplifies. In order to see this, we need to look at the energetics of these surface waves.

If we multiply the *x*-momentum equation by ρHu and the y-momentum equation by ρHv and add them together, the Coriolis terms drop out and we get an equation for the kinetic energy of the waves:

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho H(u^2+v^2)\right]=-\rho g\left[Hu\frac{\partial h}{\partial x}+Hv\frac{\partial h}{\partial y}\right]$$

The quantity in brackets on the left hand side is the kinetic energy of the wave motion. The potential energy of the wave motion is obtained by multiplying the continuity equation by ρgh to give

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho gh^2\right]=-\rho g\left[h\frac{\partial Hu}{\partial x}+h\frac{\partial Hv}{\partial y}\right]$$

If we add these two equations, we get the total energy of the wave motion:

$$\frac{\partial E}{\partial t}+\frac{\partial F^x}{\partial x}+\frac{\partial F^y}{\partial y}=0,\ where$$
$$E\equiv\left[\frac{1}{2}\rho H(u^2+v^2)\right]+\left[\frac{1}{2}\rho gh^2\right],\ and$$
$$\vec{F}=\rho gH\vec{u}h$$

If we imagine that we "average" the wave over one wavelength, we get an average value for the total energy and energy flux. We will denote this average by <…>. Now to simplify things, suppose the wave is propagating in the negative *y*-direction (southward from the Aleutians towards Hawaii) and its frequency is much higher than the inertial frequency, the relationship between the vertical motion at the surface (h) and the horizontal velocity, (u,v), is readily given by the above equations:

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial h}{\partial x} \rightarrow -i\sigma\hat{u} - f\hat{v} = -gk\hat{h} = 0\ (k=0)$$
$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial h}{\partial y} \rightarrow -i\sigma\hat{v} + f\hat{u} = -gl\hat{h}$$
$$\frac{\partial h}{\partial t} + H[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}] = 0 \rightarrow -i\sigma\hat{h} + il\hat{v}H = 0$$

Note that even though the *x-wavenumber* is zero, the *x-velocity* is not because of rotation. The energy flux terms are the important ones to consider here. Waves are weakly dissipated (by bottom friction) and energy, averaged over a wavelength is conserved. This means that the energy flux is constant in the *y*-direction (recall that the wave is propagating in this direction). This flux, F^y, averaged over a wavelength can be written using the above relations as:

$$< F^y > = < \rho gHvh > = < \rho gh^2(\sigma / l) > \approx \rho gc < h^2 >,\ for\ (\sigma / f)^2 >> 1$$

So as the wave slows down as it gets into shallow water (since $c^2=gH$) it must have a constant energy flux and thus its amplitude goes up. The root mean square (rms) amplitude goes as the ¼ *th* power of the depth according to

$$h_{rms} \equiv \sqrt{< h^2 >} \propto (\rho gc)^{-1/2} \propto H^{-1/4}$$

Imagine a 1 meter amplitude wave at the site of generation in 4000m of water. When this wave gets into shallow water (e.g. H= 4m), its rms amplitude is 5.6m. And the peak amplitude may be a few times larger than the rms. That is why tsunami warnings send residents to higher ground. It is also why storm surges, with amplitudes of 50 cm in the deep ocean (due to a low pressure in a hurricane of about 50 mbar) can become significant at the shoreline. When ocean swell propagates into shallow water, it can be studied within the approximation of "shallow water" which we have made. Thus, this amplitude amplification above also applies to ocean swell and explains wave steepening near the shore; it cannot explain breaking, which must include non-linearity and dissipation.

In shallow water, of depth 100m, the phase speed, c, takes on values of 30 m/s. For waves of semi-diurnal tidal period ($\tau \approx 12$ hrs.) the wavelength, $\lambda = c\tau$, in the direction of propagation is approximately 1300 km. A quarter wavelength is approximately 300 km. A large bay, like the Bay of Fundy, has a length and depth such that a quarter wavelength of a free wave can "fit" in it. In this case, forcing at the opening of the bay can be amplified by what is called a "*resonance*", much like an organ pipe can be forced to make an acoustic tone of wavelength 4 times the length of the pipe. In this case, *weak* forcing at the opening can cause a *large* response. Tides in the Bay of Fundy reach maximum amplitudes in the head of the bay of 16 meters and are among the largest in the ocean. Strong currents flowing into the Bay form hydraulic bores which are impressive to see.

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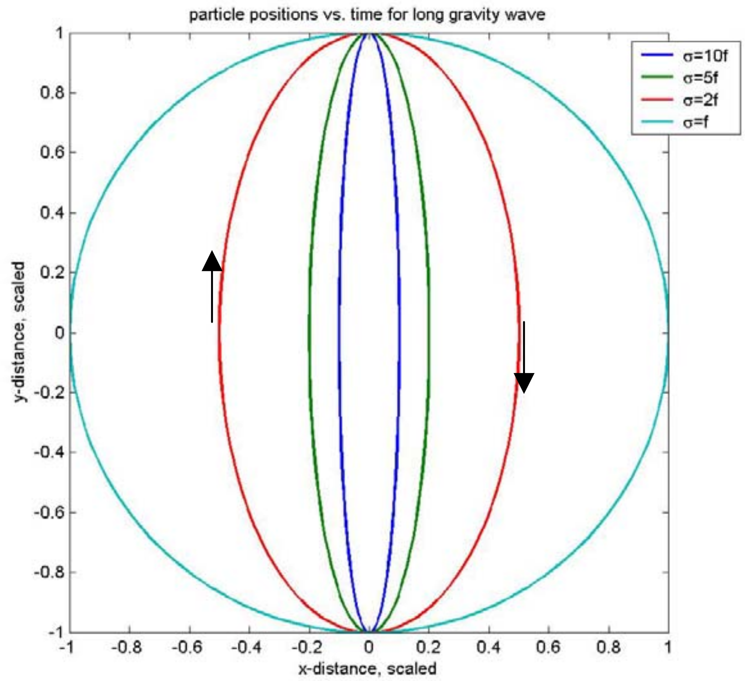
Returning now to the case of a long surface wave propagating to the south (*x-wavenumber, k*=0), we can see from the above that the ratio of the velocity in the x-direction to that in the y-direction is proportional to (f/σ), and that the two are out of phase by 90^o. If we integrate the velocity to track what a particle of fluid will experience during one cycle of the wave, we get the plot at the right. Particles will undergo an elliptical 'orbit' with the major axis aligned with the direction of wave propagation. For high frequencies (compared to the inertial frequency) the ellipse will be narrow, with most of the displacements in north/south direction in which the wave is propagating. As the frequency approaches the inertial frequency, the orbits become more circular, until at the inertial frequency, the orbits are a perfect circle, much like our old "pucks on ice" model. Whether the wave is propagating from north to south or south to north, the direction of rotation is always clockwise for the northern hemisphere. We see for this simple example that the horizontal velocity is never zero due to rotation. At very high frequencies, the motion becomes two dimensional in the direction of propagation, with horizontal and vertical velocities only in this plane. We cannot tell which direction the wave is propagating from a plot of the two horizontal velocities. To do this, we need to look at the phase of the horizontal velocity compared to the vertical displacement of the free surface. The relation between the frequency and the wavenumber of waves is called the dispersion relation. For long gravity waves, this is simply the following:



$$\sigma^2 = f^2 + \kappa^2 c^2, \ or$$
$$\sigma^2 = f^2(1+\kappa^2 a^2), \ where$$
$$\kappa^2 \equiv (k^2 + l^2), \ and$$
$$a^2 \equiv c^2 / f^2$$

recall that a is the Rossby radius. Thus, it is for wavelengths that are short (wavenumbers that are large) compared to the Rossby radius that rotation becomes unimportant. The minimum frequency is the inertial frequency, but there is a class of gravity wave for which there is no lower frequency limit: it is a Kelvin Wave, which we will examine now.

Kelvin Waves

In the case that waves are propagating next to a solid boundary, it is clear from the above that even if the boundary is parallel to the direction of wave motion, there will be a velocity into the boundary. If this boundary is the western boundary of an ocean basin, say at $x = 0$ in a rectangular ocean, we can get a class of waves that satisfy the condition that there be no flow into the boundary: just set u to be zero *everywhere* in the wave equations.

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial h}{\partial x} \rightarrow -fv = -g\frac{\partial h}{\partial x},$$
$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial h}{\partial y} \rightarrow \frac{\partial v}{\partial t} = -g\frac{\partial h}{\partial y},$$
$$\frac{\partial h}{\partial t} + H[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}] = 0 \rightarrow \frac{\partial h}{\partial t} + H\frac{\partial v}{\partial y} = 0.$$

This wave propagates in the y-direction with a phase speed c as before, but has no velocity in the east/west direction.

From the above equations one can immediately see that the dynamics in the x - direction (normal to the ‘boundary at $x = 0$) are geostrophic, with slopes in the free surface balanced by pressure gradients. As before, we can eliminate variables to get a single wave equation for the free surface, h , which we can write as follows:

$$(h,v) = (h(x), v(x))e^{i(ly-\sigma t)},$$

$$c^2 \frac{\partial^2 h}{\partial y^2} - \frac{\partial^2 h}{\partial t^2} = 0, \; giving$$

$$\sigma^2 = c^2 l^2, \; where$$

$$c^2 = gH$$

Because the dispersion relation is quadratic, there are two possible roots: one with a wave propagating to the north (*c > 0*) and one with the wave propagating to the south (*c < 0*). However, we will see that only one of these two roots is possible. The potential vorticity equation, as before reduces to

$$\frac{d}{dt}[\frac{f+\zeta^z}{H+h}] = 0 \approx \frac{1}{H}[\frac{\partial \zeta^z}{\partial t} - \frac{f}{H}\frac{\partial h}{\partial t}], \; or$$

$$\frac{\partial \zeta^z}{\partial t} = \frac{\partial}{\partial t}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = \frac{\partial}{\partial t}\frac{\partial v}{\partial x} = \frac{f}{H}\frac{\partial h}{\partial t} = -f\frac{\partial v}{\partial y}$$

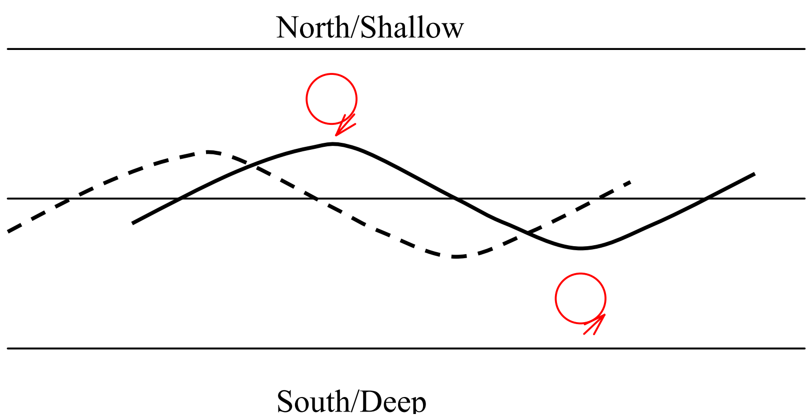
Where we have made use of the fact that $u = 0$, and the continuity equation (third equation at the bottom of the previous page). We can substitute in the above "wave" structure for (h,v) and get a very simple equation for the unknown structure in the x –direction

$$\frac{\partial v}{\partial x} - \frac{f}{c}v = 0.$$

If a wave is propagating to the north ($c > 0$) in the northern hemisphere, the has a solution exponentially grows for positve values of x and exponentially decays for negative values of x . We reject any solution which grows to be infinite as 'unphysical' leaving only one choice for the direction of propagation of a Kelvin wave: with the coastline at its right (in the northern hemisphere). Imagine a square basin in which a Kelvin wave is propagating around the boundary. In each hemisphere it can propagate in only one direction: with the coast on its right (left) in the northern (southern) hemisphere. This particlar type of wave can exist at frequencies below the local inertial frequency and is a special sort of gravity wave in this regard. The scale at which the wave decays away from the boundary is the Rossby radius of deformation The other class of waves which exist at subinertial frequencies is Rossby waves, which we will consider next for mid-latitudes only, due to limitation on our available time. However, it should be noted that the Kelvin wave can exist at the equator. When $f = \beta y$ (near the equator), this wave can only propagate from west to east along the equator in either hemisphere, but not across it. The Kelvin wave is discussed in the text of *Ocean Circulation* in section 5.3.1, although one can get confused by the discussion because at some points the text is describing *internal* Kelvin waves, not *barotropic* or *surface* Kelvin waves as we have just covered. Internal Kelvin waves will be discussed in a later section. We now turn to Rossby waves, in which vorticity is the dominant restoring 'force', not gravity.

Mid-latitude Rossby Waves

Looking down at the surface of the ocean (see figure) consider a thick, black 'line' of displaced fluid as shown. We have drawn some simple lines of constant planetary vorticity (f/H) which represent variations either due to changes in latitude or to changes in water depth. We are in the northern hemisphere because in the southern hemisphere, shallow water would be equivalent to the "south" and vice versa for deep water. In the absence of external forces or friction, potential vorticity must be conserved, so particles on the line that have been displaced to the north must develop anticyclonic (negative) relative vorticity and those to the south cyclonic (positive) relative vorticity. This is represented by the circles with arrows, showing the flow consistent with this relative vorticity. What will happen to this system? If vorticity conservation acts as a restoring force, it should try to move this structure back towards zero. Looking at the crest on the left, the circulation will try to drag down the fluid line to the right of the crest and drag up the fluid line to the left of the crest. Looking at the trough to the right, fluid motion will try to push up the trough to the right and pull it down to the left. The net result of the pushing and pulling is the dashed curve, which indicates that the initial disturbance has 'moved' to the



west or equivalently, moved with the shallow water on the right. For a southern hemisphere, the wave would still move to the west but it would also move with shallow water on its *left*. The restoring 'force' of potential vorticity conservation thus acts to produce westward motion of a disturbance over water of constant depth. This is called a Rossby wave. In cases in which variations of topography dominate over variations of latitude, it is called a topographic Rossby wave. We will examine how fast this wave propagates and what are its properties for a case of constant depth.

The equations of motion should look familiar as they have been used for each of the types of waves discussed up until now. They are:

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial h}{\partial x},$$

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial h}{\partial y},$$

$$\frac{\partial h}{\partial t} + H[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}] = 0.$$

$$\frac{d}{dt}[\frac{f+\zeta^z}{H+h}] = 0 \approx \frac{1}{H}[\frac{\partial \zeta^z}{\partial t} - \frac{f}{H}\frac{\partial h}{\partial t} + \beta v]$$

We are going to look for waves with frequencies much less than the local inertial frequency, unlike gravity waves. In this case, we will want to neglect the acceleration terms on the momentum equations compared to the coriolis terms. We call this approximation ***quasigeostrophic***. Thus

$$\frac{(\partial u/\partial t)}{fv}, \frac{(\partial v/\partial t)}{fu}, \frac{(\partial u/\partial t)}{g(\partial h/\partial x)} << 1$$

The PV equation (last one) will be used without any approximations. With these assumptions, we can obtain a single equation in a single unknown (a wave-equation), which we will write down for the surface elevation, h .

$$\left[a^2\nabla_H^2 - 1\right]\frac{\partial h}{\partial t} + \beta a^2 \frac{\partial h}{\partial x} = 0, \; where$$
$$a^2 \equiv \frac{c^2}{f^2} = \frac{gH}{f^2}$$

You will notice that unlike the other 'wave equations' so far, this one is only first order in time (only one time derivative). Again, we will look for wavelike solutions and use the above to get a dispersion relation between frequency and wavenumber:

$$u, v, h \approx (\hat{u}, \hat{v}, \hat{h})e^{i(kx+ly-\sigma t)} = (\hat{u}, \hat{v}, \hat{h})[\cos(kx+ly-\sigma t) + i\sin(kx+ly-\sigma t)]$$
$$\sigma = -\frac{\beta a^2 k}{1+\kappa^2 a^2}, \; where \; \kappa^2 = k^2 + l^2.$$

Only westward phase speeds are possible in this solution since $\sigma / k < 0$. These are the Rossby waves pictured in the above diagram. For wavenumbers much smaller than the radius of deformation, a (very long wavelengths), the second term in the above denominator is small and the Rossby waves are non-dispersive: they all have the same westward phase speed independently of wavenumber. In fact, they only propagate to the west with no north/south wavenumber. For wavenumbers much larger than the radius of deformation, of deformation (short waves), phase speed actually decreases with increasing wavenumber. At the radius of deformation, these waves attain their maximum frequency, which we will now estimate. Take (k , l) = (a^{-1}, 0), making $ka = 1$. We will now insert some 'typical' values for the remaining variables to see what we have.

$$\sigma = -\frac{\beta a^2 k}{1+\kappa^2 a^2} \rightarrow -\beta a / 2.$$
$$\beta = 2x10^{-11}(ms)^{-1}; a = 2x10^6 m, \rightarrow$$
$$\sigma = 2x10^{-5}s^{-1} \rightarrow \tau = 2\pi / \sigma = 3.14x10^5 s \approx 3.6 \; days$$

This is not much greater than the local inertial period for mid-latitudes (ca. 1 day), bringing our quasigeostrophic assumption into question, but this frequency is meant to represent the largest frequency possible. What is actually observed in lower in frequency than this upper limit. We will illustrated this with some real data taken during the local dynamics experiment in POLYMODE in the late 70s (J. Price & T. Rossby, JMR, supp. to vol. 40, 1982).

A number of SOFAR floats were released at a depth of 1300m in the sound channel near 31N, 70W in the N. Atlantic. Over a period of 9 weeks about 12 of the 18 floats took part in a textbook example of a Rossby wave oscillation. This is shown in the next figure (below).

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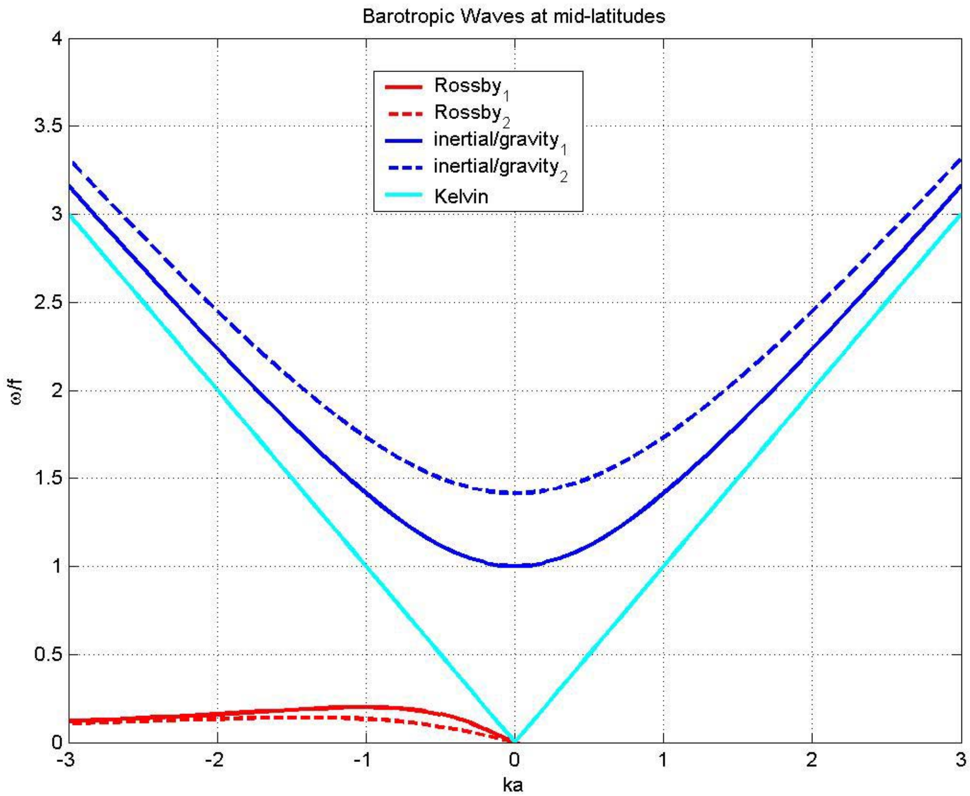
In this case, the authors have accounted for (weak) variations in the bottom bathymetry which give a slight 'tilt' to the lines of constant f / H, which are dashed in the above figure (with percent changes from a reference). As the floats reach their maximum 'northern' excursion, they develop anticyclonic vorticity, which is clearly indicated in upper right 2 panels. At their 'southernmost' extreme, cyclonic vorticity is also clear (see the 20 June panel). The authors estimate a period of about 61 days, a wavelength of ca. 340 km. And a direction of propagation towards 300 degrees true. What is amazing in an extended timeseries is that after the period shown in the figure, there is little evidence for their presence. The interlude of their passage was brief! While the crests of these waves were propagating to the WNW, their energy was actually propagating to the east, away from a likely source in the Gulf Stream. Were there an altimeter, it might have been difficult to track them because of their small scale and the small signal at the ocean surface (several cm. amplitude). But as we shall see later, these waves can be tracked from space. We haven't talked about energy propagation since our discussion of long surface gravity waves, but for Rossby waves, it is fairly complicated and beyond the scope of this course. Simply said, energy propagates with something called the *group velocity* which is defined as follows:

$$\vec{c}_g \equiv \nabla_k \sigma = \left(\frac{\partial \sigma}{\partial k}, \frac{\partial \sigma}{\partial l}\right), \; where$$

$$\sigma = -\frac{\beta a^2 k}{1 + (k^2 + l^2)a^2}$$

We will plot this dispersion diagram below including ***all*** the types of waves discussed thus far. It is conventional to only plot positive frequencies, but allow wavenumbers to have either sign. We have 'constructed' one such diagram based on a waves course (12.802) taught by Dr. Joe Pedlosky. It shows the inertial/gravity (also called Poincaré waves) dispersion diagram together with Kelvin and Rossby waves that are propagating at mid-latitudes: all waves are barotropic. Notice that Rossby waves exist only on one side (negative) of the wavenumber axis for positive frequencies. Also, Kelvin waves must propagate with the coast on their right (in the N. Hemisphere). Thus the Kelvin wave on the right (left) side of the diagram must be propagating to the east (west) with a side boundary on its right.



In the above, we have scaled the horizontal wavenumbers by the radius of deformation, and the frequency by the inertial frequency. We have chosen a couple of wave modes of each type by setting the scaled meridional wavenumber (*la*) to be 0 or 1.