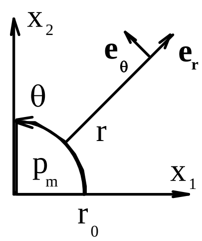
12.520: Problem set 3
Due 10/18/06

1) (35%) As discussed in class for the Spanish Peaks example, the stress perturbation $\boldsymbol{\sigma_m}$ that results from intruding a cylindrical magma body of radius r_0 aligned along the z axis, subjected to an excess magma pressure p_m, is most easily expressed in cylindrical coordinates (r, θ, z) as:

$$\sigma_{rr} = -p_m\left(\frac{r_0}{r}\right)^2$$

$$\sigma_{\theta\theta} = p_m\left(\frac{r_0}{r}\right)^2$$

for $r > r_0$



We will consider the stress tensor in "geographic" coordinates, with the x_1 axis pointing East, the x_2 axis pointing North, and the x_3 axis pointing upward (coincident with the z axis of the cylinder). Although in general the change of coordinates, tensors, etc., going from curvilinear coordinates to Cartesian coordinates is cumbersome, for this specific situation, along certain special directions, this transformation can be made by inspection.

a) Write out the perturbed stress tensor $\boldsymbol{\sigma_m}$ (in x_1 , x_2, x_3 coordinates) for points located on the x_1 axis, i.e.,

$$\sigma_{ij}^m\Big|_{x_2=0} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$$

b) Write out expressions for the stress tensor $\boldsymbol{\sigma_m}$ (in x_1 , x_2, x_3 coordinates) for points located on the x_2 axis.

$$\sigma_{ij}^m\Big|_{x_1=0} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$$

c) Use a Mohr's circle construction to determine the expressions for the stress tensor $\boldsymbol{\sigma_m}$ (in x_1 , x_2, x_3 coordinates) for points located along a line oriented N 45° E. [Hint: sketch the principal stresses along this line and use the Mohr's circle to rotate into Cartesian coordinates.]

$$\sigma_{ij}^m\Big|_{x_1=x_2} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$$

Suppose that, as may have been the case at Spanish Peaks, that a cylindrical magma body intrudes a region that had been under a state of stress that was lithostatic, plus an E-W horizontal "tectonic" compression of magnitude τ.

d) Write the preexisting stress tensor (tectonic + lithostatic).

$$\sigma_{ij}^{lith.+tect.} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$$

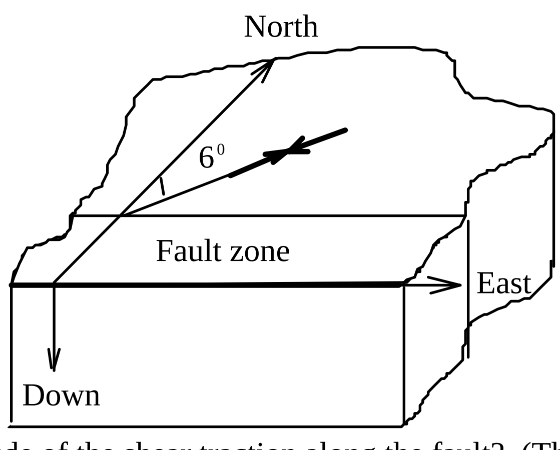
e) Suppose that the new stress field consists of the sum of the preexisting stress field plus that of the intrusion. At some point, the horizontal stress field will become isotropic, i.e.,

$$\sigma_{11} = \sigma_{22}. \text{ and } \sigma_{12} = 0.$$

What is the location of this point and what is the value of σ_{11} there?

f) Suppose that we know (e.g., from the physics of magmas) that the magnitude of the magma pressure p_m for the Spanish Peaks example is 100 bars. Using the map in the lecture notes and your calculation of the location of the isotropic point to *estimate* τ. Explain what you do.

2) (50%) We can learn the basic principles of stress rotation near "weak" fault zones by looking at the implications of the following simple model. Consider a fault zone embedded in a region. Outside the fault zone, the rock is dry, with the mean stress $\sigma_c = (\sigma_1 + \sigma_2)/2$ equal to the lithostatic stress. Here the two horizontal "tectonic" stresses are σ_1 and σ_2, with $\sigma_1 > \sigma_2$. Both the orientation and magnitude of the tectonic stress outside the fault zone are measured, with $\sigma_1 - \sigma_2 = 100$ MPa (constant with depth) and the axis of maximum compression is oriented 6° to the normal to the fault zone, as shown.



a) What is the magnitude of the shear traction along the fault? (The fault runs parallel to the fault zone, with the normal to the fault pointing north.) Does the shear traction vary with depth?

b) At a depth of 10 km, the depth at which earthquakes originate in the region, the lithostatic stress is -300 MPa. What is the magnitude of the normal traction across the boundary between the fault zone and the surrounding region at this depth?

c) Suppose that the fault zone behaves as a Coulomb material, with failure given when

$$\tau = \mu\left|\sigma_n^{eff}\right| = \mu\left|\sigma_n + p_f\right|; \text{ where } \sigma_n^{eff} \equiv (\sigma_n + p_f)$$

with p_f the pore fluid pressure. Assume $\mu = 0.6$ (and no intrinsic strength). The fault begins to rupture (on a plane with its normal pointing north) at a depth of 10 km, under the normal and shear tractions calculated in a) and b). Show on a Mohr circle diagram for the fault zone:

i)   the Coulomb failure criterion;
ii) the state of the "effective" stress;
iii) the orientation of the principal stresses in the fault zone (sketch in both Mohr and physical space);
iv) the state of the actual (not the "effective") stress.

Note: The fault zone is a different region, with a different stress state, different principal stresses, different failure law, etc., than the surrounding region.

d) What value of the pore fluid pressure p_f is required for the fault to break, as in c)?

3) (15%) We are about to become engaged in a theoretical study of sandbox tectonics. Before we begin, it might be a good idea to get some empirical observations to constrain the theory. Fortunately, there is an experiment in sandbox (dirt pile?) tectonics being carried out almost under our noses.

Somewhere near MIT there is a big pile of dirt. (We will choose which pile in class.)

a) How do you think that this pile of dirt was made (by pushing from the side ["bulldozer"] or by dumping from above ["conveyer belt"]. Justify your conclusions.

b) Before making any measurement, estimate the slope of this dirt pile. (The purpose of this question is to give you experience comparing estimates to measurements.)

c) Using a Brunton Compass (available from your TA or any competent geologist), or other suitable technique, measure the slope of the dirt pile.

NOTE: DO NOT TRESPASS ONTO THE DIRT PILE TO DO THIS MEASUREMENT. To do so would risk injury and arrest.