12.510 Introduction to Seismology
Spring 2008

Earthquake

Social Issues

Detailed study of earthquakes gives rise to more information about societal issues (such as seismic hazard or risk), tectonics (in the broad sense, e.g. stress fields, as well as on a smaller scale such as in hydrocarbon reservoirs and geothermal energy plants), and the wave motion generated by the earthquake.

✧ The "beach ball" diagrams of focal mechanisms can give information about the type and orientation of earthquakes such as the tsunami triggering earthquake suffered by Sumatra.

✧ In the San Francisco Bay Area, there are several kilometers of separation between the San Andreas Fault and the Hayward Fault, creating a wide rupture zone.

✧ The East African Rift System creates extensional stress in East Africa as well as Atlantic and South Atlantic spreading. This system also causes motion of the Indian plate. Also note that extension implies that the African plate must grow, so the plate geometries are changing.

Difference between seismic hazard and risk

✧ Seismic hazard: what is the probability of occurrence.

✧ Seismic risk: what damage could it do.

Earthquake Location and Focal Mechanisms

Once seismic information is recorded, two questions must be answered.

1. How do we locate earthquakes?

2. How do we reconstruct focal mechanisms?

A. Tectonic Frame

First, let's get back into the tectonic frame. The equation of motion (Cauchy or Navier) is given by

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \delta_j \sigma_{ij} + f_i \tag{1}$$

where f_i represents the body forces such as gravity and electromagnetism due to coupling between the seismic and electric fields. Setting the body forces to zero leads to

$$\rho \frac{\partial^2 u_i}{\partial t^2} - \delta_j \sigma_{ij} = 0 \tag{2}$$

In theory, we try to find the equivalent body forces that best describe the fault motion and put

them into equation (1). This is formally known as the "Representation Theorem" which is further discussed in, e.g., Ahi and Richards.

B. Green's Function

For a point source at (x',t'), the solution for the equation of motion is given by a ***Green's Function*** $G(x,t)$. The force can be represented as follows

$$f_i(x',t') = A\delta(x-x')\delta(t-t')\delta_{in} \tag{3}$$

where A is the amplitude, $(t-t')$ is time, $(x-x')$ is position, and n is the direction.

Putting this into the equation of motion and solving for u_i (the displacement field resulting from wave motion due to a point source) gives the Green's Function $G(x,x',t,t')$

$$\rho \frac{\partial^2}{\partial t^2} G_{in} = \delta(x-x')\delta(t-t')\delta_{in} + \frac{\partial}{\partial x_j}\left(c_{ijkl} \frac{\partial}{x_l} G_{kn} \right) \tag{4}$$

Note that

$$G_{in} = 0, \quad \frac{\partial G_{in}}{\partial t} = 0 \quad \text{if } x \neq x' \text{ and } t < t' \tag{5}$$

If $t' = 0$, that is

$$f_i(x',0) = B(t)\delta(x-x')\delta(t)\delta_{in} \tag{6}$$

The solution gives the Green's Function $G(x,x',t,0)$. Then we integrate it over time and space

$$u_n(\underline{x},t) = \int_{-\infty}^{\infty} dt' \iiint_{Volume} f_i(x',t')G(x,x',t,0)dV(x') \tag{7}$$
$$\qquad\qquad\qquad\qquad + \text{ term w./surface integral}$$

Thus, a seismogram can be built by integrating over the Green's Functions. For simplicity, we can also denote equation (7) as

$$u(\underline{x},t) \cong \int_{-\infty}^{\infty} f_i(x',t')G_{ij}(\underline{x},t-t',x',0)dt' \tag{8}$$

or

$$u(\underline{x},t) \cong f_i(x',t') G_{ij}(x,t,x',t') \qquad (9)$$

Equation (9) is an important relationship because it summarizes how the source can be used. For example, if we know the source, we can calculate displacement using forward modeling (i.e. calculating a synthetic seismogram). If the interest is the source and not the wave propagation, and we know the Green's Functions, we can use the displacement to get information about the seismic source (i.e. inversion for seismic source parameters). A full inversion involves the inversion of the Green's Functions (the waveforms). For details see Stein and Wysession, Chapter 4.

Now, let's answer the questions of earthquake locations and focal mechanisms using classical seismology. We are looking for the locations of the epicenter (in terms of latitude θ and longitude φ) and the hypocenter m (in terms of θ, φ, time t, and depth h). Location can be found using the travel time difference $T_S - T_P$. We can choose the arrival times of S and P waves and compare them to distances on travel time curves, i.e. take the distance $T_S - T_P$ from the seismogram and find the corresponding Δ value on the plot of the travel time curve. The epicenter is calculated using Δ.

In 1920s, Wadati and Benioff determined that earthquakes occur deeper than previously assumed and discovered inclined seismicity zones "Wadati-Benioff zones".

C. Depth Phases

Earthquake location can also be found using depth phases. A depth phase is a seismic phase that only occurs if the earthquake has a certain depth. pP, sP, sS, pS, etc. have depth phases related to P and S.
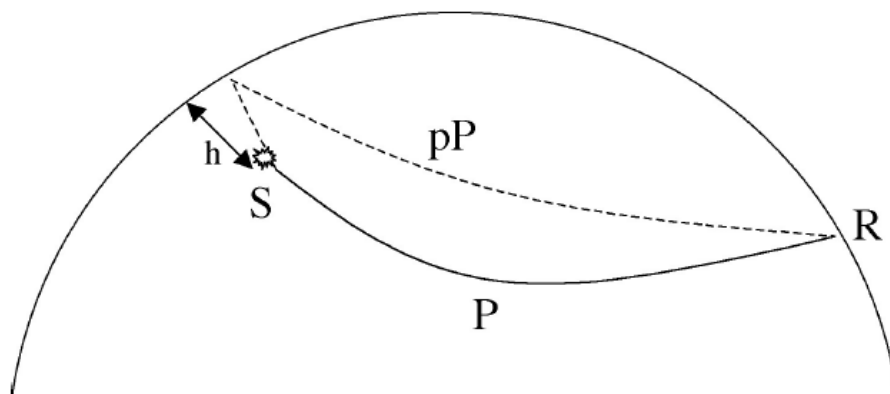

Figure 1

Figure 1 shows paths of both a P wave and a pP wave for $h \neq 0$. At very large surface distance, the two curves travel on nearly the same path, so $T_{pP} - T_P = \delta t \sim \dfrac{2h}{v}$. Note that this corresponds with the travel time curves discussed previously.
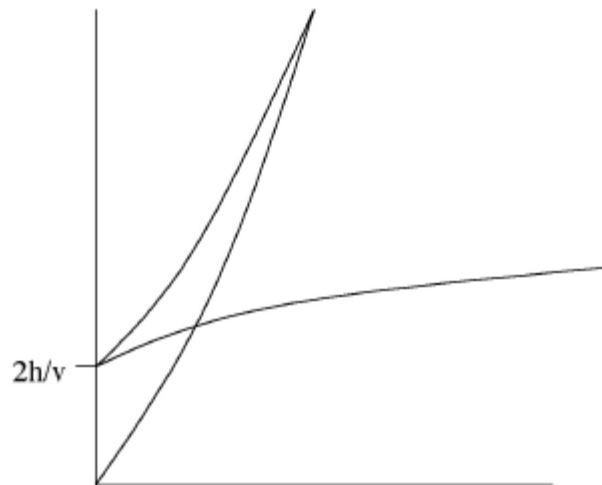
Figure 2

Problems arise when the source is too shallow, giving a low h value. When this happens, the travel time difference is small and makes it difficult to distinguish between the two arrivals. Using a combination of depth phases and travel time differences can give a good estimate of the hypocenter of the earthquake.

D. Refinement of Source Location

If we have tthe initial guess of hypocenter $\mathbf{m}_0 = \left(t_0, h_0, \theta_0, \varphi_0\right)$, then for the actual source $\mathbf{m}$

$$T_{obs} = T\left(\mathbf{m}\right) \doteq T\left(\mathbf{m}_0\right) + \delta T \tag{10}$$

where δT is the travel time residual. From travel time tomography, we have

$$\delta T = T_{obs} - T_{ref} = \delta T_{\text{3D structure}} + \delta T_{\text{Source mislocation}} + errors \tag{11}$$

The linearized residual for source mislocation is

$$\begin{aligned}\delta T &= \frac{\partial}{\partial t_0}\delta t + \frac{\partial}{\partial h_0}\delta h + \frac{\partial}{\partial \theta_0}\delta\theta + \frac{\partial}{\partial \varphi_0}\delta\varphi \\ &= \left(\nabla' \cdot T\right) \cdot \delta\mathbf{m}\end{aligned} \tag{12}$$

where

$$\delta\mathbf{m} = \left(\delta t, \delta h, \delta\theta, \delta\varphi\right) \tag{13}$$

Combine this with residual from 3D structure, we have

$$\left(\mathbf{A}_{3D} \vdots \mathbf{A}_{eq}\right)\begin{pmatrix}\mathbf{m}_{3D} \\ \cdots \\ \mathbf{m}_{eq}\end{pmatrix} = \mathbf{d} \tag{14}$$

That is

$$\underline{\underline{\mathbf{A}}}\,\underline{\mathbf{m}} = \underline{\mathbf{d}} \tag{15}$$

The solution is

$$\hat{\mathbf{m}} = \left(\mathbf{A}^{\mathbf{T}}\mathbf{A}\right)^{\mathbf{-1}}\mathbf{A}^{\mathbf{T}}\mathbf{d} \tag{16}$$