12.510 Introduction to Seismology
Spring 2008

Problem Set 4

1. Receiver Function

A radial receiver function is generated from the *P*-coda of a single event recorded at a three-component, broadband seismometer located at the surface. The background velocity model used to analyse the receiver function is isotropic and made up of a horizontal layer over a half space.

a) Show that for a 1-D, horizontal discontinuity located at a depth h within the top layer, the delay time $\delta t = t_s - t_p$ between the *Ps* and *P* arrivals is given by: $\delta t = h(\eta_s - \eta_p)$, with η the vertical slowness. (10 points)

b) Now consider a background velocity model with the following parameters:

	Thickness(km)	V_p(km/s)	V_s(km/s)	ρ (g/cm^3)
top layer	25	6.0	3.2	2.8
1/2 space	--	8.0	4.5	3.4

and an incident P wave with a ray parameter of 0.06 s/km. Calculate the delay time δt for a *Ps* wave converted at a horizontal discontinuity located exactly at the interface between the top layer and the half-space (i.e., h=25km). (5 points)

c) For the same background velocity model and incident *P* wave as in (b), what would be the depth of a horizontal discontinuity if the receiver function displayed a *Ps* phase arriving i) 2s, ii) 6 s after the *P* wave? (15 points)

2. Surface Waves

1) On a seismogram of dispersed Rayleigh waves, the times in seconds of the peaks and troughs of the waves (starting with a peak) are 30, 86, 134, 174, 200, 226, 244, 262, 276, 290, 302, 312, 320, 330, 338, 346, and 352. The starting time (t=0) of the seismogram is 10h 21m 55s. The station is a distance of 5000 km from the epicenter and the origin time of the earthquake is 10 h 10 m 0s.

   (a) Reconstruct approximately the seismogram. (5 points)

   The same earthquake is observed at a second station along a great circle path at a distance of 400 km from the first. The trend of dispersed Rayleigh waves starts from t=0 at 10h 23m 35s. Peaks and troughs are at times in seconds (the first is a peak) of 36, 100, 146, 185, 215, 240, 270, 284, 304, 318, 330, 334, 355, 364, 375, 384, and 394.

   (b) Reconstruct approximately the seismogram. (5 points)

   (c) Calculate and draw the dispersion curve of the group velocity U versus the period. (10 points)

   (d) Calculate and draw the dispersion curve of the phase velocity c versus the period. (10 points)
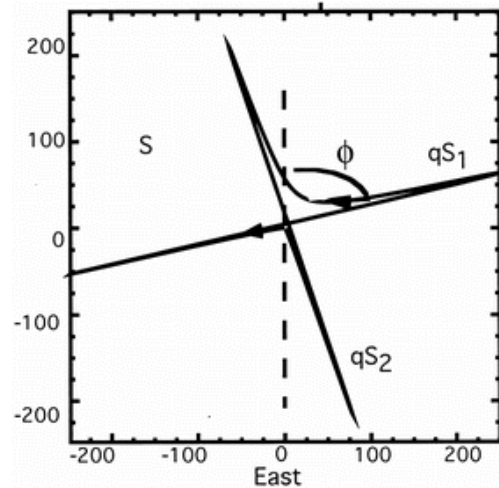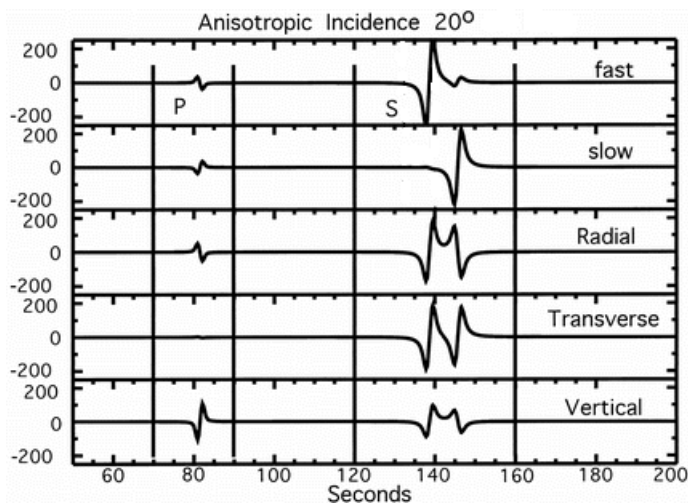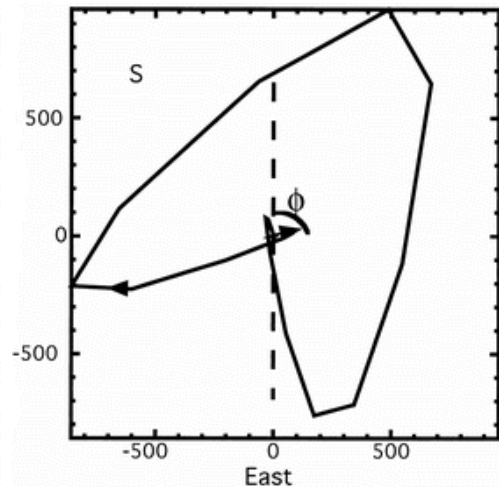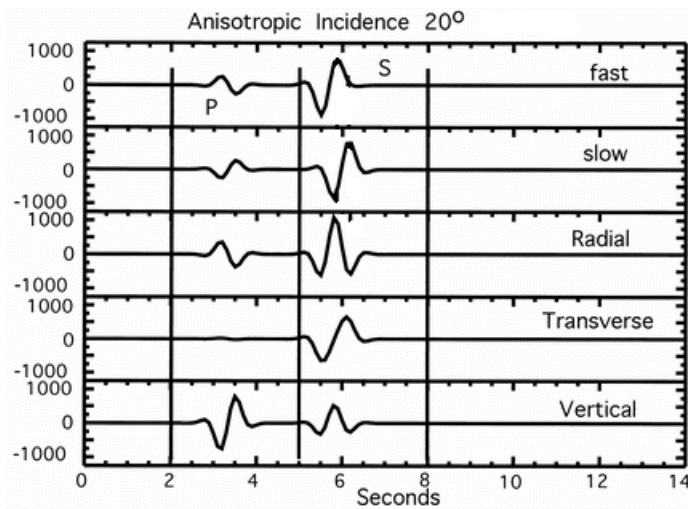
   Hints: you can refer to 12.5 section of **Principles of Seismology** by Agustin Udias for details of measurement of group and phase velocity.

2) For a Rayleigh wave in a half-space, if the P-wave velocity is 6 km/s and Poisson's ratio is 1/4, determine for a wave of Period T=20s
   (a) the depth at which the amplitude of u_1 equals to zero (5 points)
   (b) the depth at which the amplitude of u_1 is larger than that of u_3, (10 points)
      where u_1 and u_3 are the x, and z displacements respectively.

3. Anisotropy

1) In shear wave splitting studies of upper mantle anisotropy, recordings of *SKS* or *SKKS* phases are most commonly used. What are the advantages and disadvantages of using *SK(K)S*? What are the advantages and disadvantages of using direct *S* phases? (5 points)
2) Olivine, the principal upper mantle constituent, has an orthorhombic symmetry system. Why, then, are splitting measurements due to mantle anisotropy usually interpreted with hexagonal symmetry in mind? (5 points)
3) The figure below (from *Savage*, 1999) shows two synthetic seismograms, with corresponding particle motion diagrams, for an incoming shear wave passing through an anisotropic region. Explain why the particle motion diagrams look so different. What is the difference between the anisotropic models used to create the synthetic seismograms? Which situation is more relevant to studies of upper mantle anisotropy using shear wave splitting? (10 points)

Adapted from Savage, 1999