Course 12.425. Problem Set 3. Due 25 Oct. 2007.

1. **Star and Planet Radii.**

a. The smallest star is a so-called M9 dwarf star which has a radius $0.09R_{\rm sun}$ and a mass $0.08M_{\rm sun}$. How does Jupiter compare in radius and mass to such a star?

b. How can a star be smaller than a planet when the star is so much more massive?

2. **Equations of State**.

In class we discussed equations of state and phase diagrams.

a. Define equation of state.

b. Which phase diagram shown in class did you find most interesting? Write one to two paragraphs on that phase diagram. You may cross-reference with the web.

3. **Planetary Interior Structure I.**

For this question and the next you will develop a numerical code to integrate the equations of planetary interiors.

In class we reviewed the equations for planetary structure. Mass of a spherical shell

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r); \tag{1}$$

the equation of hydrostatic equilibrium

$$\frac{dP(r)}{dr} = \frac{-Gm(r)\rho(r)}{r^2}; \tag{2}$$

and the equation of state (EOS)

$$P(r) = f(\rho(r), T(r)). \tag{3}$$

a. Solve the equations numerically for a polytropic equation of state. Use the polytropic index $n = 3/2$.

$$P \sim \rho^{(n+1/n)}. \tag{4}$$

Plot the result of ρ vs. r, P vs. r, and m vs. r.

b. Show that for constant density $M_p \sim R_p^3$.

c. For a pure electron degenerate gase (index $n = 3/2$) does $M_p \sim R_p^{-3}$?

4. **Planetary Interior Structure II.**

Use the equations in the above question.

a. Consider the modified polytropic EOS, $\rho = \rho_0 + cP^n$. Why is there no analytical solution to the equations of planetary interiors with this EOS?

For parts b and c you will use your numerical code to integrate the equations of planetary interiors. You may use the modified polytropic EOS above. See the below table for ρ_0, c, and n.

b) Find the radius of a homogeneous water planet of $5M_\oplus$. What is the central pressure? List your boundary conditions.

c) What is the central pressure of a homogeneous planet with $10M_\oplus$ and $2R_\oplus$? First use the central pressure approximation derived in class. Next use your code to find the central pressure.

Material	ρ_0 [kg m^{-3}]	c [kg m^{-3} Pa^{-n}]	n
Fe(α)	8300.00	0.00349	0.528
$MgSiO_3$ (perovskite)	4100.00	0.00161	0.541
H_2O (Ice VII)	1460.00	0.00311	0.513