12.307 Weather and Climate Laboratory
Spring 2009

12.307
Project 3
Convection

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1 Background

The Earth is bathed in radiation from the Sun whose intensity peaks in the visible. In order to maintain energy balance the Earth must radiate energy away. Because the Earth is so much colder than the Sun (255K compared to 6000K) the outgoing terrestrial radiation occurs at much longer wavelengths, in the infra-red. Terrestrial radiation emanates primarily from the upper troposphere, rather than the ground. Solar radiation warms the ground, but because of the enveloping water vapor layer, terrestrial radiation starting out from the surface is absorbed by the watervapor layer in a mechanism that has become known as the 'Greenhouse effect'. Equilibrium is not established solely by radiative processes. Instead, warming at the surface triggers convection which transports heat vertically upward to the emission level where, because the atmosphere above this level is transparent in the infra-red, energy can be beamed out to space. In this project we enquire in to the nature of the convective process.

We will simulate convection in the laboratory using a tank of water with a heating pad at its base and study convection in the atmosphere using thermodynamic diagrams. Note that because the atmosphere is compressible it is the potential temperature (ϑ) of the air which is equivalent to the temperature (T) of the water in the tank.

Background notes are attached. Further informations can be found in Chapter 4 of 12.003 notes: http://paoc.mit.edu/labweb/notes/chap4.pdf.

1.1 Convection

When a fluid is heated from below (or, in fact, cooled from above, as in the ocean), it often develops overturning motions. It may seem obvious that this must occur, as the tendency of the heating (or cooling) is to make the fluid top-heavy. Consider a horizontally infinite fluid as depicted in Fig. 1. Let

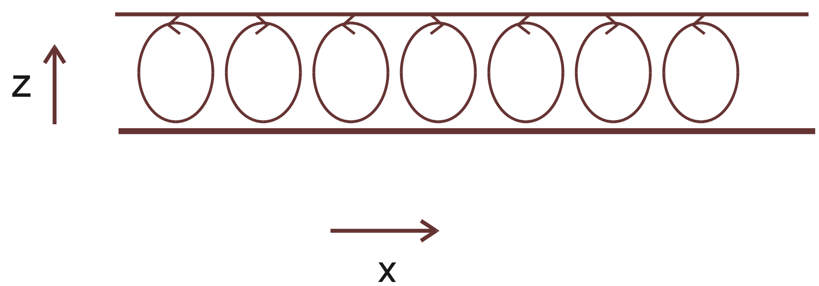

Figure 1: Schematic of convective overturning of a fluid

the heating be applied uniformly at the base; then we may expect the fluid to have a horizontally uniform temperature, so $T = T(z)$ only. This will be top-heavy (warmer, and therefore lighter, fluid below cold, dense fluid above). We will see in our laboratory experiment that motion develops from an *instability* of the fluid in the presence of the heating. As sketched in the figure the horizontal length scale of the convective motion that develops is comparable to the depth of the unstable layer.

1.2 Convection in an almost-incompressible liquid

Let us consider the stability of a parcel of fluid in an incompressible liquid. By "incompressible" we mean that density is independent of pressure. However, density will depend on temperature; a good approximation for water in typical circumstances is

$$\rho = \rho_{ref}(1 - \alpha T) \ , \tag{1}$$

where ρ_{ref} is a constant reference value of the density and α is the coefficient of thermal expansion. Now, consider a horizontally uniform state with temperature $T(z)$, and $\rho(z)$ then defined by (1). We focus attention on a single fluid parcel P, initially located at z_0. It has temperature $T_0 = T(z_0)$ and density $\rho_0 = \rho(z_0)$, the same as its environment; it is therefore *neutrally* *buoyant*, and thus in equilibrium. Now let us displace this fluid parcel a small vertical distance to $z_1 = z_0 + \delta z$, as shown in Fig. 2. The question we are

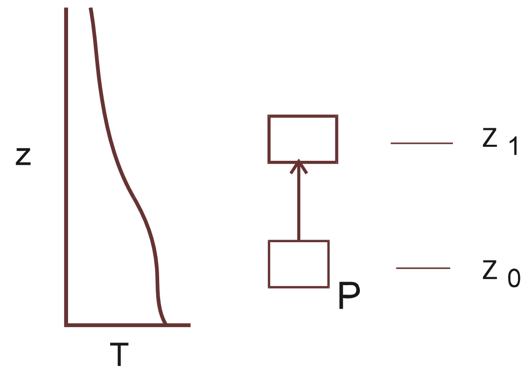

Figure 2: Parcel stability

going to investigate is the buoyancy of the parcel when it arrives at B. Now, if the displacement is done sufficiently quickly so that the parcel does not lose or gain heat on the way, it will occur *adiabatically*. The internal energy of a liquid depends only on its temperature, so T will be conserved during the displacement. Therefore the temperature of the perturbed parcel at z_1 will still be T_0, and so its density will still be ρ_0. The environment, however, has density

$$\rho(z_1) \simeq \rho_0 + \left(\frac{d\rho}{dz}\right)_e \delta z \ ,$$

where $(d\rho/dz)_e$ is the environmental density gradient. The buoyancy of the parcel just depends on the difference between its density and that of its environment; therefore it will be

$$\left.\begin{array}{c}\text{positively}\\ \text{neutrally}\\ \text{negatively}\end{array}\right\} \text{buoyant } if \ \left(\frac{d\rho}{dz}\right)_e \left\{\begin{array}{l} > 0 \\ = 0 \\ < 0 \end{array}\right. . \tag{2}$$

If the parcel is positively buoyant, it will keep on rising at an accelerated rate. Therefore an incompressible liquid is **unstable if density increases with height**. It is this instability that leads to the convective motions we discussed above. The condition for instability is just the "top-heavy" condition.

In the laboratory experiment we now describe, a stable stratification is set up and destabilized by warming from below.

2 Experiment

We can study a turbulent convective layer in a laboratory setting using the apparatus sketched in Fig. 3 comprising a tank of water which is heated from below. Heating at the base is supplied by a heating pad, whose power can be controlled with a transformer.

The motion of the fluid is made visible by

- sprinkling a VERY SMALL amount of potassium permanganate evenly over the base of the tank after the stable stratification has been set up.
- shining a light from a projector through the evolving convection layer on to a screen (made from tracing paper) attached to the side of the tank.

Quantitative information can be obtained by recording temperature time-series from thermometers arranged vertically at the side wall of the tank.

- After switching on the heating at the base, thermals will be seen to rise from the base. Successive thermals rise higher as the layer deepens.

We will carry out two experiments. In one we study convection in to an initially unstratified fluid of constant depth; in the other we study convection in to an initially stably stratified fluid.

A stable stratification can be set up by slowly filling the tank with water whose temperature is slowly increased with time. This is done using (i) a mixer which mixes hot and cold water together and (ii) a diffuser which floats on the top of the rising water and ensures that the warming water floats on the top without generating turbulence - see Fig.3. Using the hot and cold water supply in the laboratory we can achieve a temperature difference of $\sim 20^\circ$ C or more over the depth of the tank. The temperature profile can be measured and recorded using the thermocouples provided.

2.1 Experimental procedure and observations

2.1.1 Convection in to a linearly-stratified layer

1. Establish a linear stable stratification within the tank and initiate convection by turning on the heating pad. Monitor the development of the

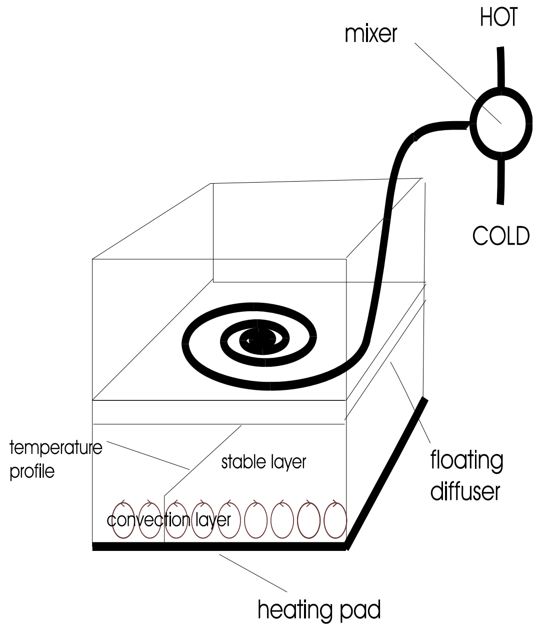

Figure 3: Convection experiment

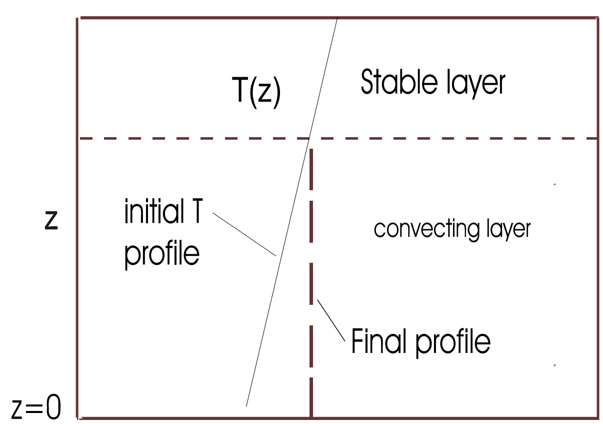

Figure 4: Evolving convective boundary layer above heating pad

boundary layer visually and through the temperature recorded by the sensors. Plot graphs of the depth and the temperature of the boundary layer as functions of time and interpret in terms of the following theory.

The thermodynamic equation (horizontally averaged over the tank) can be written:

$$\rho c_p \frac{dT}{dt} = \frac{\mathcal{H}}{h} \tag{3}$$

where h is the depth of the convection layer - see fig.4 - $\mathcal{H}$ is the heat flux coming in at the bottom from the heating pad, ρ is the density and c_p is the specific heat.

We observe that the temperature in the convection layer is almost homogeneous and 'joins on' to the linear stratification in to which the convection is burrowing. Thus $T = \overline{T_z} h$ (see fig.4) and so the above can be arranged thus:

$$\frac{\rho c_p \overline{T_z}}{2} \frac{d}{dt} h^2 = \mathcal{H}$$

The solution of the above is:

$$h = \left(\frac{2\mathcal{H}t}{\rho c_p \overline{T_z}} \right)^{\frac{1}{2}}$$

Thus h and T of the convective boundary layer should evolve like $\sqrt{t}$.

2. Plot the theoretical prediction along with your observations and discuss.

3. Investigate the diameter and speeds of the thermals as a function of the depth of the mixed layer. The thermals have a size spectrum for a given depth, so several observations will be required to enable averages to be taken. Use eqs(5) and (6) derived in the theory section to estimate typical w's and $\delta T's$ of the boundary layer in terms of the applied heating rate. Are they in accord with the observations.

2.1.2 Convection in to a homogeneous fluid of constant depth

Modify your experiment and the theory developed in the previous section to discuss convection in to a homogeneous fluid of constant depth (h =constant). How does the T evolve in time in this case?

3 Theory

3.1 Energetics of convection

3.1.1 Available Potential energy

Show that the change in potential energy resulting from the interchange of the two small elements (of incompressible) fluid shown in fig.2 (of equal volume but differing densities) is given by:

$$\Delta P = P_{initial} - P_{final} = -gz^2 \left(\frac{d\rho}{dz}\right)_e \tag{4}$$

where $P_{initial}$ is the potential energy of the two particles before they are swapped and P_{final} is there potential energy after they are swapped.

Interpret the stability condition (2) in terms of eq(4)[1].

3.1.2 Law of vertical heat transport

By equating the released potential energy to the acquired kinetic energy of the convective motion, show that:

$$w^2 \sim \alpha g z \delta T \tag{5}$$

where δT is the difference in potential temperature between a parcel and its environment after it has been displaced a height z, and w is a typical vertical velocity.

Hence show that a "law" of vertical heat transfer appropriate to the convection in our tank is:

$$\mathcal{H} = \rho c_p \overline{w \delta T}^{time} = \rho c_p (\alpha g z)^{\frac{1}{2}} \delta T^{\frac{3}{2}} \tag{6}$$

[1] ΔP is called available potential energy: if $\left(\frac{d\rho}{dz}\right)_e < 0$, then potential energy is available for conversion in to kinetic energy of the growing convection cells: see chapter 7 of 12.003 notes.

4 Convection and Atmospheric Thermodynamics

As an introduction to this assignment read notes on dry and moist convection from 12.307 web site — Convection in a compressible atmosphere (http://paoc.mit.edu/12307/convection/convection_in_air.pdf)

When a fluid is heated from below (or cooled from above), it often develops overturning motions. We are all familiar with summer time convective clouds and thunderstorms. They develop because solar heating during the day warms the surface making the air in contact with it buoyant.

Here we will use observed temperature profiles from radiosonde soundings to study the onset of convection in the atmosphere.

4.1 Dry Convection in a compressible fluid

Let's consider the stability of a parcel of air in a compressible fluid such the atmosphere.

As the air parcel rises, it moves into an environment of lower pressure. The parcel will adjust to this pressure; in doing so it will expand and thus cool adiabatically. The rate at which the temperature decreases with height under adiabatic displacement is called:

$$\text{the dry adiabatic lapse rate } \Gamma_d = -g/c_p \sim 10^o K/\text{kilometer}$$

The stability of the profile depends on how dT/dz of the environment varies relative to Γ_d. We find:

$$\left.\begin{array}{c}\text{UNSTABLE}\\ \text{NEUTRAL}\\ \text{STABLE}\end{array}\right\} \text{ if } \left\{\begin{array}{l}\left(\frac{dT}{dz}\right)_E < -\Gamma_d\\ \left(\frac{dT}{dz}\right)_E = -\Gamma_d\\ \left(\frac{dT}{dz}\right)_E > -\Gamma_d\end{array}\right. .$$

The non-conservation of T under adiabatic displacement makes it a less than ideal measure of atmospheric thermodynamics. However we can define a temperature called potential temperature which is conserved in adiabatic displacement. The potential temperature of an air parcel, denoted by θ, is the temperature it would have if it were compressed adiabatically from its existing p and T to a standard pressure.

The definition of potential temperature is:

$$\theta = T \left(\frac{p_0}{p} \right)^{\kappa} \tag{7}$$

with $k = R/c_p = 9/7$ and conventionally $p_o = 1000mb$.

The stability criterion, expressed in terms of potential temperature becomes:

$$\left. \begin{array}{c} \text{UNSTABLE} \\ \text{NEUTRAL} \\ \text{STABLE} \end{array} \right\} \text{ if } \left\{ \begin{array}{l} \left(\frac{d\theta}{dz}\right)_E < 0 \\ \left(\frac{d\theta}{dz}\right)_E = 0 \\ \left(\frac{d\theta}{dz}\right)_E > 0 \end{array} \right. . \tag{8}$$

In meteorology it is customary to study the stability of observed vertical profiles of temperature using "thermodynamic diagrams". The most common form of thermodynamic diagram used is one in which pressure forms the ordinate and temperature the abscissa. While the temperature scale is linear the pressure scale is logarithmic because $logp$ is proportional to altitude in an isothermal atmosphere. Fig. 5 shows an example of a commonly used diagram, the $skewTlogp$ diagram in which the isotherms runs diagonally across the diagram rather than vertically. This is done so that typical atmospheric soundings take up less area on a piece of paper. In addition to the aforementioned coordinates, lines of constant potential temperature (adiabats) are marked.

4.1.1 Stability of 'standard' atmosphere to dry processes

Plot the appended tabulated temperatures for 'standard' atmospheric conditions in middle latitude ($40^o N$) and the tropic ($10^o N$). Make two separate plots.

From each plot:

1. compare the observed tropospheric lapse rate to the dry adiabatic lapse rate. You will see that the atmosphere is stable to dry adiabatic displacements.

2. if a parcel is displaced dry adiabatically from its equilibrium condition it will return. From your data estimate the period of the resulting buoyancy oscillations in mid-troposphere (see 12.003 Notes, Chapter 4 - Eqs. 4.20 and 4.21).

3. identify the tropopause for the standard atmosphere - is it higher in the tropics or middle latitudes?

4.1.2 Stability of observed radiosonde profiles

Case Study: Yuma, AZ June 18, 2007 Yuma is in the Arizona desert and a great location for dry convection during the hot summer months. Here you will study the evolution of the boundary layer temperature during July 18, 2007, using observed radiosonde profiles every two hours. See instructions at: http://paoc.mit.edu/12307/convection/yuma_instructions.htm.

This is a great analogue to the tank experiment.

Your own profiles Examine the latest surface maps and satellite image. Where would you expect dry convection to occur? Plot temperature profiles at selected locations over the US and look for examples of dry convection.

Study the profiles and identify:

1. tropopause height
2. regimes of static stability (stable, unstable, neutral)
3. note any temperature inversions and its diurnal variation

Comment on where and when you might expect to observe dry convection in the atmosphere.

4.2 Moist Convection in a compressible fluid

As we have seen in Section 4.1, the atmosphere is almost always stable to dry adiabatic displacements. On the other hand we know that the atmosphere is a mixture of dry air and water vapor. It is because of the presence of water vapor that the atmosphere is convectively unstable.

Let's consider what happens if we displace an air parcel vertically. If the air is unsaturated, no condensation will occur and so our previous results for dry air hold. However, if condensation occurs, the release of latent heat will make the air parcel warmer and therefore more buoyant. The atmosphere is destabilized by the presence of moisture.

4.2.1 Standard atmosphere

From the profiles of temperature for the standard atmosphere we can estimate the amount of water vapor that the air can hold and study how it depends on height and latitude (i.e. assuming that the air is saturated):

The atmosphere is a dry air and water vapor mixture. The equation of state of dry air is:

$$p = \rho R T$$

and that for water vapor is:

$$e = \rho_v R_v T$$

where e is the vapor pressure, R_v is the gas constant for the vapor and ρ_v is the density of water vapor.

The specific humidity 'q' is a common measure of the amount of water vapor defined by

$$q = \frac{\rho_v}{\rho} = \frac{R}{R_v}\left(\frac{e}{p}\right) = \varepsilon\left(\frac{e}{p}\right)$$

where

$$\varepsilon = \frac{\text{mol. wt. water vapor}}{\text{mol. wt. air}} = \frac{18}{29} = 0.62.$$

Note that q is unchanged until condensation takes place. Then it becomes q_*, the saturation specific humidity, which is a function of p and T:

$$q_* = \frac{\varepsilon e_s\left(T\right)}{p}$$

where e_s is the saturation vapor pressure (only a function of T) given by:

$$e_s = A \exp\left(\beta T\right)$$

with A and β constants and T is in ^oC.

Some measured values of e_s (in mb) are:

$e_s(mb)$	6.11	13.0	23.4	1000
$T(^oC)$	0	10	20	100

Evaluate β and A to fit the data in the table.

From your tables and the information above deduce the value of q_* at $850mb$ and hence how much water vapor the atmosphere can hold at saturation in middle latitudes and in the tropics. Quote your answer in g/kg of dry air. Comment on your result.

Since q_* is only function of temperature and pressure, lines of constant saturation specific humidity can be plotted on a thermodynamic diagram. These are the dotted lines bending to the right in Fig.6. They are labeled in g/kg.

Given an observed temperature profiles, we can use the q_* lines to estimate the maximum amount of water vapor the air can hold before condensation occurs.

As the water vapor condenses, latent heat is released and the parcel is warmed, thus we expect the lapse rate of the moist parcel to be less than if it were dry. As shown in the attached 12.307 Notes (Moist Convection), for a saturated parcel undergoing adiabatic displacement:

$$dT/dz = \Gamma_s < \Gamma_d$$

where Γ_s is the saturated adiabatic lapse rate.

Compare the observed lapse rate in the tropics and in middle latitudes for the given standard atmosphere profiles.

In a thermodynamic diagram lines showing the decrease in temperature with height of a parcel of air, which is rising or sinking under saturated adiabatic conditions are called saturated adiabats (or pseudo-adiabats). In Fig. 7 they are the dashed lines slightly curved to the left.

A saturated atmosphere is unstable if:

$$dT/dz < -\Gamma_s$$

where $\Gamma_s < \Gamma_d$. This instability is known as conditional instability, since it is conditional on the air being saturated. Typically $\Gamma_s \sim 0.7\ \Gamma_d \sim$7K/km.

4.2.2 Stability of observed radiosonde profiles

Examine the latest surface maps and satellite image. Where would you expect moist convection to occur? Plot temperature profiles at selected locations over the US and look for examples of moist convection.

Study the profiles and identify:

1. tropopause height
2. regimes of moisture (dry, moist)
3. regimes of static stability (stable, unstable, neutral)
4. if cloud is expected, estimate pressure at cloud base and cloud top

Comment on the occurrence of moist convection in the atmosphere.