Lab 1: Stress, Mohr's circles
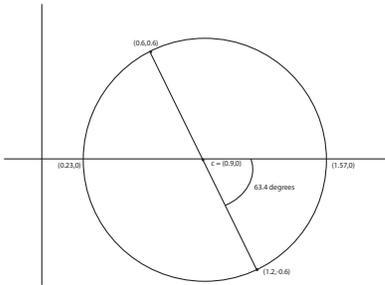# SOLUTION KEY

Fall 2005

1 Mohr circles for stress

For the problems that you use a Mohr circle construction, please show the circles you draw, labeling the points on the construction corresponding to the relevant planes.
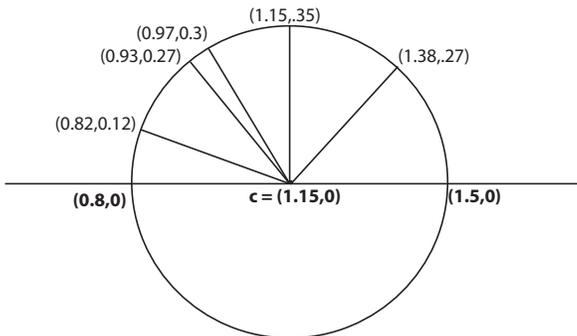
1.1

$\sigma_N = 8.66 \times 10^5$ Pa
$\tau = 5 \times 10^5$ Pa

1.2

$\sigma_1 = 1.57$kbar, $\sigma_3 = 0.23$kbar. The angle between the normal to the first plane and σ_1 is one-half of $63.4°$, i.e. $31.7°$. Note how the angle is measured.
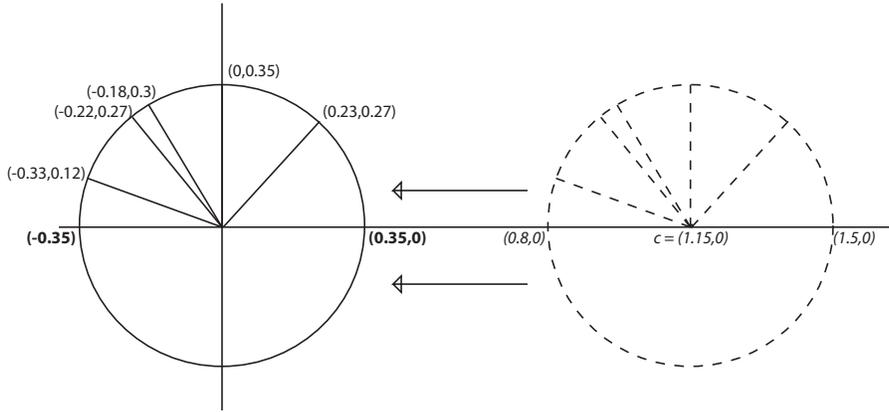
1.3

1.4

The stress tensor can be decomposed into two parts: a lithostatic component and a deviatoric component:

$$\begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} = \begin{bmatrix} p & 0 \\ 0 & p \end{bmatrix} + \begin{bmatrix} \sigma_1 - p & 0 \\ 0 & \sigma_2 - p \end{bmatrix}$$
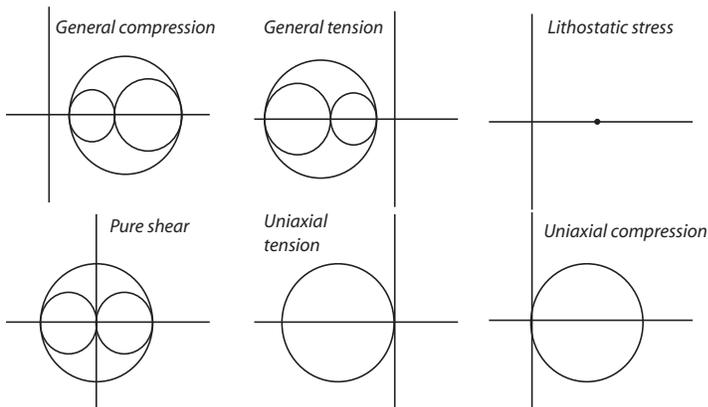
On the Mohr diagram, this has the effect of shifting the Mohr circle to the right. Note that the new Mohr circle – the deviatoric Mohr circle – can have negative (that is, tensional) normal stress. Note also that the shear stresses for the deviatoric stress are the same as for the original stress state.

The lithostatic stress (mean stress) for question 2 is 0.9 kbar; for question 3 it is 1.15 kbar. The deviatoric stresses for the various planes are:

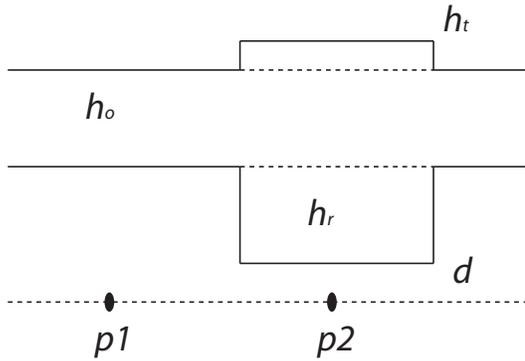|	Deviatoric normal stress	Deviatoric shear stress
Q2, plane 1	0.3	-0.6
Q2, plane 2	-0.3	0.6
Q3, plane (i)	0.23	-0.27
Q3, plane (ii)	0.00	-0.35
Q3, plane (iii)	-0.18	-0.30
Q3, plane (iv)	-0.22	-0.27
Q3, plane (v)	-0.33	-0.12

1.5

Note that the above figures show the Mohr circle representation of 3 dimensional stress: that is, there are three circles corresponding to the σ_1, σ_3; σ_1, σ_2; σ_2, σ_3 pairs. The latter two circles are inscribed within the first, largest circle. We only made passing reference to the three dimensional representation of stress in lecture, so drawing plane stress circles is ok for the purposes of the assignment. Nevertheless, you should examine the figures (labeling the principal stresses as appropriate) and make sure you understand them.

2 Isostasy

2.1

The figure shows the setup. If the mountain range is in isostatic equilibrium, the pressure at point 1 will equal the pressure at point 2. The additional mass of the mountain range of height h_t is compensated by the negative mass anomaly of the crustal root (thickness h_r). To set up the equations, we equate the pressure at point 1 and point 2, and solve for the ratio h_t/h_r.

$$\rho_c g h_o + \rho_m g(h_r + d) = \rho_c g h_t + \rho_c g h_o + \rho_c g h_r + \rho_m g d$$
$$\rho_m h_r = \rho_c h_t + \rho_c h_r$$
$$(\rho_m - \rho_c) h_r = \rho_c h_t$$
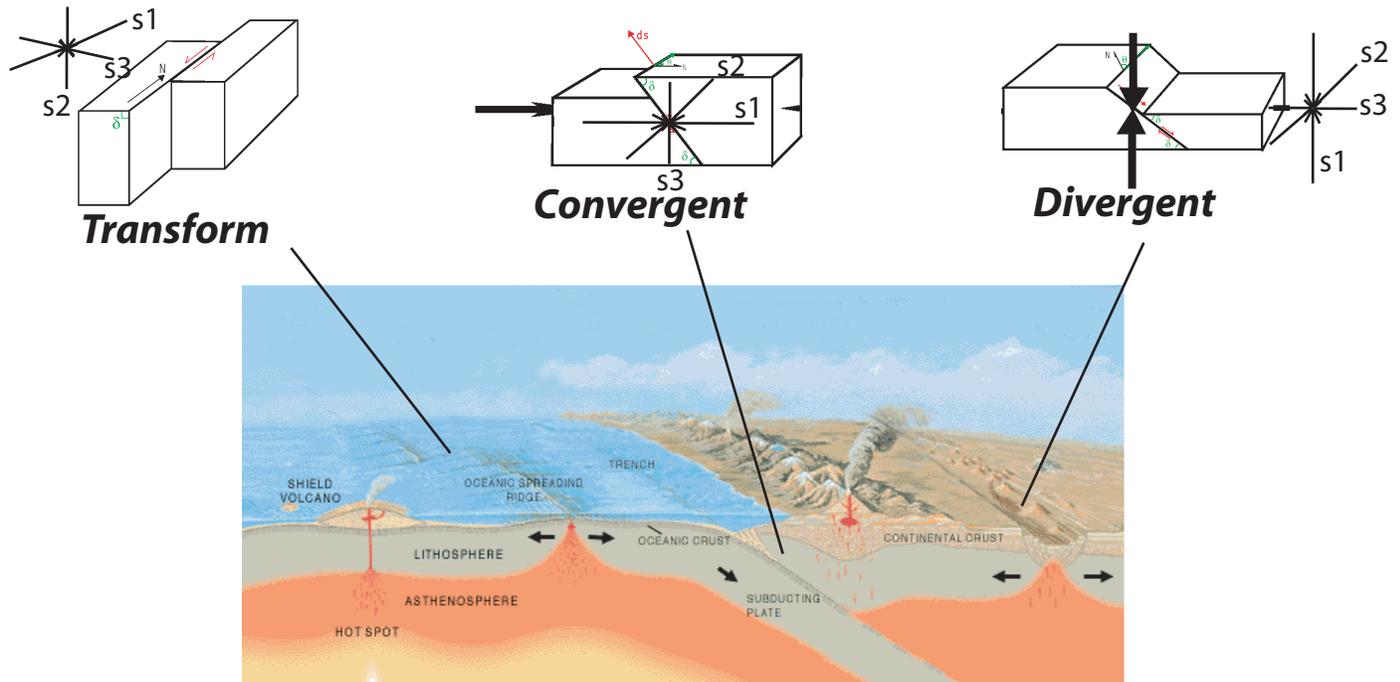$$\frac{h_t}{h_r} = \frac{(\rho_m - \rho_c)}{\rho_c}$$

Plugging in the values, this implies that every kilometer of topography is compensated by 5km of extra crustal thickness in the root. A 5km high mountain range, therefore, may have a 25 thick root, so the total crustal thickness would be 65km (5 + 35 + 25).

2.2

The calculated ratio between the height of topography and the thickness of the crustal root implies that in order to eliminate one kilometer of topography, you need to thin the crust by six kilometers (one kilometer of topography, and five kilometers of root). In order to eliminate five kilometers of topography, then, you need to thin the crust by 30 kilometers. If crustal thinning proceeds only by erosion from the top of the crustal column, rocks originally at 30km depth will be exposed at the surface once all excess topography has been completely eliminated. The pressure at 30km depth is $P = \rho g(30\text{km})$, or roughly 840 MPa (= 8.4 kbar). The actual mineral assemblage that would then be exposed will also depend on the temperature, but for a relatively cool geothermal gradient of 25°per kilometer, temperatures should be around 750°C. This corresponds to upper amphibolite / lower granulite grade rocks. In thick continental crust, temperatures increase faster with depths, so even higher metamorphic grades might be expected. Although these grades are not unheard of in exposures of older, deeply eroded rocks, nowhere on Earth are rocks of that grade exposed over an area the size of Tibet.

Leaving aside the possibility that something like Tibet is unprecedented in over 4 billion years of Earth history, we might find a solution to this apparent problem by asking whether there are processes apart from erosion that might thin the continental crust. Thickening and mountain building in both the Appalachian and Cordilleran mountain belts was quickly followed by normal faulting and crustal extension. In the Appalachians, crustal extension eventually thinned the continental crust so much that an ocean opened up, floored by basaltic crust created at a mid-ocean ridge. The so-called "late extensional collapse" of mountain belts is a strong possible explanation for the thinning of thickened continental crust, with only local exposure of very high grade rocks.

Another possibility is if the lower or middle crust becomes so weak that it flows laterally out of the thickened crust into adjoining, thinner and lower crust. This process may be occurring today along the eastern margin of the Tibetan plateau.

Transform   *Convergent*   *Divergent*

3 The orientation of principal stresses

The above figure crudely shows the expected orientation of the maximum principal stress for transform (eg. San Andreas fault), convergent (eg. Cascadia subduction zone) and divergent (eg. Atlantic mid-ocean ridge, or Basin and Range) plate boundaries. The minimum principal stress is oriented vertically for convergent boundaries and thrust faults; horizontally for divergent boundaries and normal faults. We expect that both maximum and minimum principal stresses are horizontal for transform boundaries or strike-slip faults. In particular, we expect faults to be at roughly 30°to the maximum principal stress. That is, thrust faults should, theoretically, be shallower than normal faults.

As deformation progresses, the stresses also change. In fact, it is useful to think of deformation as a processes whereby stresses are relieved. In extensional environments, crustal thicknesses decrease as a consequence of normal faulting. Therefore, the vertical stress should also decrease. Eventually, all else being equal, the vertical stress should no longer be greater than the horizontal confining stress, and normal faulting ought to cease. Similarly, thrust faults thicken the crust and increase the vertical stress. That is, as a consequence of deformation (thrust faulting), the minimum principal stress increases. Eventually, the horizontal principal stress driving thrust faulting will no longer be able to overcome the vertical stress, and faulting will cease (or, more likely, move to somewhere else). Consequently, we expect that there ought to be a limit to how thick the crust can get.