MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Civil and Environmental Engineering

1.731 Water Resource Systems

Problem Set 2 – Optimality Conditions, GAMS
Due: Tuesday, Sept. 26, 2006

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1. Put all of the following optimization problems in a consistent standard form (i.e. maximize the objective function and write all constraints in the form $g_i(x) \leq 0$).

2. Provide GAMS and graphical solutions to each problem. Please append GAMS code and output to document your GAMS solutions. Note that GAMS may produce much more output than really needed. Please excerpt only those parts needed to document your calculations.

3. Explicitly show that all applicable necessary (Kuhn-Tucker) conditions are met at each local minimum or local maximum you identify (make sure that these conditions are relevant for each problem before trying to apply them). Use MATLAB to check definiteness for the Kuhn-Tucker curvature condition.

4. Use the theorem relating local and global optima to determine if the local optimum you have identified is also a global optimum.

If you are unfamiliar with GAMS I suggest that you reproduce the example problem in the GAMS Tutorial in the GAMS documentation (download from http://www.gams.com/docs/document.htm) and also that you look over the rest of the material in the tutorial. There are also some examples and good discussion in Bruce McCarl's GAMS Users Guide at the same link.

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1. Minimize: $x_1^2 + x_2^2$
   such that: $x_1 x_2 \geq 1$
   $x_1, x_2 \geq 0$

2. Minimize: $x_1^2 + x_2^2$
   such that: $x_2 \geq 2 - 3x_1$
   $x_2 \geq 1/5$
   $x_1, x_2 \geq 0$

3. Minimize: $(x_1-1)^2 + 2x_2^2 + x_3^2$
   such that: $x_3 = 2x_1^{1/2}$

$x_2 + 2x_1 \geq 2$
$x_1, x_2 \geq 0$

4. Maximize: $x_1 + 2x_2$
   such that: $x_1 + x_2 \leq 1$
   $2x_2 \leq x_1 + 2$
   $x_1, x_2 \geq 0$

5. Maximize: $3x_1 + 7x_2$
   such that: $x_1 - x_2 \geq 0$
   $x_1 + x_2 \leq 7/2$
   where x_1 and x_2 are nonnegative integers

   You should use GAMS' integer programming capabilities in this problem.