

**MASSACHUSETTS INSTITUTE OF TECHNOLOGY**  
**Department of Civil and Environmental Engineering**

**1.731 Water Resource Systems**

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**Problem Set 1 – Linear Algebra Review**  
**Due: Tuesday, Sept. 19, 2006**

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Please work out these problems by hand and check with MATLAB for your own information (do not submit MATLAB programs or results). Make sure that you explain what you are doing and why.

1. Put the following matrix into echelon form by using elementary row operations (indicate the operations explicitly). What are the rank and determinant of this matrix?

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{bmatrix}$$

2. Determine the existence and uniqueness of the solutions of the following systems of equations by evaluating the ranks of the appropriate matrices. Also, give the solution (if there is one).

a) 
$$\begin{aligned} 2x + y &= 3 \\ x + 3y &= 1 \\ x - 2y &= 2 \end{aligned}$$

b) 
$$\begin{aligned} x + y + 2z &= 1 \\ 2x - y + z &= 2 \end{aligned}$$

3. Find all nontrivial solutions to the following homogeneous equations. Why are they unique or non-unique?

a) 
$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= 0 \\ 2x_1 + 5x_2 + 6x_3 &= 0 \end{aligned}$$

b) 
$$\begin{aligned} 2x_1 - x_2 + 3x_3 &= 0 \\ 3x_1 + 2x_2 + x_3 &= 0 \\ x_1 - 4x_2 + 5x_3 &= 0 \end{aligned}$$

4. Determine whether the following vectors are linearly dependent or linearly independent:

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1/2 \end{bmatrix} \quad \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ -1/2 \\ -1/2 \\ 1/4 \end{bmatrix}$$

5. Find the eigenvalues and eigenvectors of the following matrix:

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

6. Write the following quadratic form as a product of the form  $x_i A_{ij} x_j$ , where  $A_{ij}$  is a symmetric matrix. Compute the eigenvalues of  $A_{ij}$  and determine the definiteness of the quadratic form:

$$f(x_1, x_2, x_3) = x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$$