IV-2A TWO-DIMENSIONAL FLOW

(Objective: to know how to interpret flow nets)

Part IV - 2A TWO-DIMENSIONAL FLOW

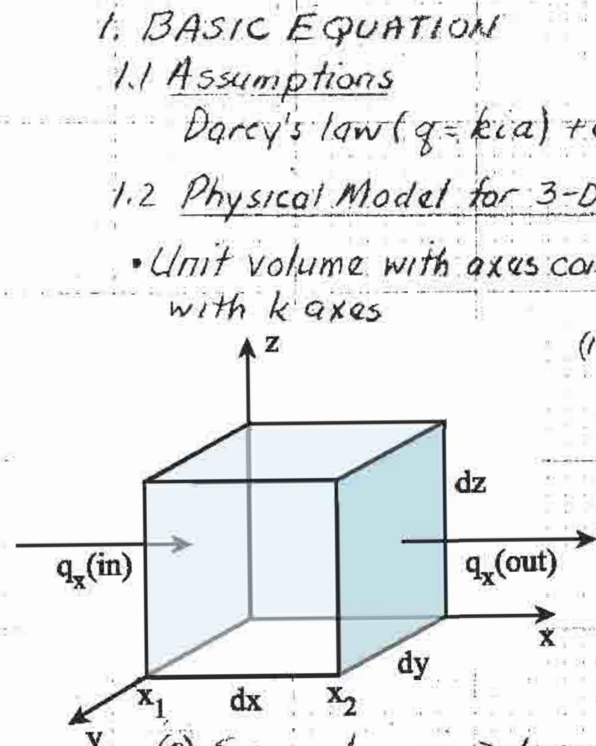
1. BASIC EQUATION

1.1 Assumptions

Darcy's law ($q = kia$) + constant k + incompressible fluid ($S = 100\%$)

1.2 Physical Model for 3-D Flow with Changing Volume

- Unit volume with axes coinciding with k axes
- Partial differentials since $h = f(x, y, z)$



(1) For flow in x direction ($a = dy\,dz$; h = total head)

$$q_x(in) = k_x \underbrace{\left(\frac{\partial h}{\partial x}\right)_{x_1}}^{i_x} a \qquad q_x(out) = k_x \left(\frac{\partial h}{\partial x}\right)_{x_2} a$$

$$\therefore \Delta q_x = k_x \underbrace{\left[\left(\frac{\partial h}{\partial x}\right)_{x_1} - \left(\frac{\partial h}{\partial x}\right)_{x_2}\right]}_{\text{Change in gradient over } dx} a$$

$$= k_x\left(\frac{\partial^2 h}{\partial x^2}\, dx\right) \underbrace{dy\,dz}_{\text{Unit volume}}$$

(2) Summation in 3 directions for unit volume

$$\underbrace{k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2}}_{\text{Net flow into element / unit time (or out of)}} = \underbrace{\frac{1}{1+e}\frac{\partial e}{\partial t}}_{\text{Volumetric change / unit time}} \quad \Big\}\ \text{3-D CONSOLIDATION}$$

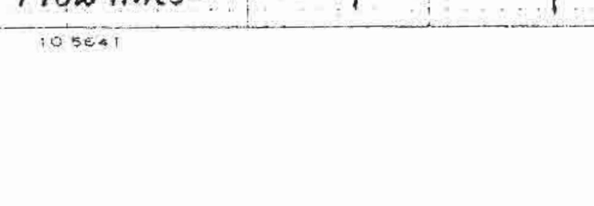
NOTE: For 1-D flow, $i_x = 0$ and $\partial^2 h/\partial z^2 = 0 \rightarrow i_z$ = constant $\therefore q_z = k_z i_z a$

1.3 Solution for 2-D Flow & No Volume Change ($\partial e/\partial t = 0$)

(1) $$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \longrightarrow \underbrace{\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2}}\ = 0 \quad \text{for isotropic soil } (k_x = k_z)$$

Change in i_x gradient per unit distance in x direction
+ " " i_z " " " " " z " = zero

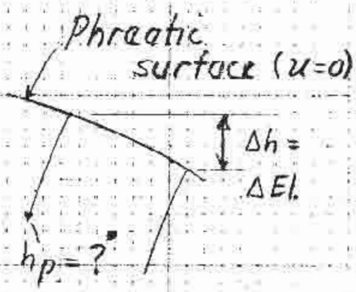
(2) LaPlace Eq. $\nabla^2 h = 0$ (also applicable to heat flow, etc.) wherein solution → series of lines intersecting at 90° ∡:

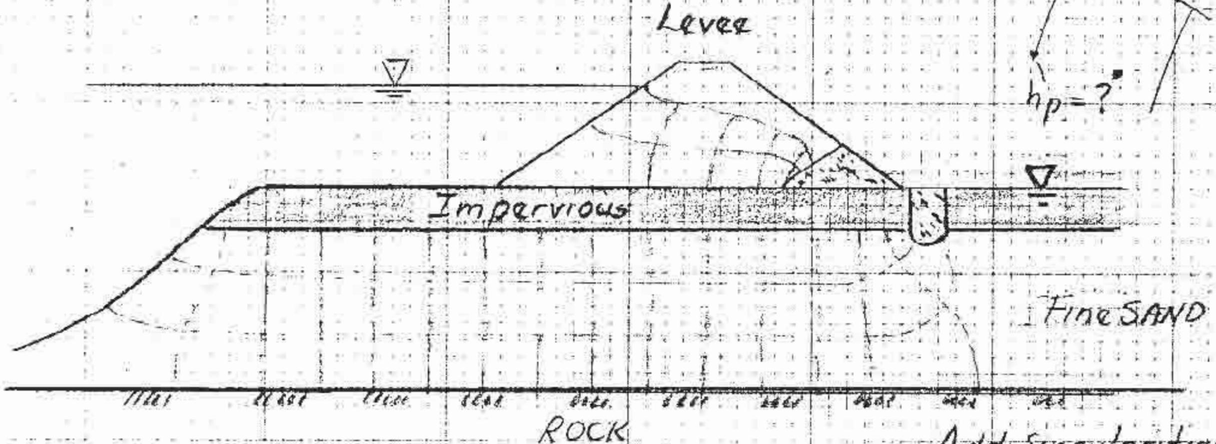


- Ratio b/ℓ = constant
- For convenience select $b/\ell = 1 \rightarrow$ "squares"
- $h_1 > h_2 > h_3$
- Same flow & Δh in each rectangle

2. DRAWING FLOW NETS

2.1 Problem ($k_h = k_v$)





Add Singularities

2.2 Steps in Drawing Flow Net (If computer program not available)

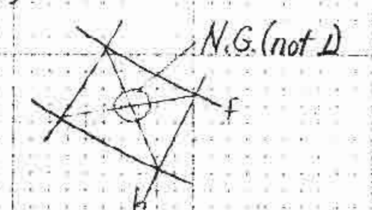
(1) Draw problem in ink

(2) Draw in known equipotential & flow boundary lines

(3) Sketch in 2 or 3 flow lines (experience helps!)

(4) Draw corresponding equipotential lines
- Check for "squares" & ⊥ intersections

(5) Keep adjusting (and adjusting ......)



2.3 Eq. For Flow Per Unit Length

- $q\left(\frac{\ell^3/t}{\ell}\right) = k\,h\left(\frac{n_f}{n_d}\right)\left(\frac{b}{\ell}\right) = k\,h\,(\text{shape factor } \$)$ $\quad \$$ independent of k & h

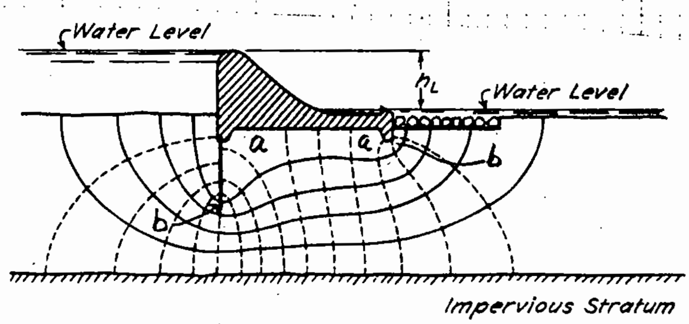
where n_f = no. of flow paths & n_d = no. of equipotential drops

- Even sloppy flow nets (such as above) → reasonable $ and flow estimates given uncertainty in k values.
- BUT sloppy nets can → large errors in $i = \frac{\Delta h}{\Delta \ell}$ at critical locations

$q_i(\text{1 flow path}) = k\,i\,a = k\frac{\Delta h}{\ell}b = k\frac{h}{n_d}\left(\frac{b}{\ell}\right) \rightarrow q = q_i \cdot n_f$ = eq. in 2.3

3. MISCELLANEOUS

3.1 Singularities



(1) When angle between flow & equipotential lines (on side containing the flow)

a $< 90^\circ \rightarrow i = 0$ (5-sided)

b $> 90^\circ \rightarrow i = \infty$ (3-sided)

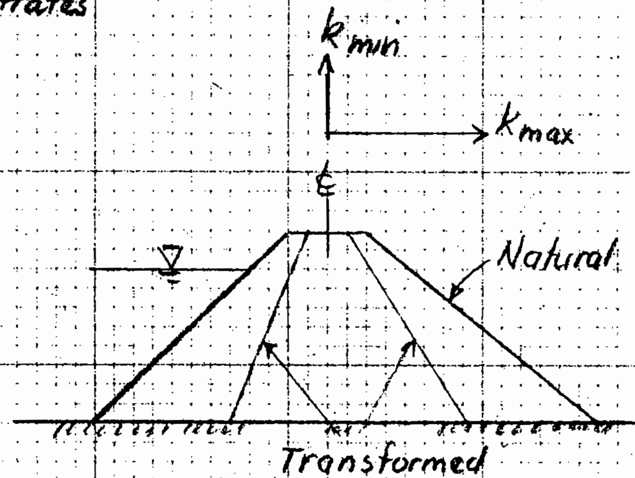
(2) When $i \rightarrow \infty$ near surface, worry about PIPING & QUICK condition

(3) Identify singularities Fig 2.1

3.2. Non-Homogeneous Soils

- See Sheet A for example that illustrates constant $ for varying n_f/n_d & b/l

3.3. Anisotropic Soil ($k_x \neq k_y$)



(1) Transformed section:

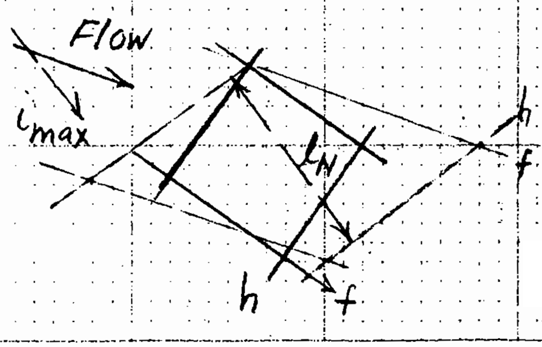
- Decrease l in direction of k_{max} by $\sqrt{k_{min}/k_{max}}$ OR
- Increase l in direction of k_{min} by $\sqrt{k_{max}/k_{min}}$

(2) Draw flow net → $

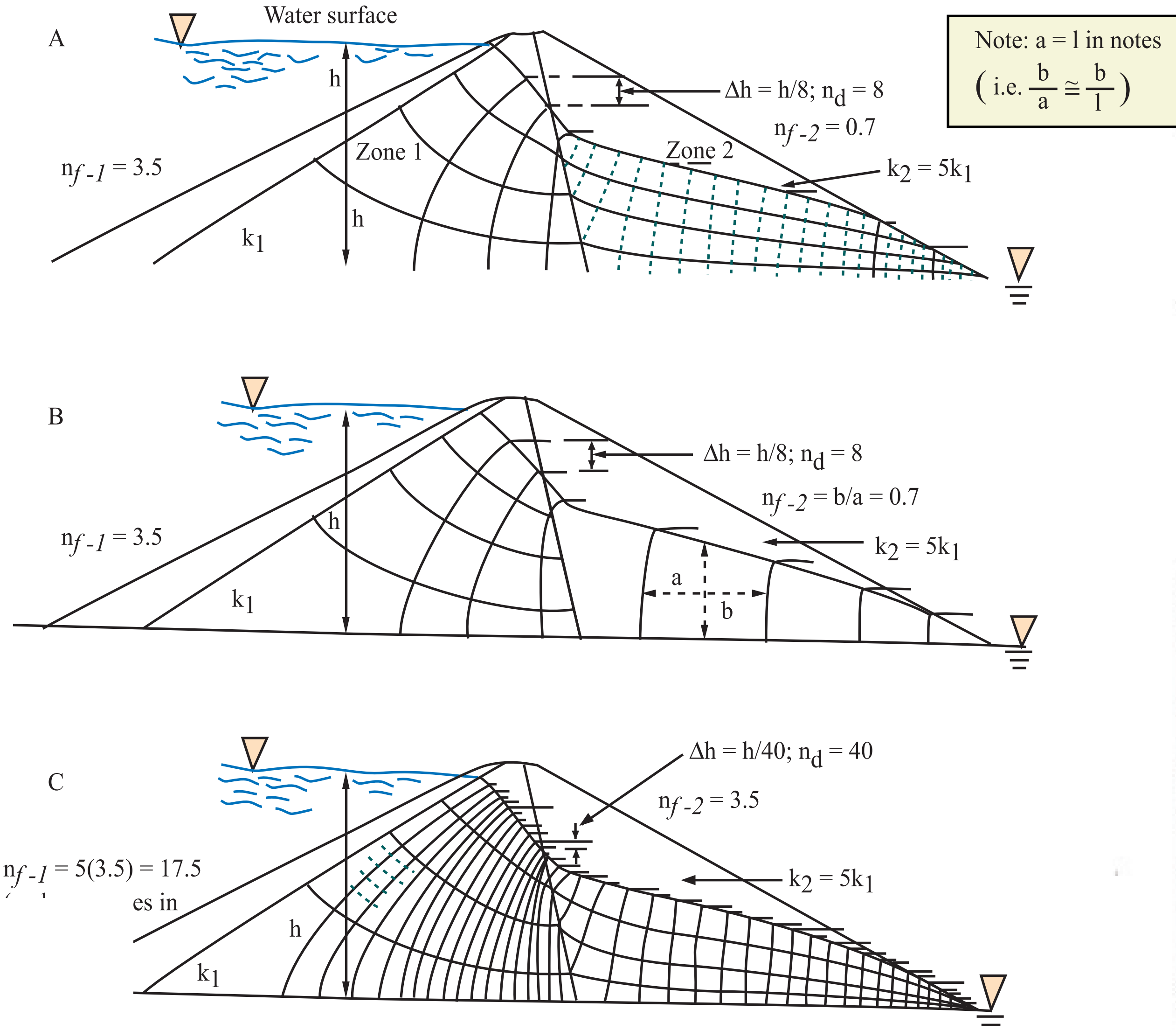
where $k_e = \sqrt{k_{min}\, k_{max}}$

$$q = \underset{\text{REAL}}{k_e\, h} \underbrace{\left(\frac{n_f}{n_d}\right)\left(\frac{b}{l}\right)}_{\text{transformed}}$$

(3) To obtain i from transformed section ($k_h \neq k_v$), need → natural



$$i = \frac{\Delta h}{l_N}$$ = shortest distance between equipotential lines



Three forms of one flow net.

4.6(b) Zone 1: $q_1 = k_1 \frac{h}{8}(3.5)(1) = 0.44 k_1 h$

Zone 2: $q_2 = k_2 \frac{h}{8}(0.7)(1) = 0.088 k_2 h = 0.088(5k_1)h = 0.44 k_1 h$

solid lines

Fig 4.6(a) Zone 2: $q_2 = k_2 \frac{h}{8}(3.5)(1/5) = 0.088 k_2 h = 0.44 k_1 h$

Fig. 4.6(c) Zone 1: $q_1 = k_1 \frac{h}{40}(3.5)(5/1) = 0.44 k_1 h$

Zone 2: $q_2 = k_2 \frac{h}{40}(3.5)(1) = 0.088 k_2 h = 0.44 k_1 h$

Example of Applying $q = kh\left(\frac{n_f}{n_d}\right)\left(\frac{b}{l}\right)$ to Seepage in Non-Homogeneous Soils

A