

# Vehicle Scheduling

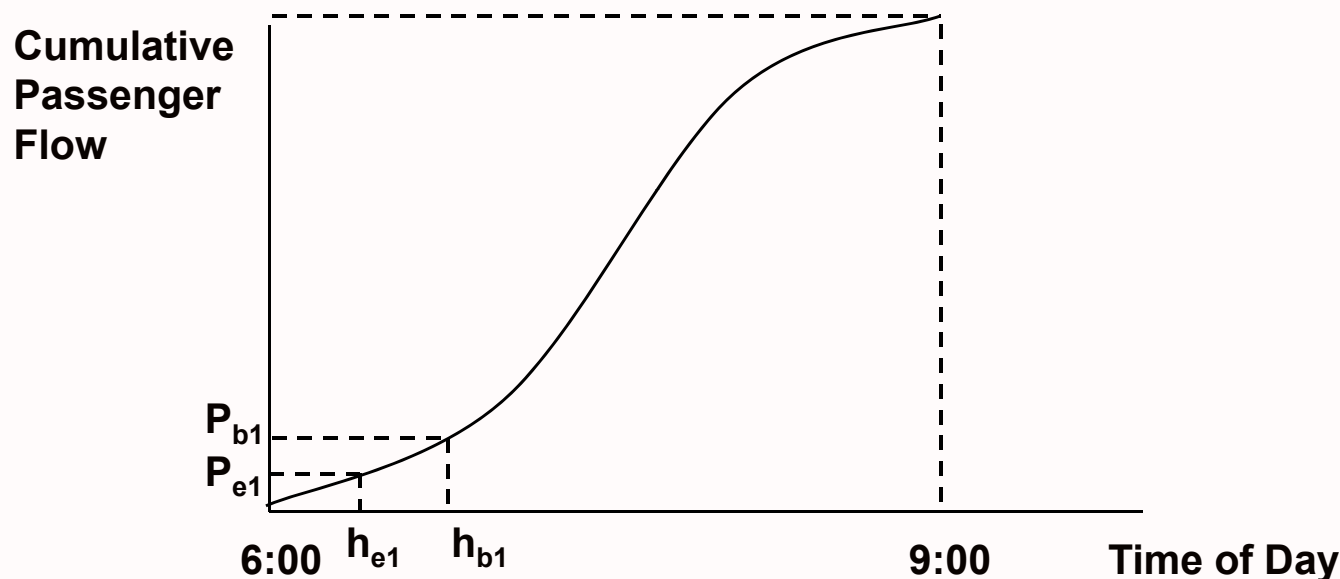
## Outline

1. **Timetable Development**
2. **Fleet Size**
3. **Vehicle Scheduling**

# Timetable Development

Can translate frequency into timetable by specifying headways as:

- equal -- appropriate if demand is uniformly distributed across period
- balanced load -- appropriate if there is substantial variation in demand over period
- clockface or not -- do headways repeat every hour



# Timetable Development

If we have  $N$  departures in peak period:

- equal headway solution: 
$$H = \frac{\text{Peak Period}}{N}$$
- balanced load solution:

$$\text{Pass Load / Departure} = \frac{\text{Total Passenger Flow}}{N}$$

# Fleet Size Requirement

## Salzborn's Fleet Size Theorem:

Given:

$l(k,t,s)$  = # of departures from terminal  $k$  by time  $t$  following schedule  $s$

$a(k,t,s)$  = # of arrivals at terminal  $k$  by time  $t$  following schedule  $s$

and:

$d(k,t,s)$  =  $l(k,t,s) - a(k,t,s)$ , deficit function at terminal  $k$  at time  $t$  following schedule  $s$

# Fleet Size Requirement

## Salzborn's Fleet Size Theorem:

Then:

**$N(s)$ , the minimum size fleet to serve schedule  $s$ , is given by:**

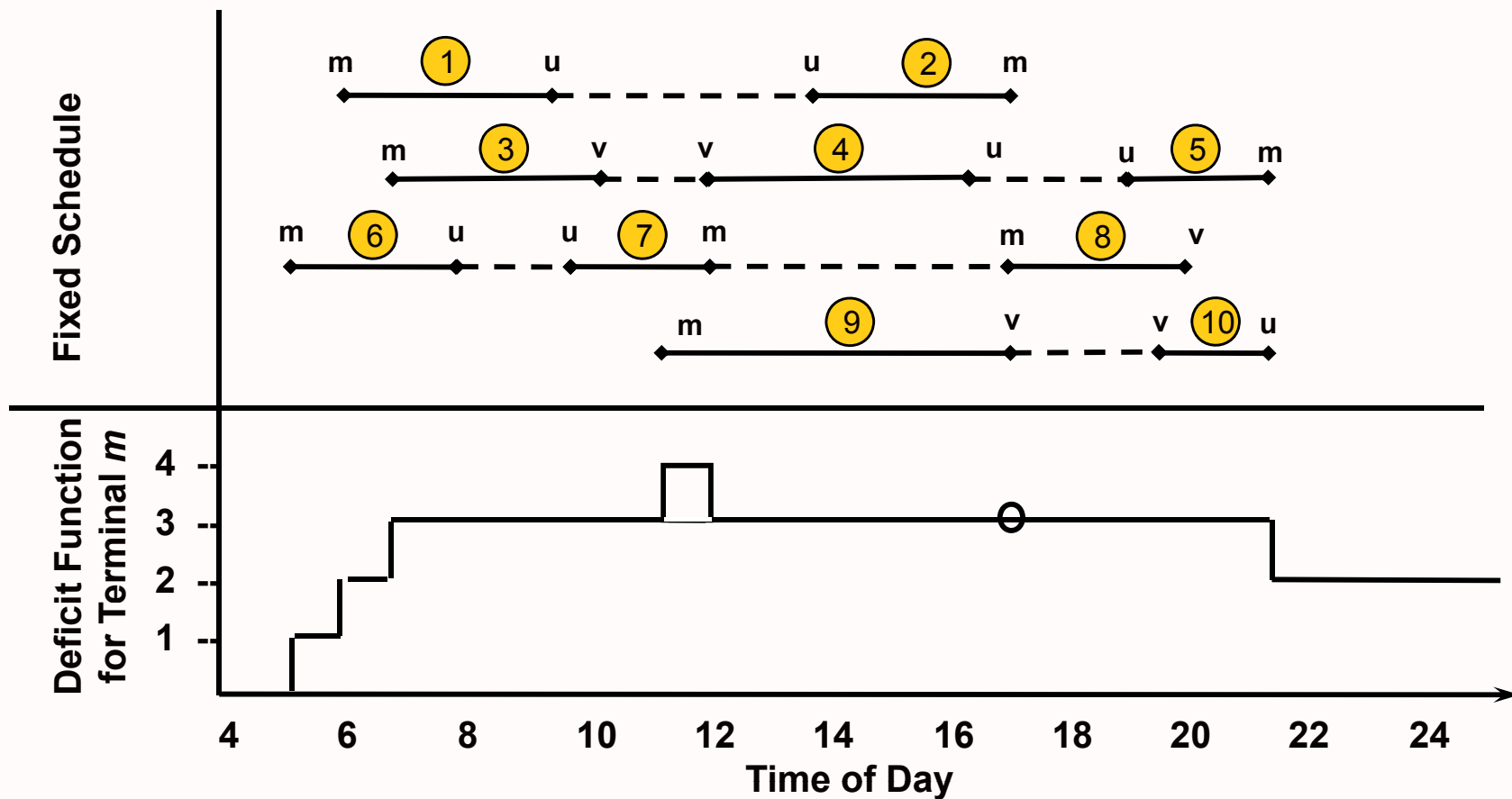
$$N(s) = \sum_{k \in T} \max_t(d(k, t, s))$$

**Also,  $N(s) \geq$  Max # of trips in simultaneous operation.**

# Fleet Size Required

The deficit function, or minimum required fleet size, may be reduced by:

- shifting departure and/or arrival times
- adding deadhead trips between terminals



# Vehicle Scheduling Problem

## Input:

- A set of vehicle revenue trips to be operated, each characterized by:
  - starting point and time
  - ending point and time
- Possible layover arcs between the end of a trip and the start of a (later) trip at the same location
- Possible deadhead arcs connecting:
  - depot(s) to trip starting points
  - trip ending points to depot(s)
  - trip ending points to trips starting at a different point

# Vehicle Scheduling Problem

## Observations:

- there are many feasible but unattractive deadhead and layover arcs, generate only plausible non-revenue arcs
- layover time affects service reliability, set minimum layover (recovery) time



# Vehicle Scheduling Problem (continued)

## Objective:

- Define vehicle blocks (sequences of revenue and non-revenue activities for each vehicle) covering all trips so as to:
  - minimize fleet size (i.e. minimize #crews)
  - minimize non-revenue time (i.e. minimize extra crew time)

## Observation:

- These are proxies for cost, but a large portion of cost will depend on crew duties which are unknown at this stage of solution.

# Vehicle Scheduling Problem (continued)

## Constraints:

- Minimum vehicle block length
- Maximum vehicle block length

## Variations:

- each vehicle restricted to a single line vs. interlining permitted
- single depot vs multi-depot
- vehicle fleet size constrained at depot level
- routes (trips) assigned to specific depot
- multiple vehicle types

# Example: Single Route AB



## Results of earlier planning and scheduling analysis:

	AM Peak Period (6-9 AM)	Base Period (after 9 AM)
Headways	20 min	30 min
Scheduled trip time (A⇒B or B⇒A)	40 min	35 min
Minimum layover time	10 min	10 min

Dominant direction of travel in AM is A⇒B

# Timetable and Vehicle Block Development

Depart A	Arrive B
6:00	6:40
6:20	7:00
6:40	7:20
7:00	7:40
7:20	8:00
7:40	8:20
8:00	8:40
8:20	9:00
8:40	9:20
9:00	9:35
9:30	10:05
10:00	10:35
10:30	11:05
11:00	11:35

# Timetable and Vehicle Block Development

<b>Depart A</b>	<b>Arrive B</b>	<b>Depart B</b>	<b>Arrive A</b>
6:00	6:40	6:50	7:30
6:20	7:00	7:10	7:50
6:40	7:20	7:30	8:10
7:00	7:40	7:50	8:30
7:20	8:00	8:10	8:50
7:40	8:20	8:30	9:10
8:00	8:40	8:50	9:30
8:20	9:00	9:15	9:50
8:40	9:20		
9:00	9:35	9:45	10:20
9:30	10:05	10:15	10:50
10:00	10:35	10:45	11:20
10:30	11:05	11:15	11:50
11:00	11:35	11:45	12:20

# Timetable and Vehicle Block Development

Veh #	Depart A	Arrive B	Depart B	Arrive A
1 x	>6:00	6:40	6:50	7:30---->
2x	6:20	7:00	7:10	7:50
3x	6:40	7:20	7:30	8:10
4x	7:00	7:40	7:50	8:30
5x	7:20	8:00	8:10	8:50
1	7:40	8:20	8:30	9:10
2	8:00	8:40	8:50	9:30-->y
3	8:20	9:00	9:15	9:50
4	8:40	9:20 -->y		
5	9:00	9:35	9:45	10:20
1	9:30	10:05	10:15	10:50
3	10:00	10:35	10:45	11:20
5	10:30	11:05	11:15	11:50
1	11:00	11:35	11:45	12:20

x = from depot

# Example: Vehicle Blocks

**Block 1: Depot - A (6:00) - B (6:50) - A (7:40) - B (8:30) - A (9:30) - B (10:15) - A (11:00) - B (11:45) - ...**

**Block 2: Depot - A (6:20) - B (7:10) - A (8:00) - B (8:50) - Depot**

**Block 3: Depot - A (6:40) - B (7:30) - A (8:20) - B (9:15) - A (10:00) - B (10:45) - ...**

**Block 4: Depot - A (7:00) - B (7:50) - A (8:40) - Depot**

**Block 5: Depot - A (7:20) - B (8:10) - A (9:00) - B (9:45) - A (10:30) - B (11:15) - ...**

# Vehicle Scheduling Model Approaches

## Heuristic approaches:

1. Define compatible trips at same terminal  $k$  such that trips  $i$  and  $j$  are compatible iff :

$$t_{sj} - t_{ei} > M_k$$

$$t_{sj} - t_{ei} < 2 D_k$$

where

$t_{sj}$  = starting time for trip  $j$

$t_{ei}$  = ending time for trip  $i$

$M_k$  = minimum recovery/layover time at terminal  $k$

$D_k$  = deadhead time from terminal  $k$  to depot

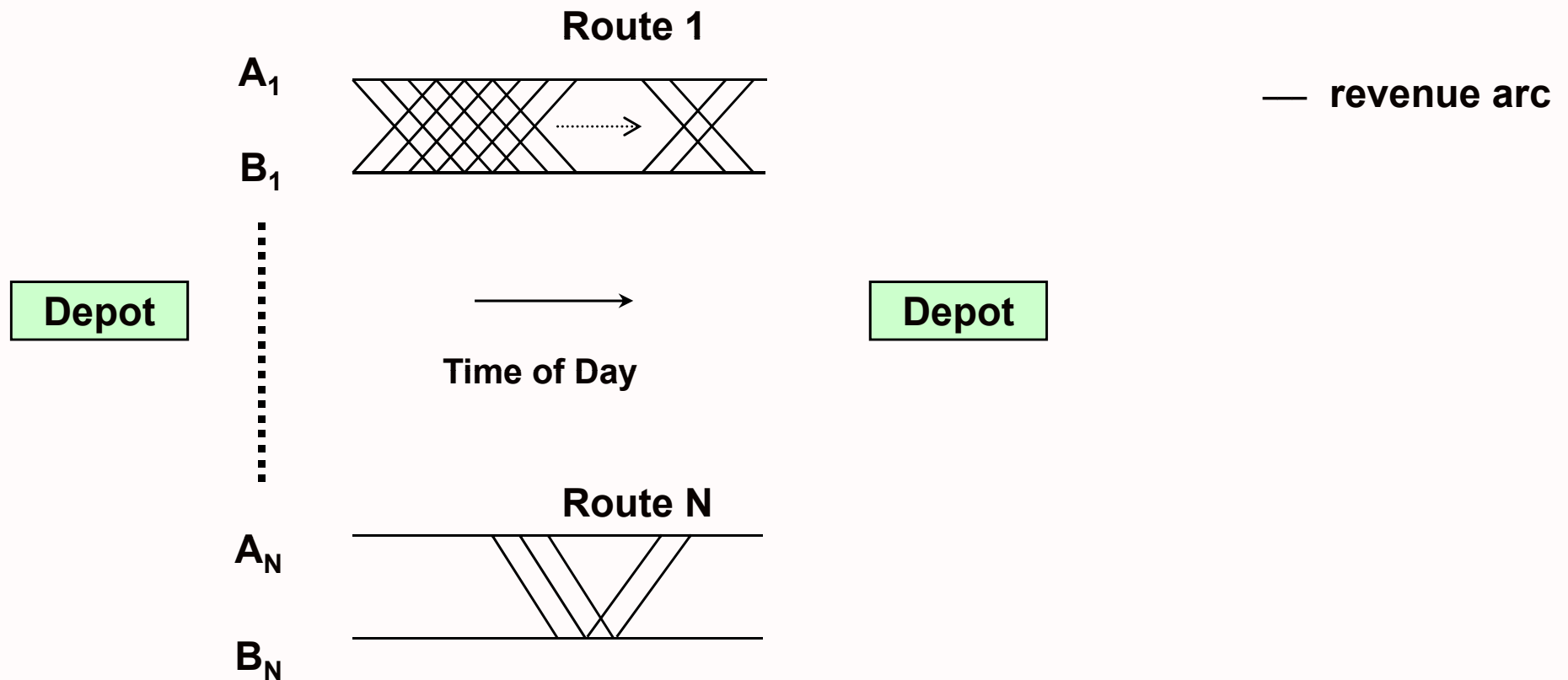


# Vehicle Scheduling Model Approaches

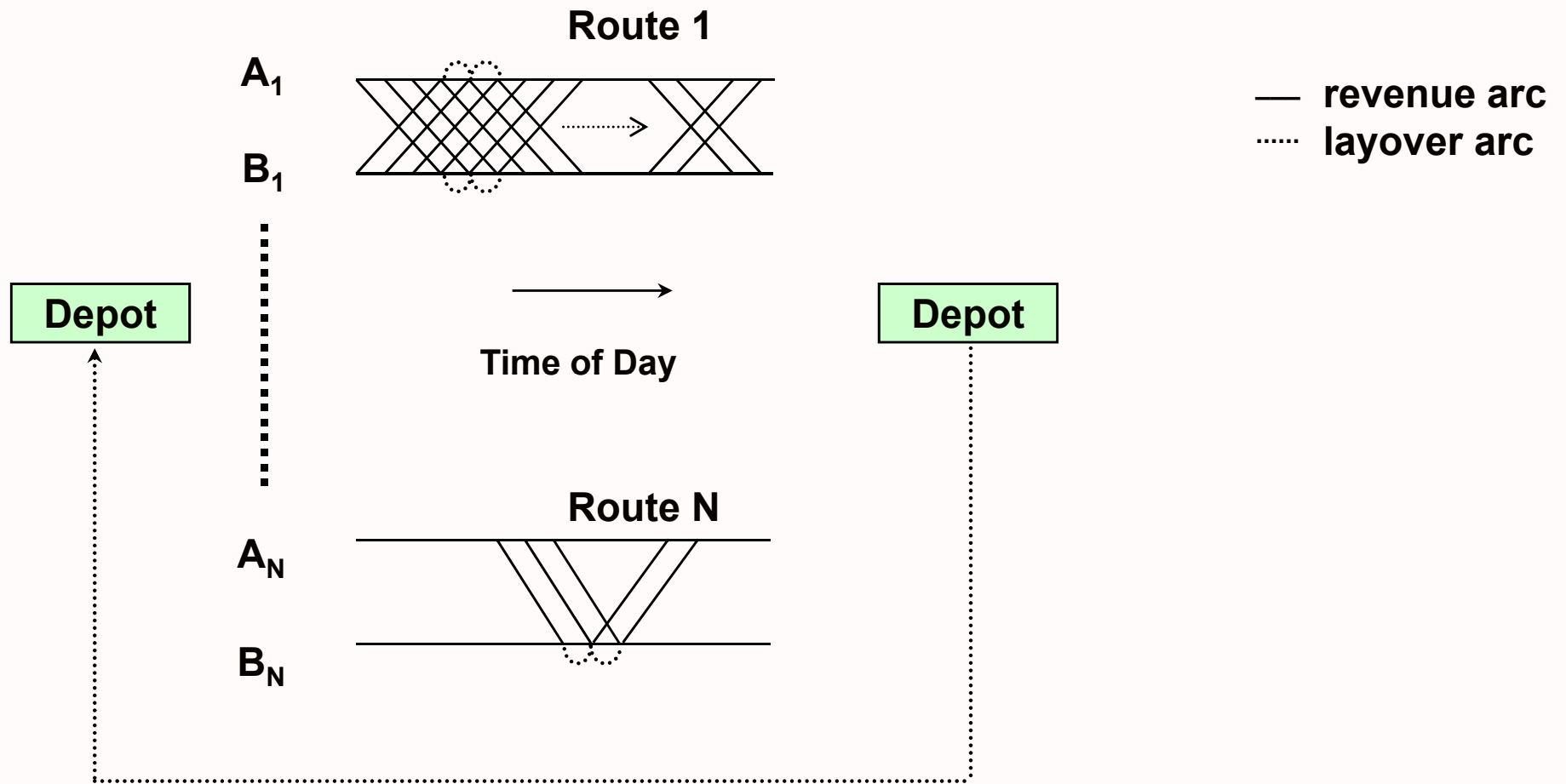
## 2. Apply Restricted First-in-First-out rules at each terminal

- a) Start with (next) earliest arrival at terminal; if none, go to step (d)
- b) Link to earliest compatible trip<sub>de</sub> parture; if none, return vehicle to depot and return to step (a)
- c) Check vehicle block length against constraint: if constraining, return vehicle to depot and return to step (a); otherwise return to step (b) with new trip arrival time
- d) Serve all remaining unlinked departures from depot

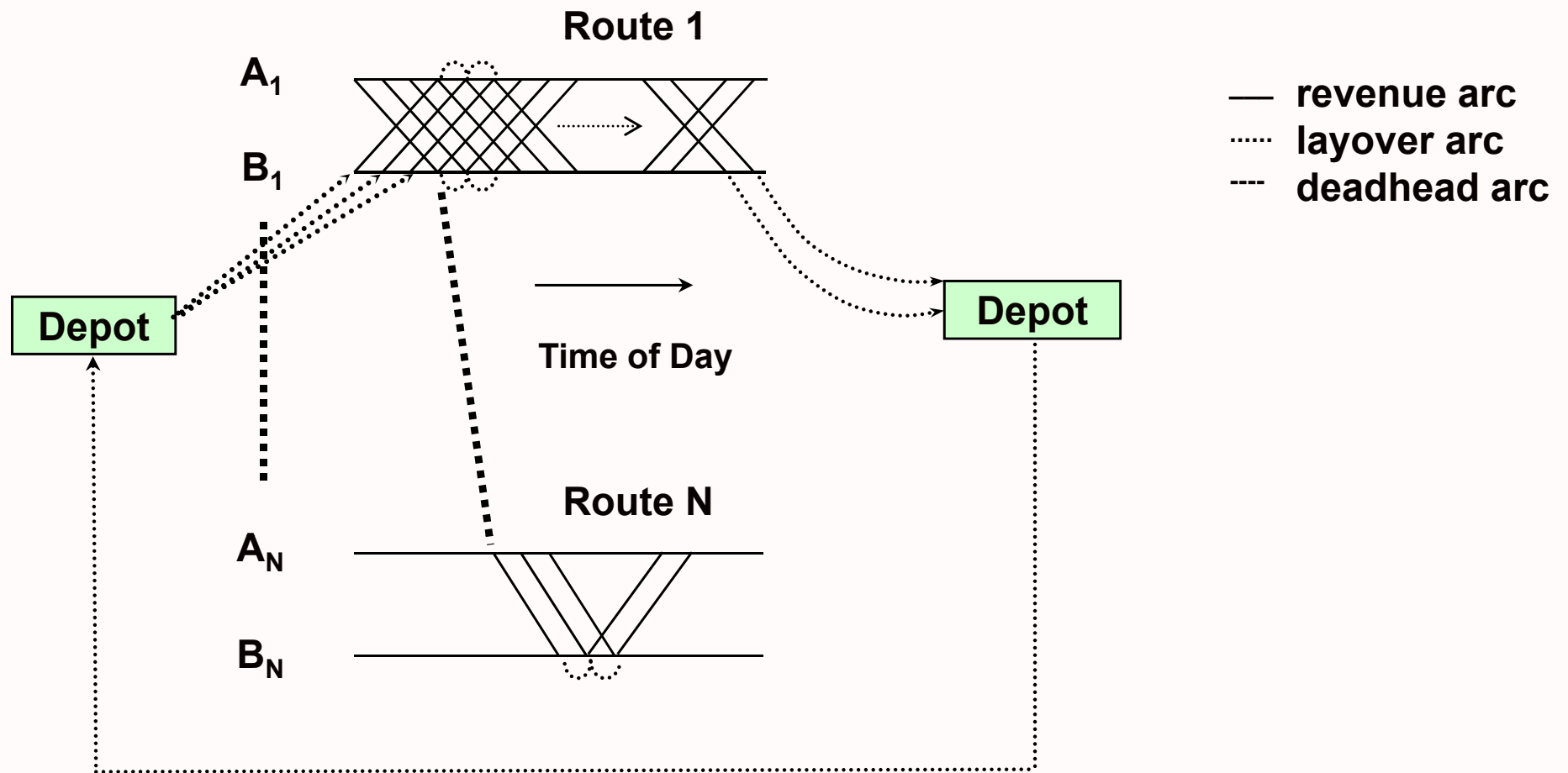
# Time-Space Network Representation



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