LECTURE #31

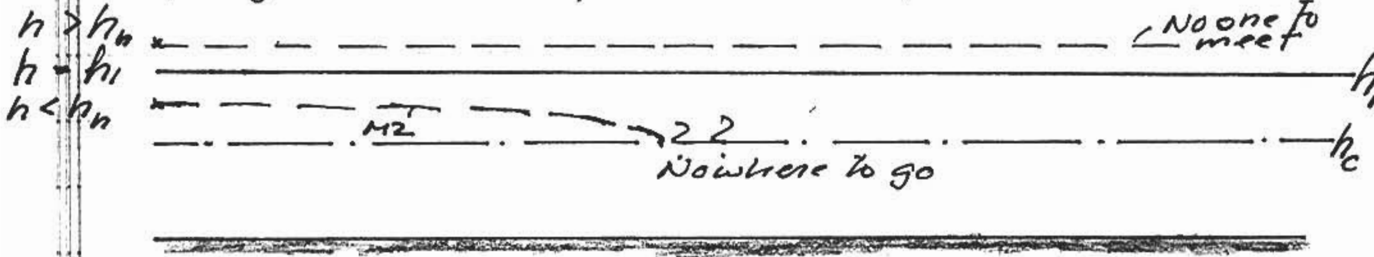
1.060 ENGINEERING MECHANICS II

EXAMPLES OF THE APPLICATION OF GRADUALLY VARIED FLOW PROFILES

Ex 1. Entrance Condition for Mild Slopes

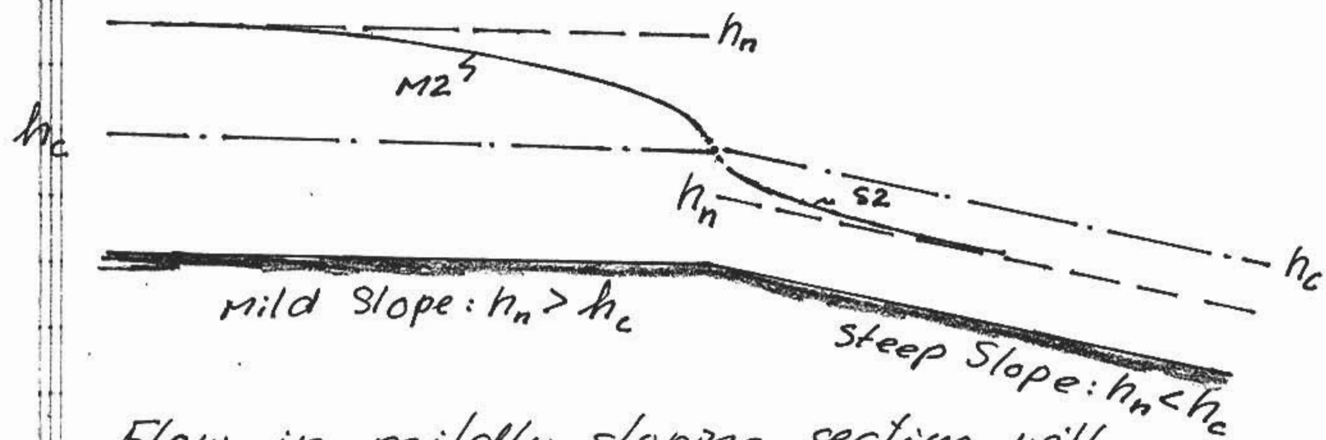
In the absence of a downstream control a subcritical flow entering a mildly sloping channel must do so at normal depth.

Proof: If the flow entered the channel at a depth $h > h_n$ it would have to continue along an M1-Profile - this would be impossible unless some downstream control were present. If the flow entered the channel at a depth $h < h_n$ it would have to follow a M2-Profile, which would bring the flow to critical depth and this flow could not continue down the channel since only normal flow can exist in a uniformly sloping channel for an "infinite" distance.



Ex. 2. <u>Occurance of Critical Flow</u>

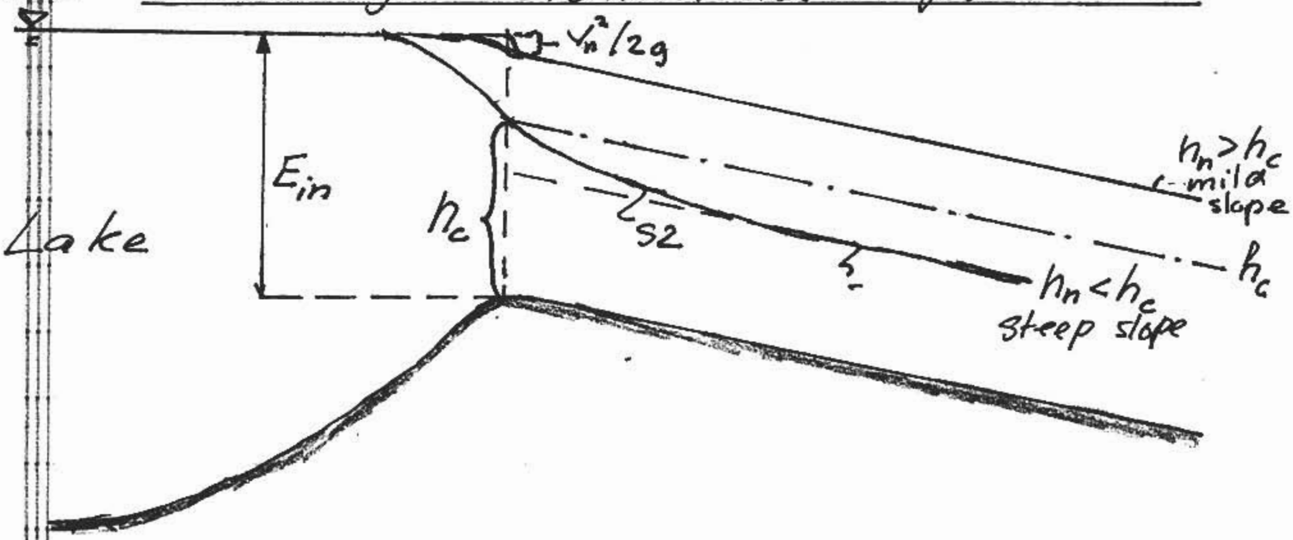
If the slope of a channel changes from Mild to Steep, the flow <u>must</u> pass through critical flow at the location of the change in slope.



Flow in mildly sloping section will be drawn down to h_c at the change in slope. Upon entering the steep channel it starts at h_c and proceeds toward h_n through a S2-profile. If the M2-profile did not hit h_c at entrance to the steep channel the flow would follow an S1-profile which would be impossible in the absence of a downstream control.

Computation would start at the change in slope where $h = h_c$; proceed upstream in the subcritical flow ($h > h_c$), and downstream in the supercritical ($h < h_c$) channels!

Ex. 3 <u>Discharge Determination from Lake</u>



a) <u>If channel slope is steep</u> this is a special case of transition from sub to supercritical flow (example 2). So flow must pass through critical at entrance and follow an S2-profile until reaching normal depth.

The discharge from the lake to the channel is obtained from

$$E_{in} = h_c + \frac{Q^2}{2g A_c^2} \quad \& \quad Fr_{in}^2 = \frac{Q^2 b_s}{g A_c^3} = 1$$

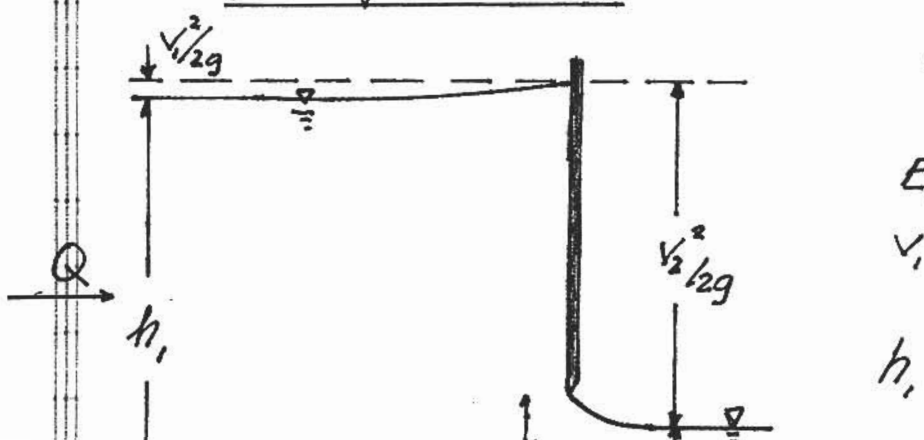
With this Q it remains to be shown that normal flow in the channel is supercritical, i.e. $h_n < h_c$, since this was assumed to be the case initially. If $h_n < h_c$ then channel is indeed "Steep" for the given Q and this <u>is</u> the discharge from the lake to the river.

b) <u>If channel slope is mild</u>, suggested by a low value $S_o < \sim 10^{-3}$, or obtained after first assuming it to be steep and finding this to be in error, then the outflow from lake to channel is a special case of example 1, i.e. inflow from lake must be at normal depth right from the start. For this case we have

$$E_{in} = h_n + \frac{Q^2}{2g A_n^2} \quad \& \quad Q = \frac{1}{n}\,\frac{A_n^{5/3}}{P_n^{2/3}}\sqrt{S_o}$$

which may be solved for h_n = normal depth and Q = discharge from lake to channel

Ex. 4. Discharge and Flow Characteristics of Underflow Gates



Free outflow

$$E_1 = E_2$$

$$V_1 h_1 = V_2 h_2 = Q$$

$$h_1 + \frac{(Q/b)^2}{2g h_1^2} = h_2 + \frac{(Q/b)^2}{2g h_2^2}$$

Q known $\Rightarrow$

h_1 & h_2 are ALTERNATE DEPTHS

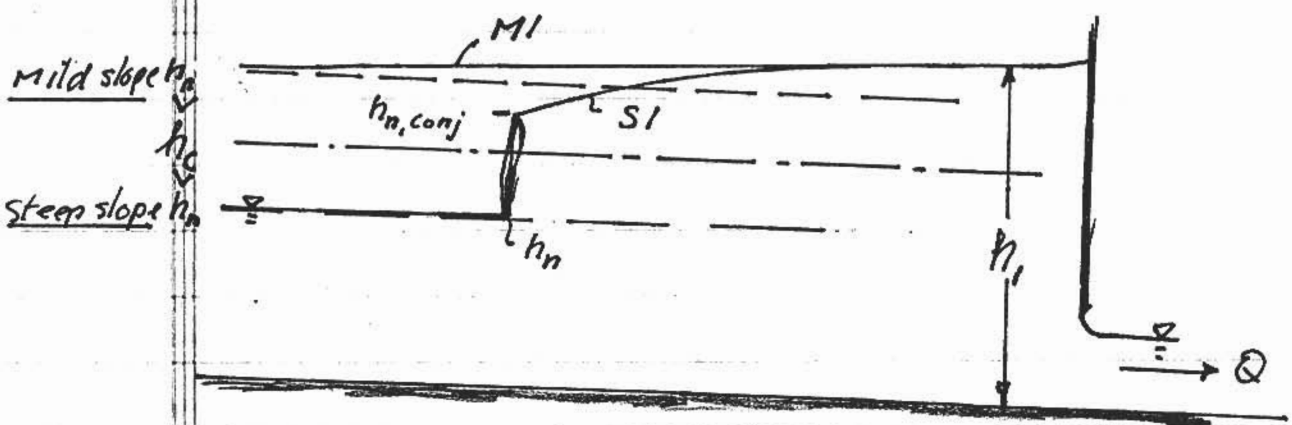
Upstream of gate $h = h_1$ is $> h_c$: Subcritical Flow

Downstream of gate $h = h_2$ is $< h_c$: Supercritical Flow

a) *Upstream Flow Characteristics*

If upstream channel's slope is mild, then h_1 upstream of gate is reached through an M1-profile (backwater curve).

If upstream channel's slope is steep, the uniform supercritical flow proceeds until a location where the normal depth, $h_n < h_c$, is conjugate to the backwater curve - the S1-profile followed by the flow upstream of the gate starting from h_1 and decreasing in the upstream direction.



Jump condition: $MP_{in} = MP_{out}$

$$\left(\rho \frac{Q^2}{A_n^2} + p_{CG,n}\right) A_n = \left(\rho \frac{Q^2}{A_{conj}^2} + p_{CG,conj}\right) A_{conj}$$

normal flow supercritical: $h = h_n$ — flow after jump is subcritical: $h = h_{n,conj}$

From h_n the conjugate depth is obtained as $h_{n,conj}$. An S1-profile goes upstream from $h = h_1$ at the gate to meet $h_{n,conj}$ at the downstream end of the jump.

b) Downstream Flow Characteristics

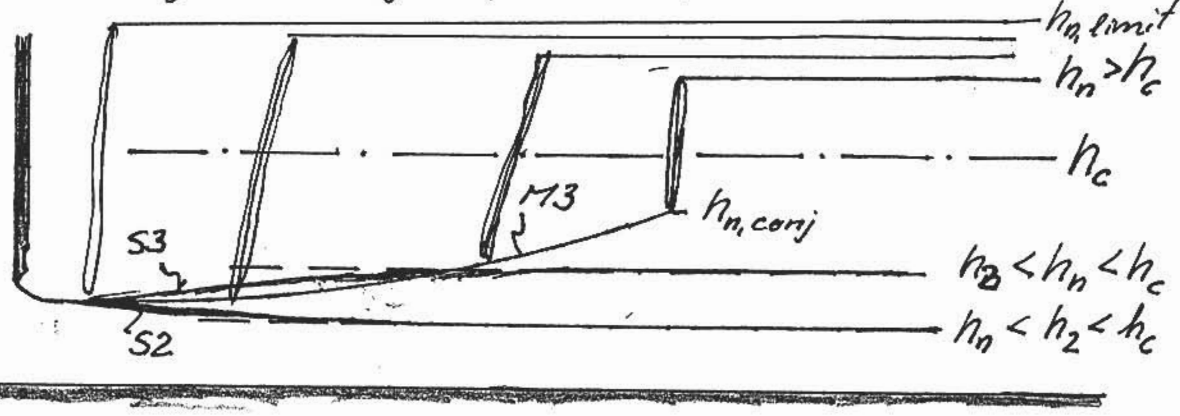
If the channel slope downstream of the gate is steep then the flow will proceed from $h = h_2$ at the exit from under the gate (it is guaranteed to be supercritical!) following an S2 or S3-profile until h_n is reached.

If the channel slope downstream of the gate is mild, then a jump forms bringing the flow from supercritical to subcritical, where the subcritical depth is $h_n > h_c$ normal depth in the channel [must be so since there is no downstream control - see example 1]. The jump condition is

$$MP_{in} = MP_{out} = MP_n$$

$$\underbrace{\left(\rho\, Q^2/A^2_{conj} + p_{CG,conj}\right) A_{conj}}_{\text{supercritical flow } h = h_{n,conj}} = \underbrace{\left(\rho \frac{Q^2}{A_n^2} + p_{CG,n}\right) A_n}_{\text{subcritical normal flow } h = h_n}$$

The flow from vena contracta after the gate, $h = h_2$, follows an M3-profile until it reaches $h_{n,conj}$ where a hydraulic jump to h_n takes place.

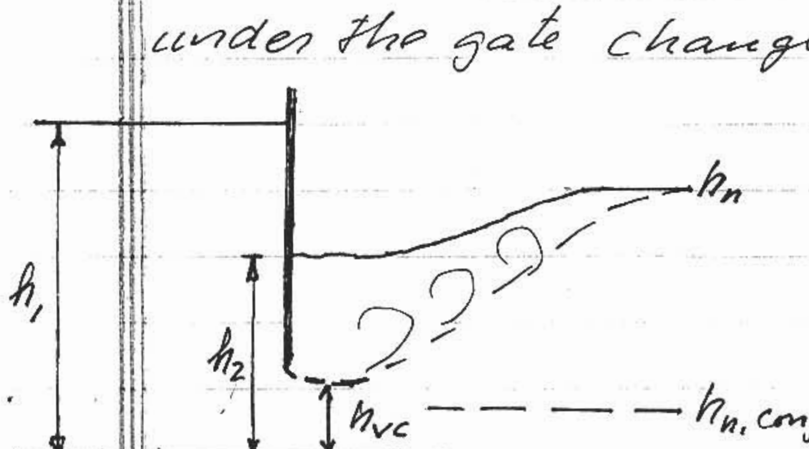


The smaller the downstream slope, the larger the normal depth h_n for the given Q [corresponding to a free outflow under the gate]. But the larger h_n is the smaller $h_{n,conjugate}$, which is the depth the M3-curve from vena contracta must reach in order for the jump to form. Thus, as h_n increases and $h_{n,conjugate}$ decreases [it will still be located on the same M3-curve since Q is the same] the location of the downstream jump moves towards the gate. The limiting case is reached when

$$h_{n,conjugate} = h_{vc} = \text{depth at vena contracta.}$$

Any further increase in h_n would make $h_{n,conjugate} < h_{vc}$ and there is no way for the flow exiting from under the gate to get <u>down</u> to this depth. Result is that the outflow from under the gate no longer is free, i.e. into the atmosphere. The outflow becomes "drowned" and the discharge under the gate changes - or if the discharge is to remain constant, the depth upstream of the gate, h_1, must increase. For this type of problem consult Test #2 and its solutions.

or see next page



Drowned outflow equations: [Rectangular Channel]

CONTINUITY: $V_1 h_1 = V_{vc} h_{vc} = V_n h_n = \frac{Q}{b}$

MOMENTUM: $\rho V_{vc}^2 h_{vc} + \frac{1}{2}\rho g h_2^2 = \rho V_n^2 h_n + \frac{1}{2}\rho g h_n^2 \Rightarrow$ gives h_2

ENERGY: $h_1 + \frac{V_1^2}{2g} = h_2 + \frac{V_{vc}^2}{2g} \Rightarrow$ gives h_1

Downstream depth h_n is obtained from

$$\frac{Q}{b} = h_n \frac{1}{n} R_{hn}^{2/3} \sqrt{S_0} \qquad \left[R_{hn} = \frac{h_n b}{b + 2h_n} = \frac{h_n}{1 + \frac{2h_n}{b}}\right]$$