LECTURE # 10

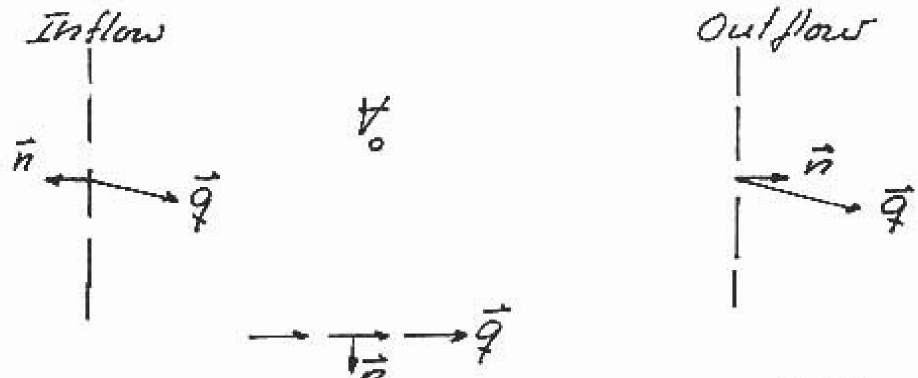
1.060 ENGINEERING MECHANICS II

REYNOLDS TRANSPORT THEOREM

$\frac{DM}{Dt}$ = Rate of change of M within $\forall(t)$ =

$$\frac{\partial}{\partial t}\int_{\forall_o} m\, d\forall - \int_{A_{in}} m q_{\perp} dA + \int_{A_{out}} m q_{\perp} dA =$$

Rate of change of M between fixed inflow & outflow Sections − Rate of inflow of M + Rate of outflow of M



$q_{\perp} = -\vec{n}\cdot\vec{q}$ $\quad$ $q_{\perp} = \vec{n}\cdot\vec{q} = 0$ $\quad$ $q_{\perp} = \vec{n}\cdot\vec{q}$

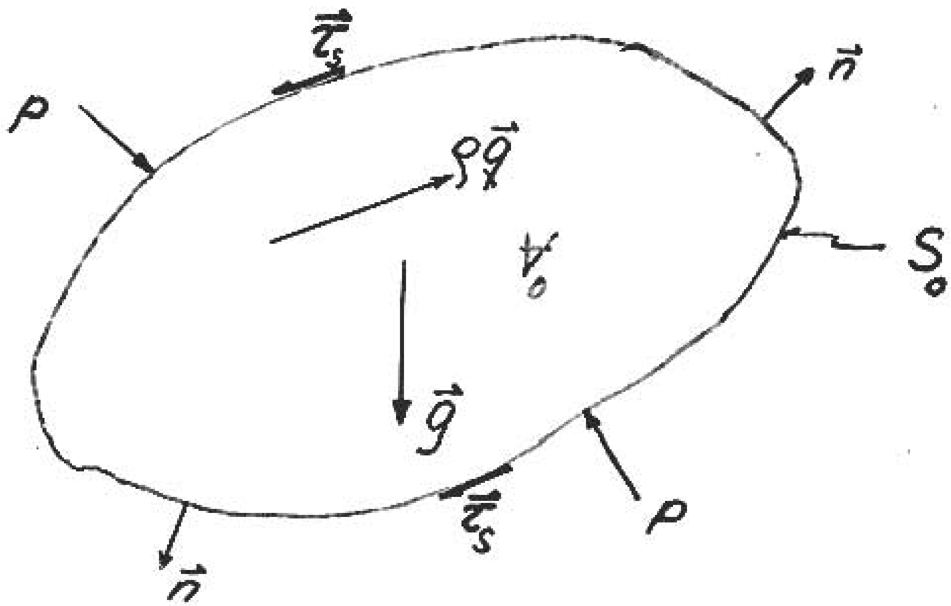
CONSERVATION OF (LINEAR) MOMENTUM: $m = \vec{m} = \rho\vec{q}$

$$\frac{D\vec{M}}{Dt} = \frac{\partial}{\partial t}\int_{\forall_o} \rho\vec{q}\, d\forall + \int_{S_o} \rho\vec{q}\,(\vec{n}\cdot\vec{q})\, dS = \sum \text{Forces on } \forall_o =$$

$$\int_{\forall_o} \rho\vec{g}\, d\forall + \int_{S_o} \left(-p\vec{n} + \vec{\tau}_s\right) dS =$$

Gravity Force on Fluid in $\forall_o$ + Pressure & <u>Shear Forces</u> ($\vec{\tau}_s$) on $\forall_o$ from surrounding fluid and/or boundaries.

CONSERVATION OF MOMENTUM



$$\frac{D}{Dt}\left\{\iint_{\forall(t)} \rho\vec{q}\,d\forall\right\} = \text{Rate of change of momentum for material volume} =$$

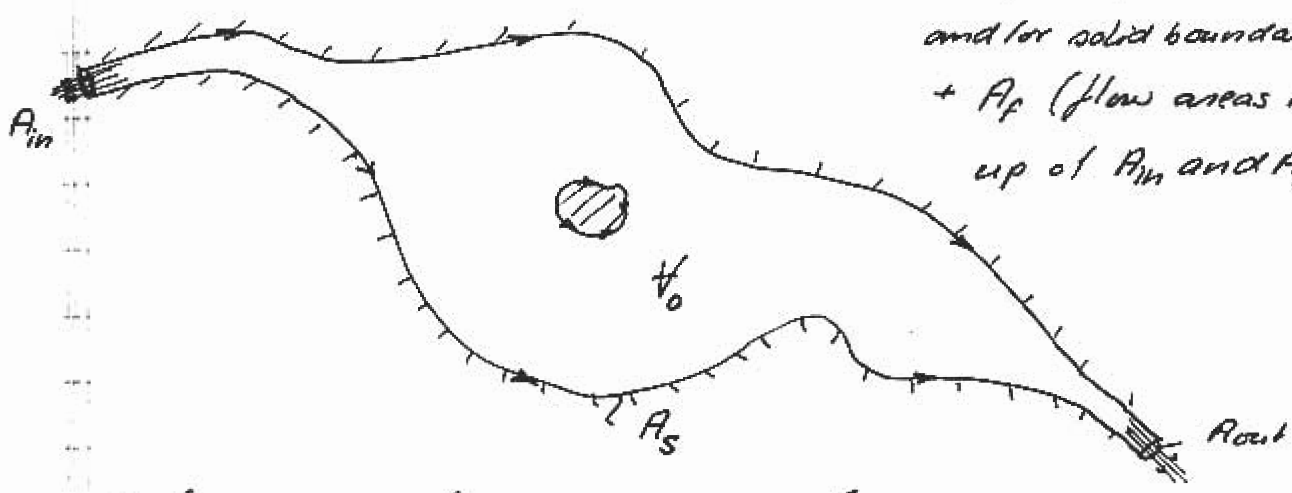
$$\frac{\partial}{\partial t}\left\{\iint_{\forall_o} \rho\vec{q}\,d\forall\right\} + \int_{S_o} \rho\vec{q}\,(\vec{q}\cdot\vec{n})\,dS =$$

Rate of change in fixed Volume + Net rate of outflow from fixed volume =

$$\int_{\forall_o} \rho\vec{g}\,d\forall + \int_{S_o} (-p\vec{n})\,dS + \int_{S_o} \vec{\tau}_s\,dS$$

Gravity + Pressure + Shear

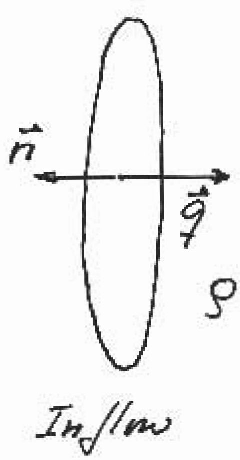
$S_o = A_s$ (streamlines and/or solid boundaries) $+ A_f$ (flow areas made up of A_{in} and A_{out})



$$\frac{\partial}{\partial t}\int_{\forall_o} \rho \vec{q}\, d\forall + \int_{A_f} \rho \vec{q}\,(\vec{n}\cdot\vec{q})\, dA = \int_{\forall_o} \rho \vec{g}\, d\forall + \int_{A_f} (-\vec{n}p + \vec{\tau}_s)\, dA + \int_{A_s} (-\vec{n}\cdot p + \vec{\tau}_s)\, dA$$

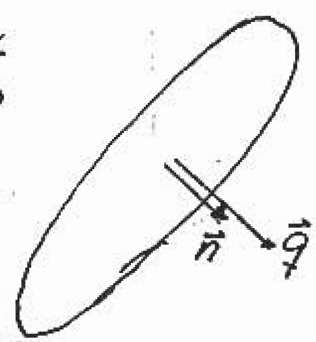
<u>PICK FLOW AREAS WHERE FLOW IS WELL BEHAVED</u>

1) Straight, parallel streamlines with $A_f \perp$ streamlines



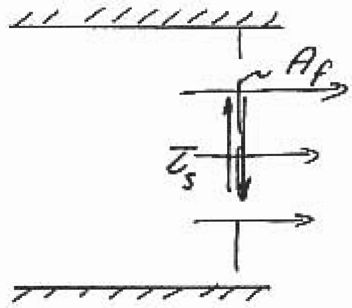
$$\vec{q} = -q_\perp \vec{n}$$
$$\vec{n}\cdot\vec{q} = -q_\perp$$
$$\rho \vec{q}(\vec{n}\cdot\vec{q}) = \rho q_\perp^2 \vec{n}$$

$\forall_o$



$$\vec{q} = \vec{n}\cdot q_\perp$$
$$\vec{n}\cdot\vec{q} = q_\perp$$
$$\rho \vec{q}(\vec{n}\cdot\vec{q}) = \rho q_\perp^2 \vec{n}$$

$\underline{\int_{A_f} \rho \vec{q}\,(\vec{n}\cdot\vec{q})\, dA = \int_{A_f} \rho q_\perp^2\, \vec{n}\, dA}$ over A_f whether In- or Outflow

2) 

Wellbehaved flow ~
little to no shear in velocity across A_f ~ $\tau_s \approx 0$ over A_f,
i.e. $\int_{A_f} \vec{\tau}_s \, dA \approx 0$

3) Wellbehaved flow ⇒ Pressure varies hydrostatically ⊥ to streamlines (i.e. pressure distribution varies LINEARLY OVER A_f).

$$\int_{A_f} -p\vec{n}\,dA = -\left(\int_{A_f} p\,dA\right)\vec{n} = -p_{CG} A_f \vec{n}$$

p_{CG} = pressure at center of gravity of flow area

$p_{CG} A_f$ = total pressure force on A_f on fluid in $\forall_o$ from surrounding fluid outside $\forall_o$

Pressure Force is ⊥ A_f and acts Inwards, i.e. towards $\forall_o$ [$-\vec{n}$-direction] if $p_{CG} > 0$.

Now the MOMENTUM EQUATION is

$$\frac{\partial}{\partial t}\int_{\forall} \rho \vec{q}\, d\forall = \text{Rate of change of momentum within} =$$

$$\int_{\forall} \rho \vec{g}\, d\forall - \int_{A_f} (\rho q_\perp^2 + p)\, \vec{n}\, dA_f + \int_{A_s} (-p\vec{n} + \vec{\tau}_s)\, dA_s =$$

Gravity force + THRUST on flow areas, Acting inwards towards $\forall_o$ and ⊥ to A_f's + Sum of all other forces acting on fluid within $\forall_o$

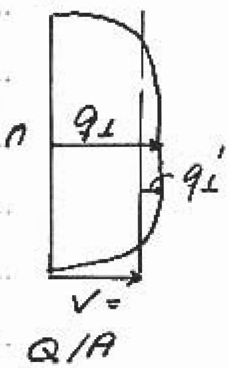
THE THRUST

$$\text{Thrust} = \left[\iint_A \left(\rho q_\perp^2 + p\right) dA\right]\left[-\vec{n}\right]$$

$\int_A p\, dA = p_{CG}\, A$ = P(ressure) force – just like hydrostatics since flow is wellbehaved

p_{CG} = pressure at CG of A.

$$\int_A \rho q_\perp^2\, dA = K_m \rho V^2 A = K_m \rho V Q$$



K_m = Momentum Coefficient = $\dfrac{\int_A q_\perp^2\, dA}{V^2 A} \approx 1$ if $q_\perp \approx V$ over A

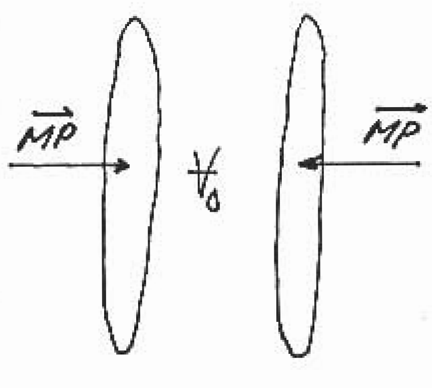
$$\int_A q_\perp^2\, dA = \int_A \left(V + q_\perp'\right)^2 dA = \int_A \Big(V^2 + \underbrace{2Vq_\perp'}_{\approx 0 \text{ since } \int_A q_\perp' dA = 0} + q_\perp'^2\Big) dA =$$

$$V^2 \int_A \Big[1 + \underbrace{\left(\tfrac{q_\perp'}{V}\right)^2}_{\approx 0}\Big] dA$$

≈ 0 If $q_\perp'/V \ll 1$ over most of A.

$$\int_A \rho q_\perp^2\, dA = \overset{\approx 1}{K_m} \rho V^2 A = \text{M(omentum) force}$$

$$\overrightarrow{THRUST} = \overrightarrow{MP} = \left(K_m \rho V^2 A + p_{CG} A\right)\left(-\vec{n}_A\right)$$



$\overrightarrow{MP}$ acts perpendicular to wellbehaved flow area and is always directed inwards towards $\forall_0$, regardless of in– or outflow and no need to worry about sign of $q_\perp$.

THE MOMENTUM PRINCIPLE

$$\frac{\partial}{\partial t}\int_{\forall} \rho\vec{q}\,d\forall = \int_{\forall} \rho\vec{g}\,d\forall + \sum\overrightarrow{MP} + \overrightarrow{\left(\text{All other forces on } \forall_0.\right)}$$

If flow is steady $\Rightarrow \partial/\partial t = 0$ and

Gravity Force, $\int_{\forall}\rho\vec{g}\,d\forall$, + Thrusts at Flow Areas, $\sum\overrightarrow{MP}$ + Sum of all other forces on fluid in $\forall_0 = 0$

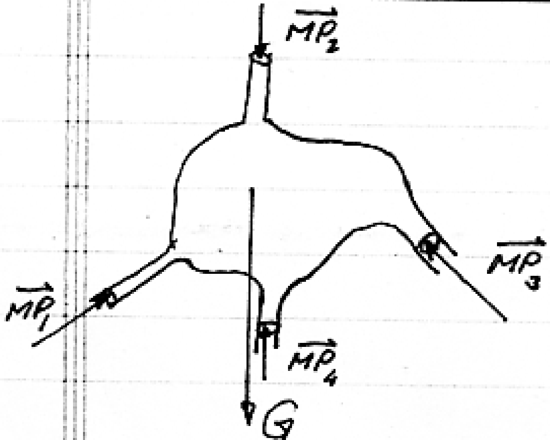
Since "Sum of $\overrightarrow{\text{forces }\underline{\text{on}}}$" = − "Sum of forces $\underline{\text{from}}$"

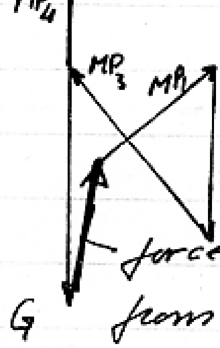
Sum of all forces from fluid in $\forall_0$ $\underline{\text{on}}$ its surroundings (including frictional forces!) =

Gravity force on fluid inside $\forall_0$ +

$\sum$ Thrusts that depend $\underline{\text{only}}$ on conditions at inflow and outflow sections to $\forall_0$

POWERFUL STUFF = THE ORIGINAL "BLACK BOX"





force on fluid inside $\forall_0$ from surroundings.