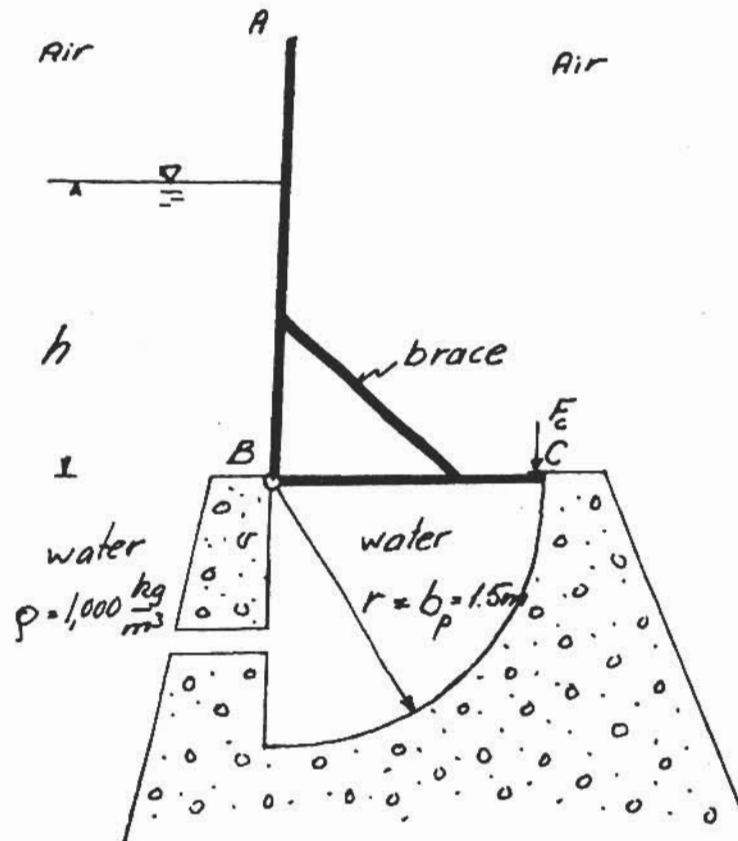


MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Civil and Environmental Engineering

1.060 Engineering Mechanics II

In-Class Examination 10 March, 2006

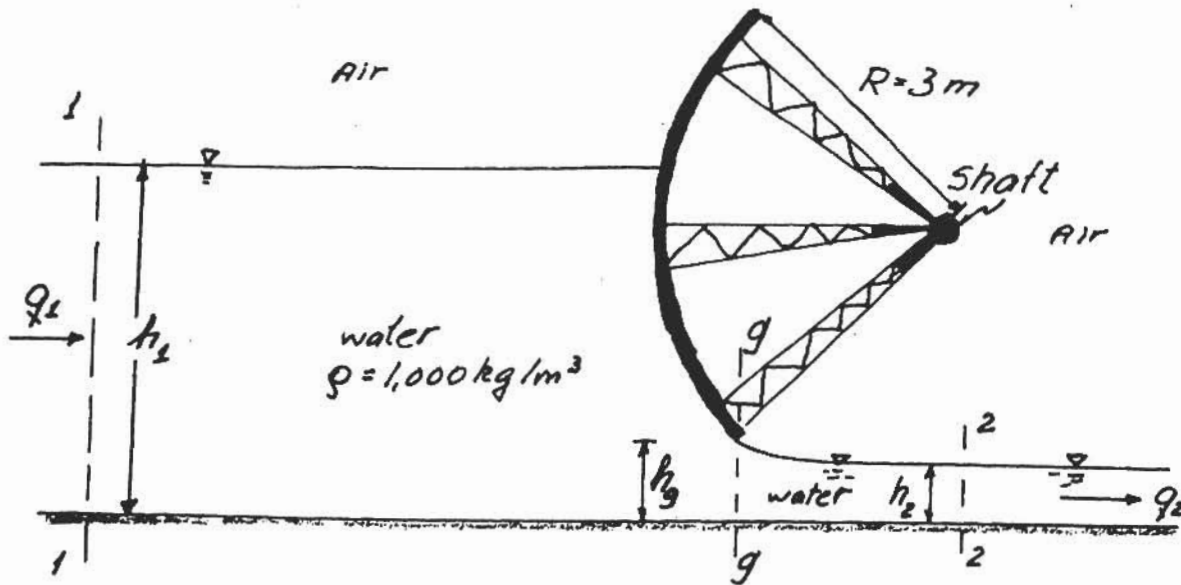
Problem No: 1 (35%)



The sketch shows a plane pivot-gate installed on a concrete dam. The gate is shown in its closed position. The vertical member (AB) is $h_p = 3.0$ m high, the horizontal member (BC) is $b_p = 1.5$ m wide, and the length of the gate (into the plane of the paper) is $l_p = 10$ m. The thickness as well as the weight of the gate members may be assumed negligibly small. The water surface in the reservoir is located a distance h above the gate's pivot point (B). When the gate is in its closed position (as shown in sketch), the compression along C forms a water tight seal.

- Sketch the pressure distribution on the vertical and horizontal gate members, AB and BC .
- Set up an equation from which the vertical reaction force, F_c , at C may be determined for given value of h .
- Noting that the support at C cannot provide tension, determine the height of water, h_0 , above B for which the gate will open.

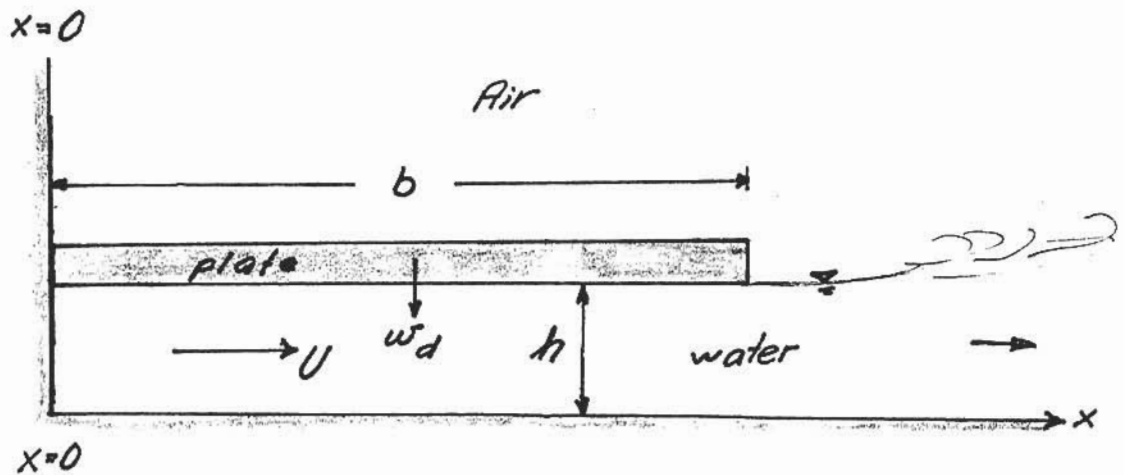
Problem No: 2 (35%)



A circular gate of radius $R = 3.0\text{ m}$ (a so-called "Tainter" Gate), used to hold back water in a large reservoir is raised (by turning it around its center, the shaft) leaving a gap of height $h_g = 1.0\text{ m}$ between the gate and the bottom through which the water in the reservoir passes as shown in the sketch above. A short distance downstream of the gate (at section 2-2) the depth reaches a constant value of $h_2 = 0.6\text{ m}$. Upstream of the gate the depth is considered constant, $h_1 = 3.6\text{ m}$. The flow is steady, and the gate is sufficiently long (into the paper) to consider the problem "plane". The fluid is water, and may be considered an ideal (incompressible and inviscid) fluid.

- Why is the depth h_2 less than h_g ?
- Why is it not reasonable to assume the pressure distribution at section g-g, immediately below the gate opening, is hydrostatic.
- Why is it reasonable to assume the pressure distribution at sections 1-1 and 2-2 to be hydrostatic?
- Determine the discharge Q [m^3/s per m length of gate into paper].
- Why must the total force acting on the circular "Tainter" gate pass through its center (i.e. the shaft)?

Problem No: 3 (30%)



The sketch shows a horizontal plate of width b (and very long in the direction into the paper). The plate is supported on tracks running along the vertical wall located at $x = 0$, and the seal between the vertical wall and the plate is water tight. The plate is forced downward at a constant velocity, w_d , towards a horizontal impermeable bottom. The gap between the bottom and the plate, of height h , is filled with water that is being squeezed out, as the plate moves downward, in such a way that it exits from below the plate as a free outflow (see sketch).

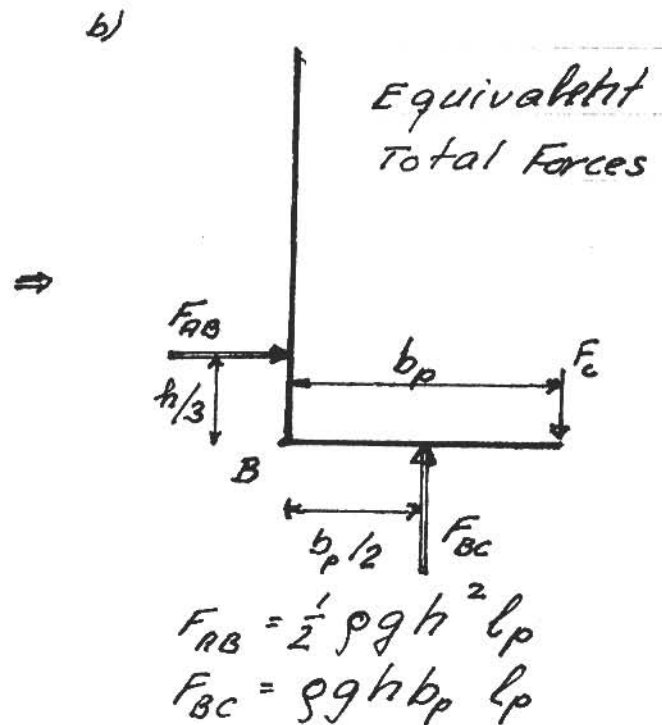
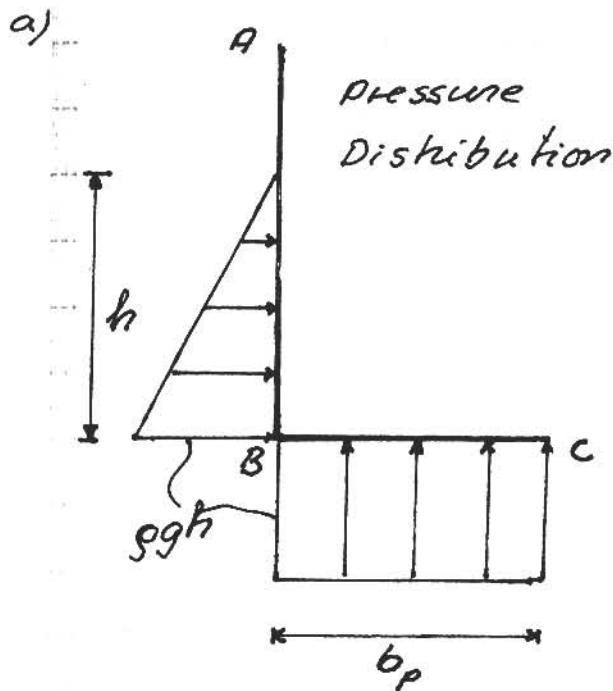
- Determine the horizontal velocity in the gap U as a function of x , w_d , and h .
- Neglecting friction and unsteadiness and assuming that U is so large that $U^2/2g \gg h$, determine the pressure distribution along the bottom of the plate.

1.060 ENGINEERING MECHANICS II

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SOLUTIONS

Problem No: 1



Moment around pivot point B:

$$F_{AB} \cdot h/3 + F_C b_p = F_{BC} \cdot b_p/2$$

$$F_C = \frac{1}{2} F_{BC} - F_{AB} \frac{1}{3} \frac{h}{b_p} = \left(\frac{1}{2} \rho g h b_p - \frac{1}{6} \rho g h^3 / b_p \right) l_p$$

c)

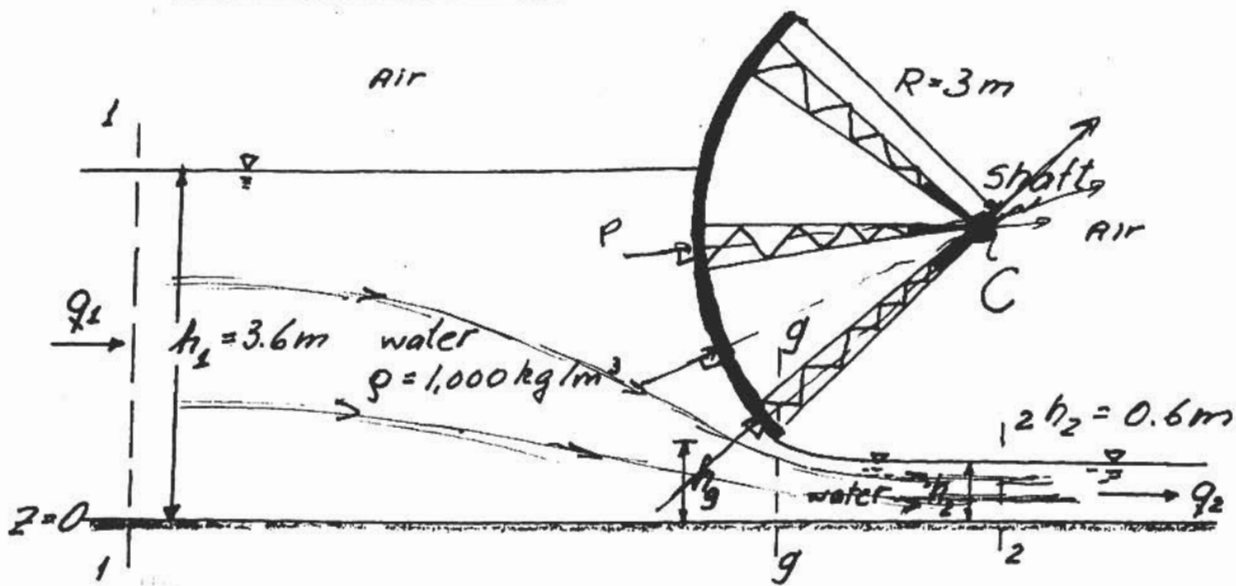
$F_C = 0$ is condition for gate opening.

$$\frac{1}{2} \rho g h b_p - \frac{1}{6} \rho g h^3 / b_p = 0 \Rightarrow \left(\frac{h}{b_p} \right)^2 = 3 \Rightarrow h = \sqrt{3} b_p$$

If

$$h > \sqrt{3} b_p = 2.60 \text{ m} \quad \text{Gate will pivot open}$$

Problem No: 2



- Flow can not turn a sharp corner. After exiting under gate takes time to turn to a horizontal direction $\Rightarrow h_2 < h_g$
- Immediately below gate streamlines are curved (flow changes direction). Pressure does not vary hydrostatically.
- At 1-1 & 2-2 streamlines are straight and parallel. Flow is well behaved and pressure varies hydrostatically \perp streamlines - here this coincides with the z -direction.

a) Conservation of volume: $Q = V_1 h_1 = V_2 h_2$

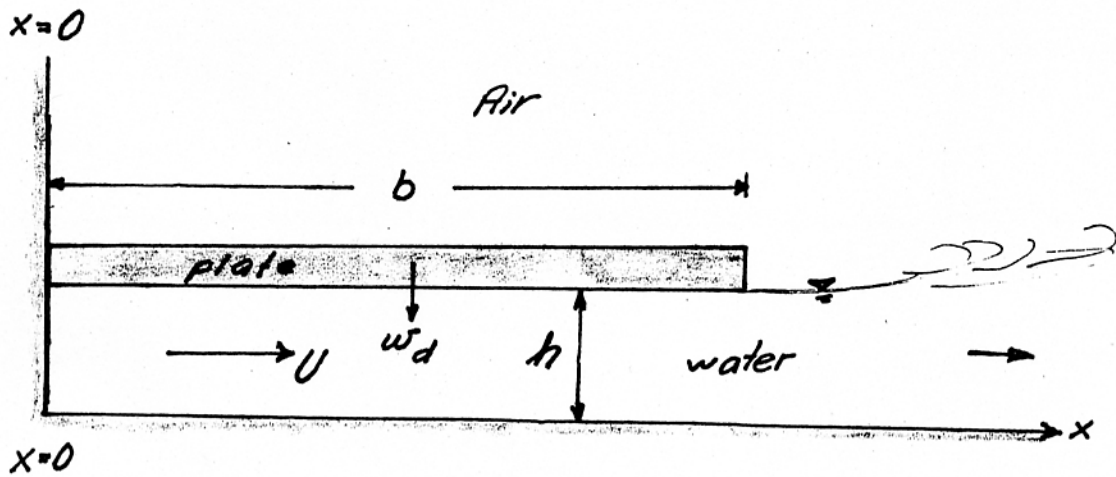
Bernoulli from 1-1 to 2-2: [$z=0$ along bottom]

$$\frac{1}{2} \rho V_1^2 + \underbrace{p_1 + \rho g z_1}_{\rho g h_1} = \frac{1}{2} \rho (h_2/h_1)^2 V_2^2 + \rho g h_1 = \frac{1}{2} \rho V_2^2 + \underbrace{p_2 + \rho g z_2}_{\rho g h_2}$$

$$V_2 = \sqrt{\frac{2g(h_1 - h_2)}{1 - (h_2/h_1)^2}} = 7.78 \text{ m/s} \Rightarrow Q = V_2 h_2 = 4.67 \frac{\text{m}^3}{\text{s}} \text{ per m}$$

- Since fluid is ideal, the only force on Tainter gate is pressure. Pressure is \perp surface which is circular, so pressure force are all in radial direction and pass through C (shaft)

Problem No: 3



a) Conservation of volume:

$$Q_{in} - Q_{out} = \frac{\partial V}{\partial t}$$

at $x=0$ at x \forall from $x=0 \rightarrow x$

$$Q_{in} = 0 \quad Q_{out} = Uh \quad \forall = h \cdot x \Rightarrow \frac{\partial \forall}{\partial t} = x \cdot \frac{\partial h}{\partial t} = -x w_d$$

$$0 - Uh = -x w_d \Rightarrow \underline{U = (x/h) w_d}$$

b) Bernoulli along bottom of plate

$$\frac{1}{2} \rho U^2 + p(x) + \rho g h = \frac{1}{2} \rho U^2(x=b) + p(b) + \rho g h$$

$$\underline{p(x) = \frac{1}{2} \rho (U(x=b)^2 - U(x)^2) = \frac{1}{2} \rho \left(\frac{w_d}{h}\right)^2 (b^2 - x^2)}$$

Patm = 0

Note: With $U = \frac{x w_d}{h} \Rightarrow \frac{\partial U}{\partial t} = -\frac{x w_d}{h^2} \frac{\partial h}{\partial t} = x \left(\frac{w_d}{h}\right)^2$

In the Bernoulli equation it is possible to include unsteady effects (see Lecture and/or Recitation #4) by evaluating the integral

$$\int_{s_1}^{s_2} \rho \frac{\partial \psi}{\partial t} ds = \int_0^b \rho x \left(\frac{w_d}{h}\right)^2 dx = \frac{1}{2} \rho \left(\frac{w_d}{h}\right)^2 (b^2 - x^2)$$

here x

i.e. UNSTEADINESS DOUBLES THE VALUE FOR $p(x)$ IF IT WERE INCLUDED!