

**1.033/1.57**

**Mechanics of Material Systems**  
(Mechanics and Durability of Solids I)

Franz-Josef Ulm

*Lecture: MWF1 // Recitation: F 3:00-4:30*

# Part IV: Plasticity and Yield Design

## 9. Limit Analysis and Yield Design

# Content 1.033/1.57

## Part I. **Deformation and Strain**

- 1 Description of Finite Deformation
- 2 Infinitesimal Deformation


## Part II. **Momentum Balance and Stresses**

- 3 Momentum Balance
- 4 Stress States / Failure Criterion

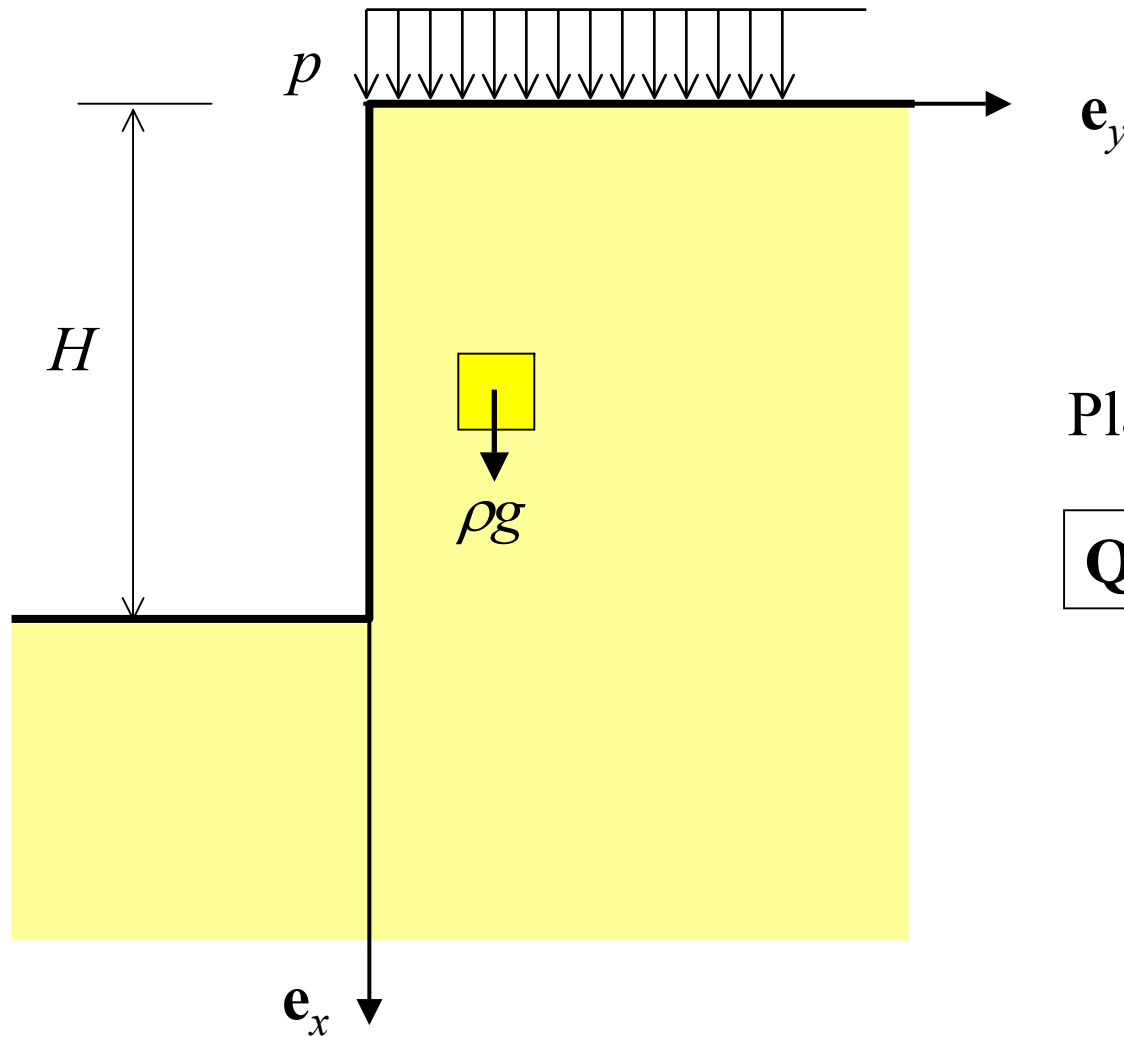
## Part III. **Elasticity and Elasticity Bounds**

- 5 Thermoelasticity,
- 6 Variational Methods

## Part IV. **Plasticity and Yield Design**

- 7 1D-Plasticity – An Energy Approach
- 8 Plasticity Models
-  9 Limit Analysis and Yield Design

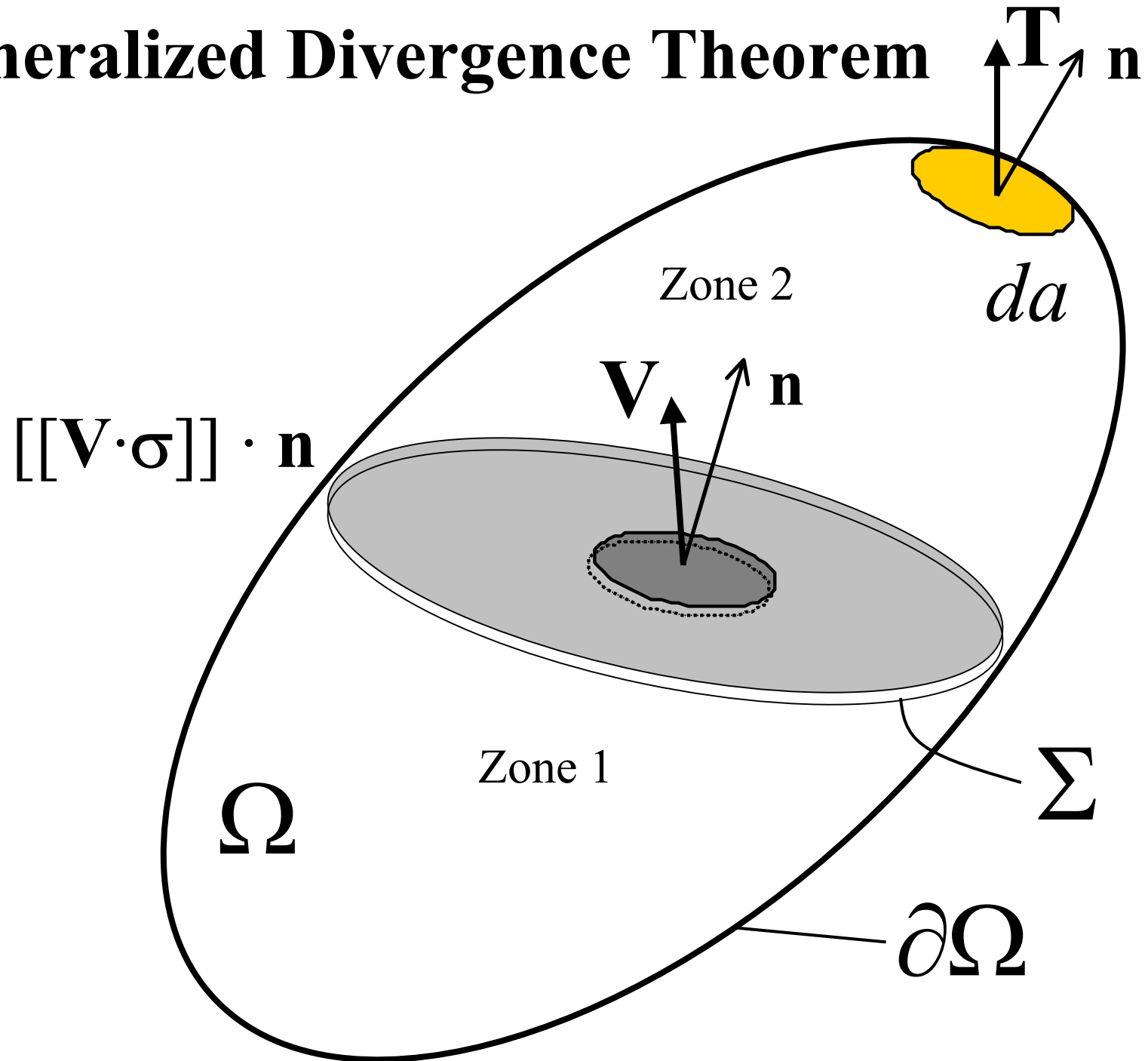
# Excavation Pit



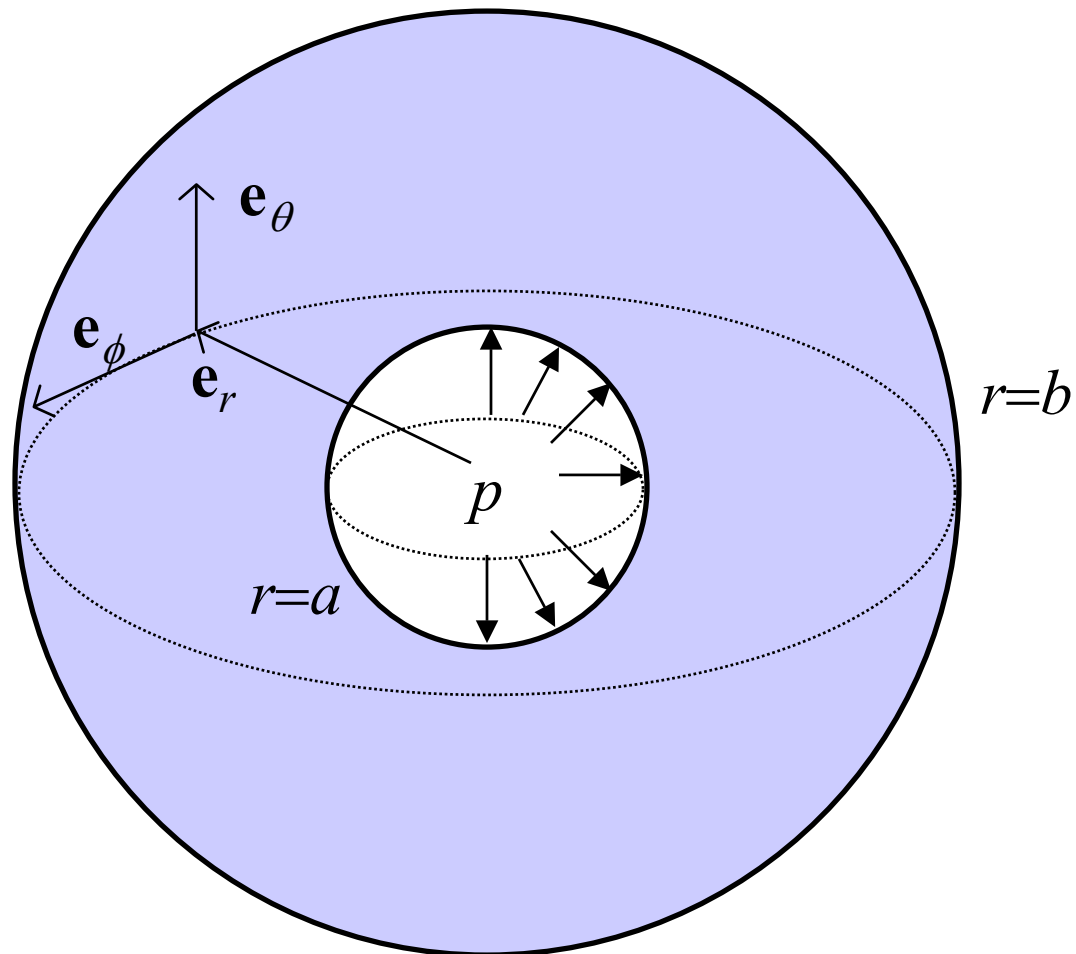
Plastic Collapse:

$$\mathbf{Q}^{\text{lim}} \cdot \mathbf{q}(\mathbf{V}) = dD(\mathbf{V})/dt$$

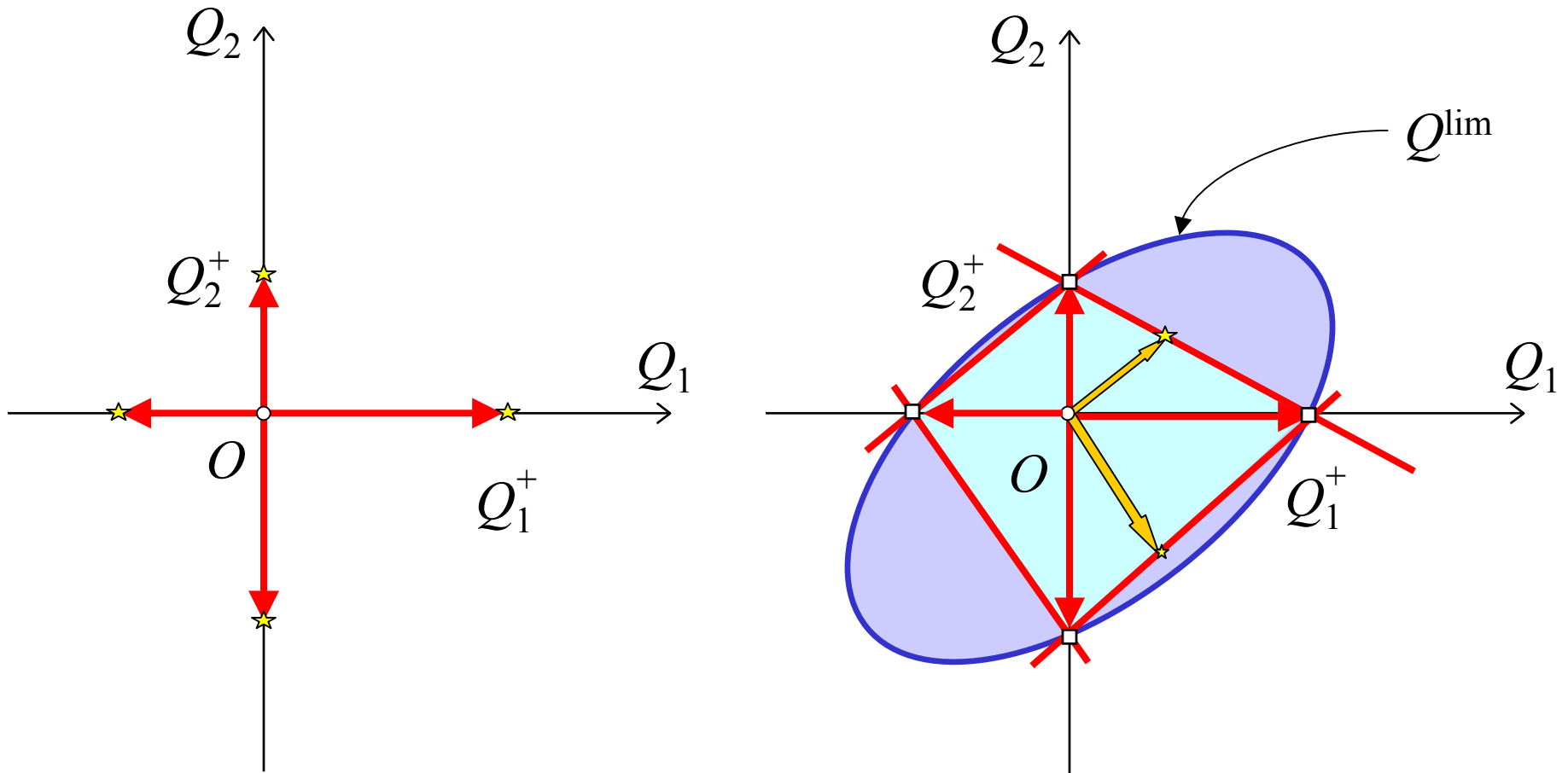
# Generalized Divergence Theorem



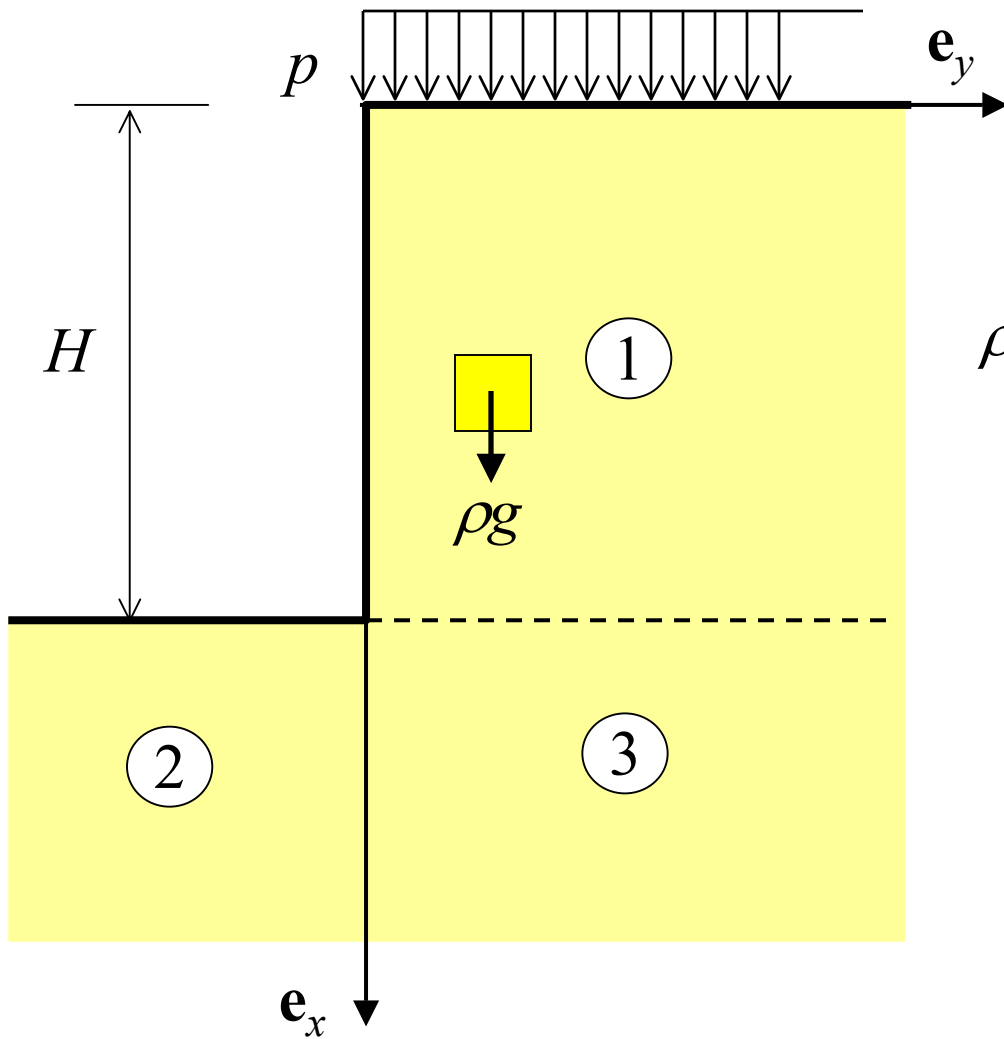
# Exercise: Hollow Sphere



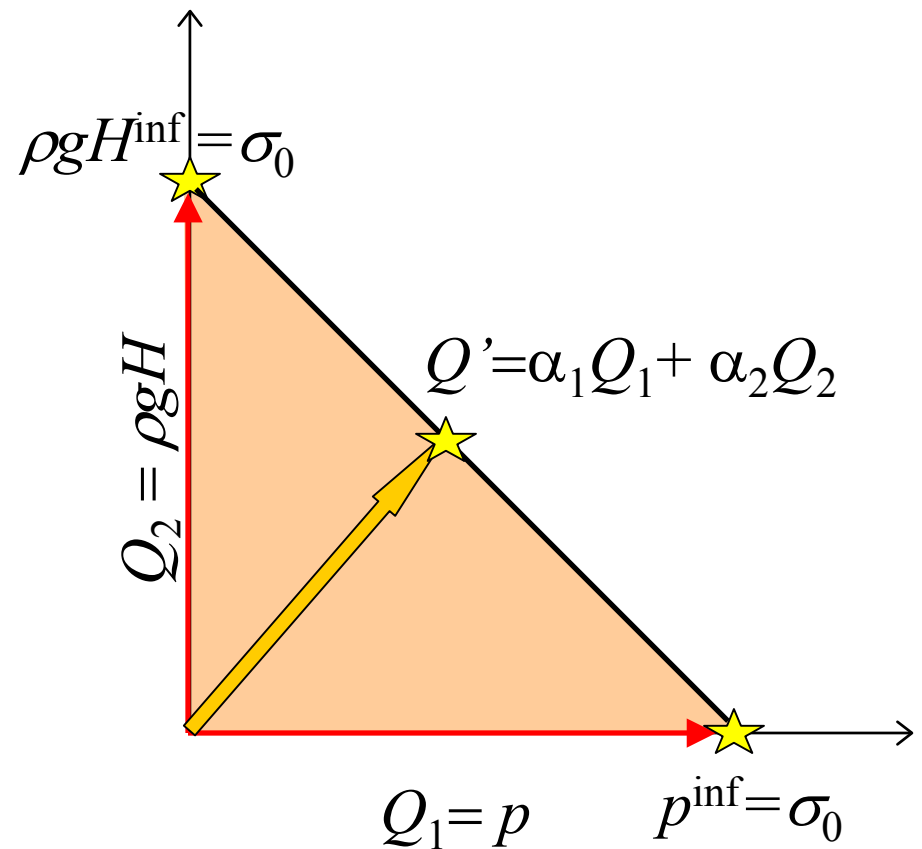
# Lower Limit Theorem: Constructing the Domain of Safe Loads from the “Inside”



# Excavation Pit: Lower Bound



(a)

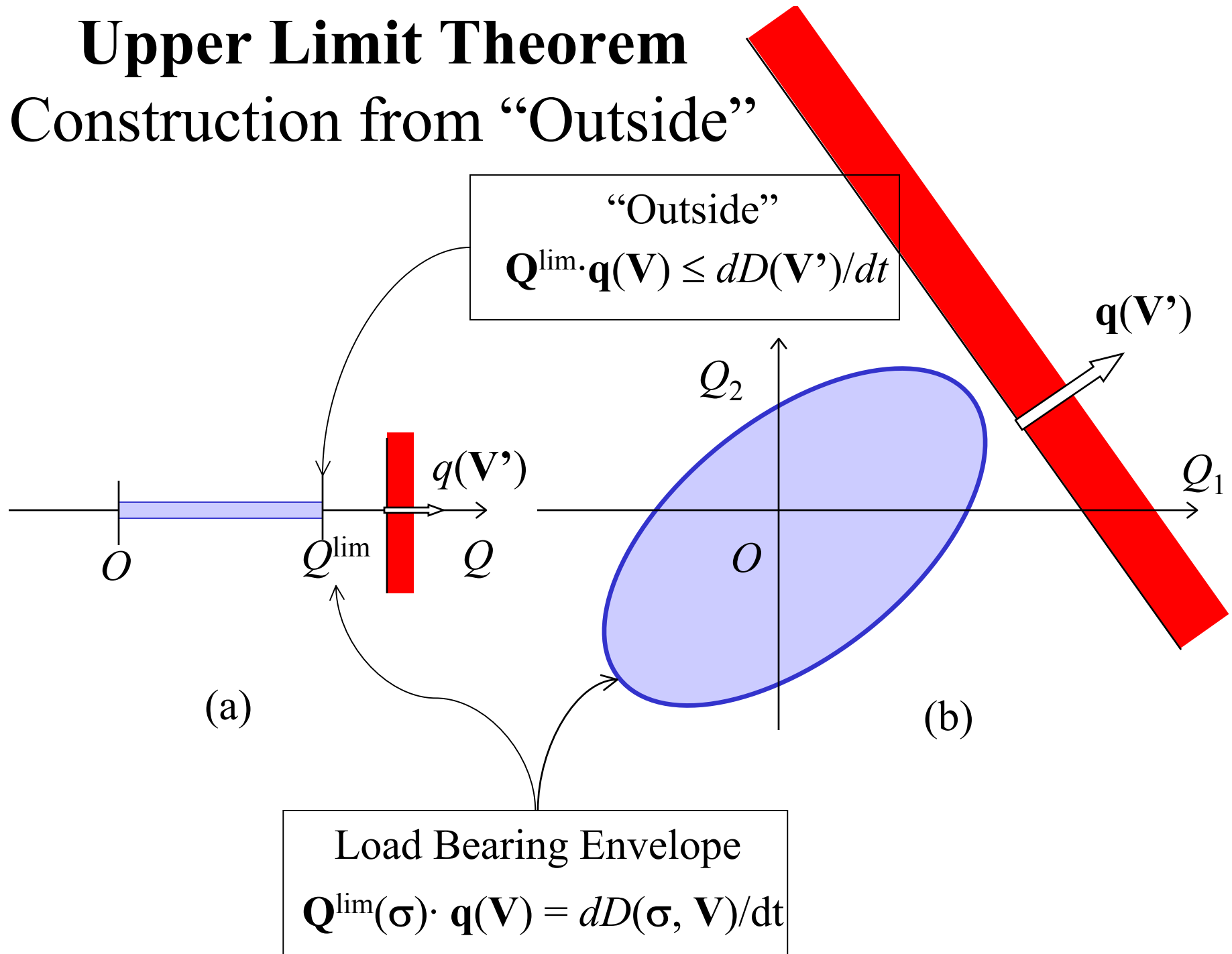


(b)

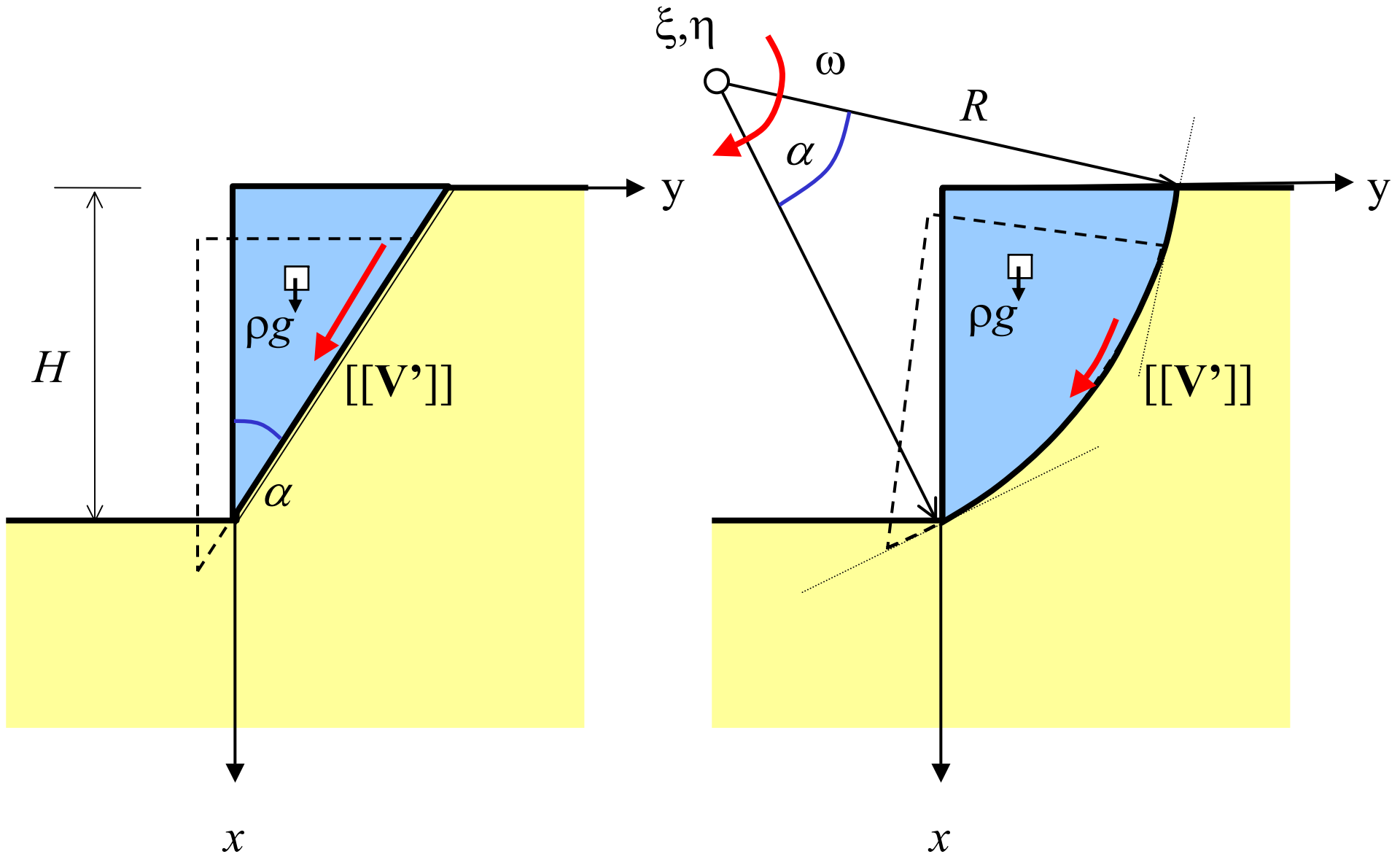


# Upper Limit Theorem

## Construction from “Outside”



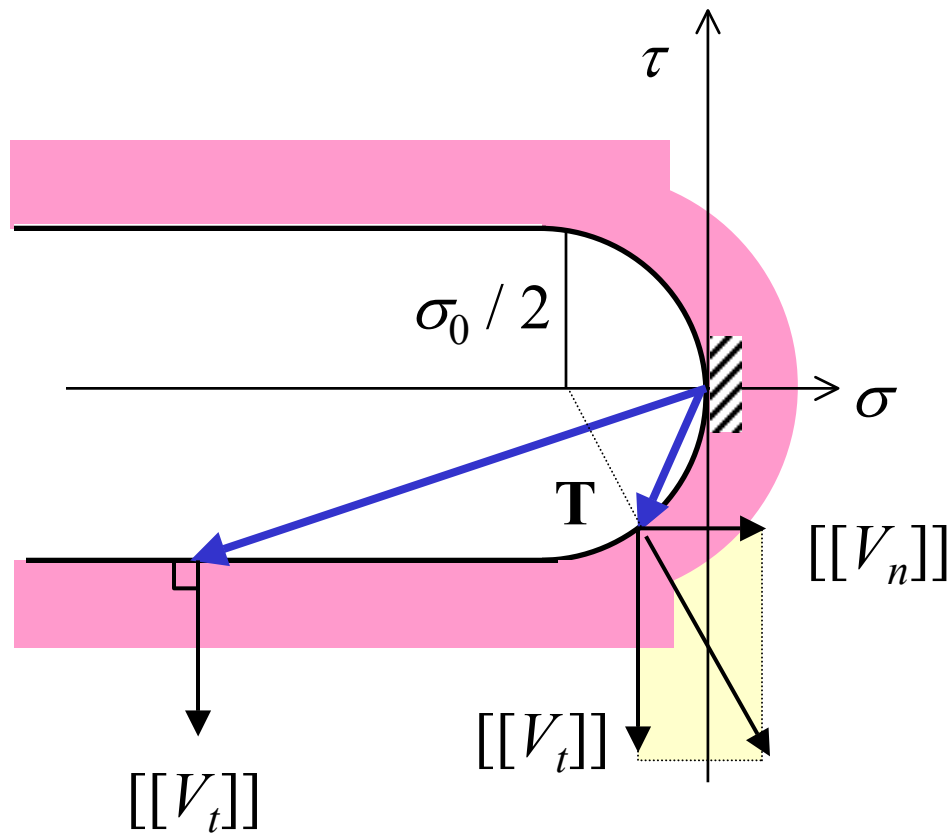
# Excavation Pit: ... from “Outside”



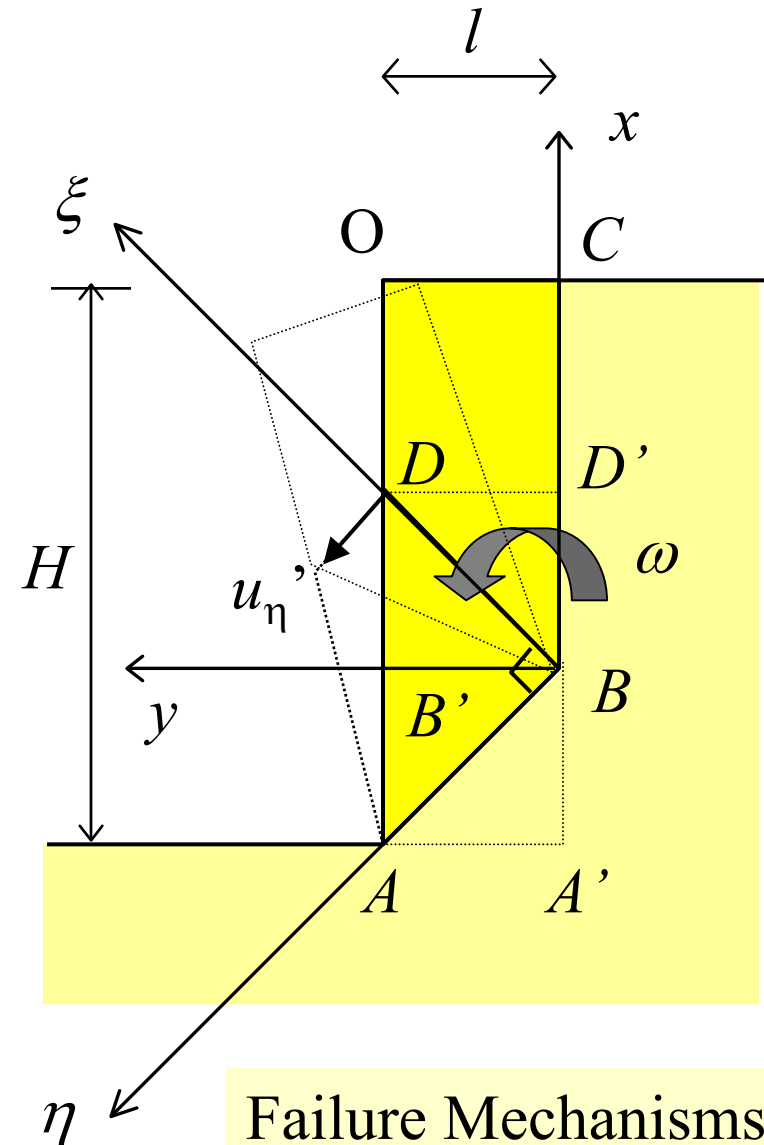
(a)

(b)

# Excavation Pit: Tension Cut-Off

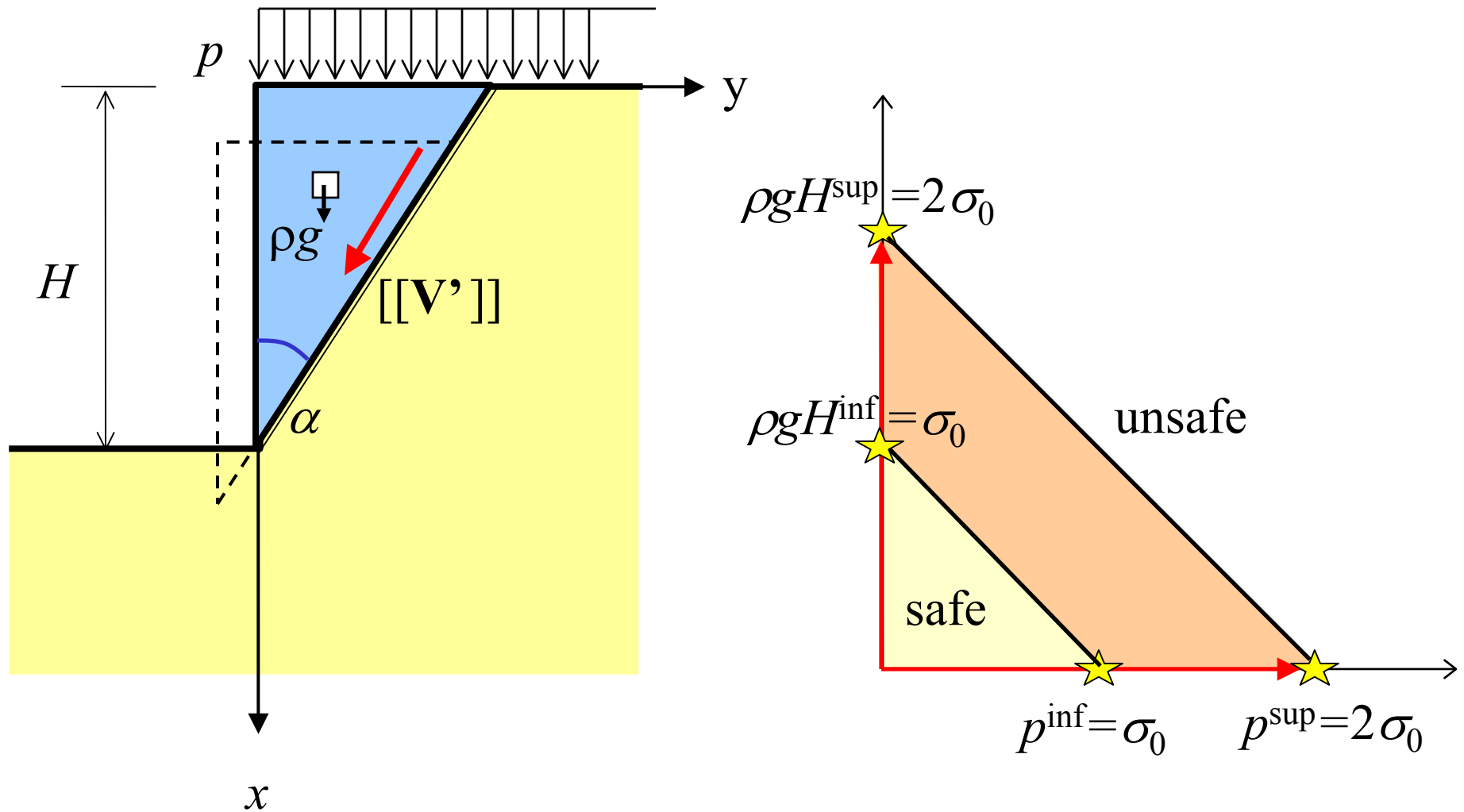


Tresca Criterion + Tension Cut-Off in the Mohr Plane

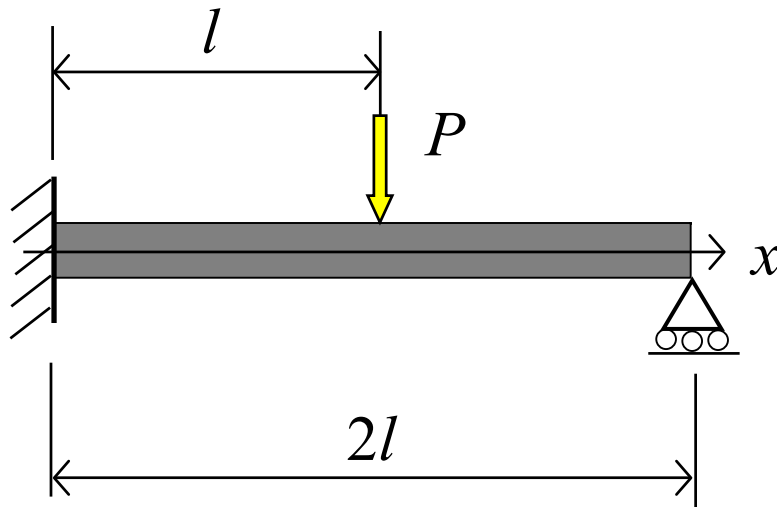


Failure Mechanisms

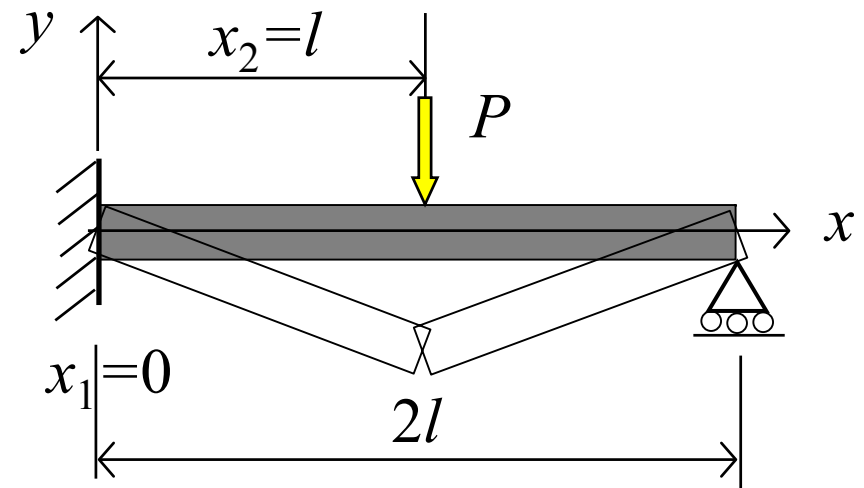
# Excavation Pit:... from “Outside”



# Lower and Upper Bound of Structural Elements

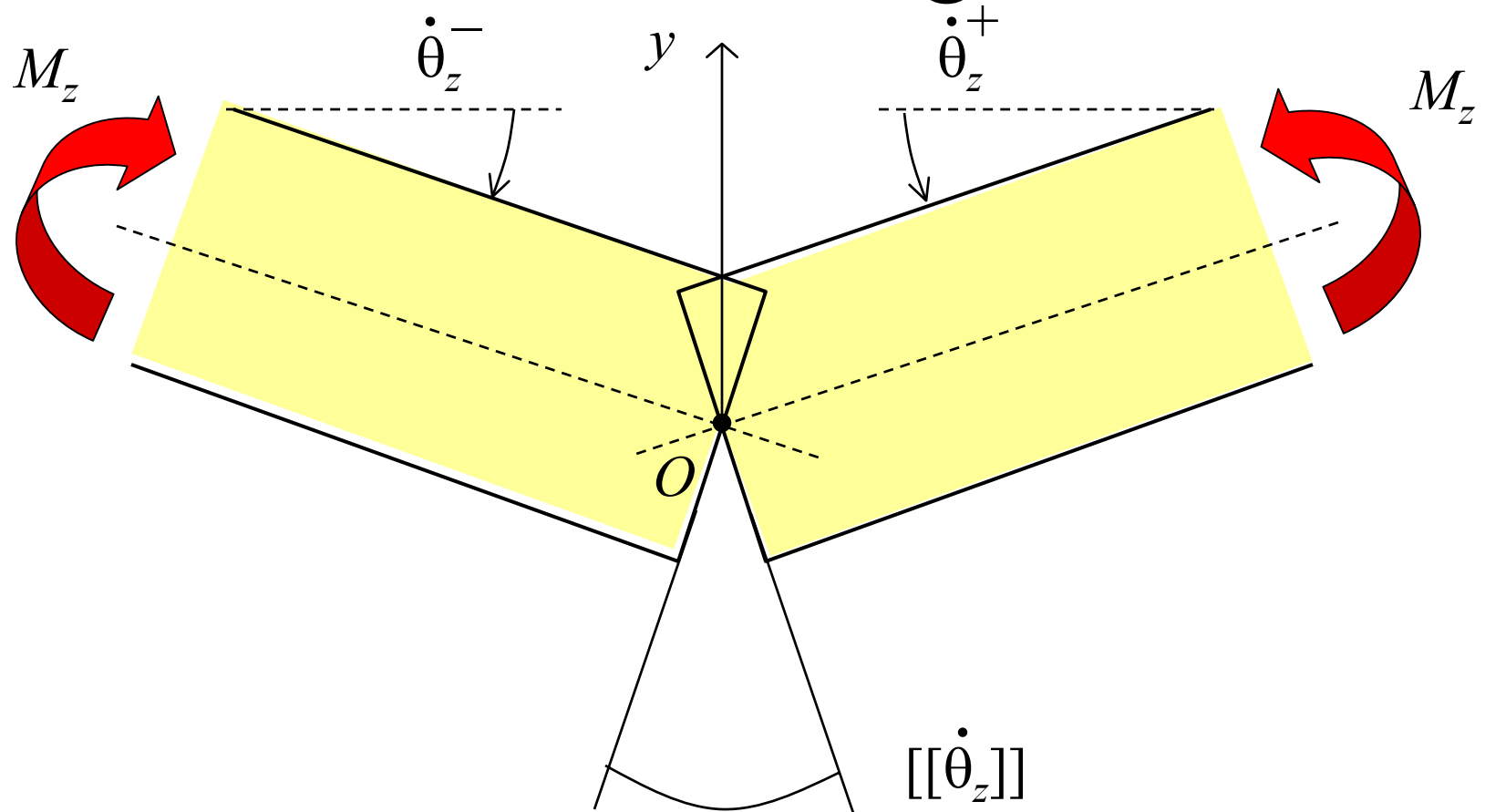


(a)

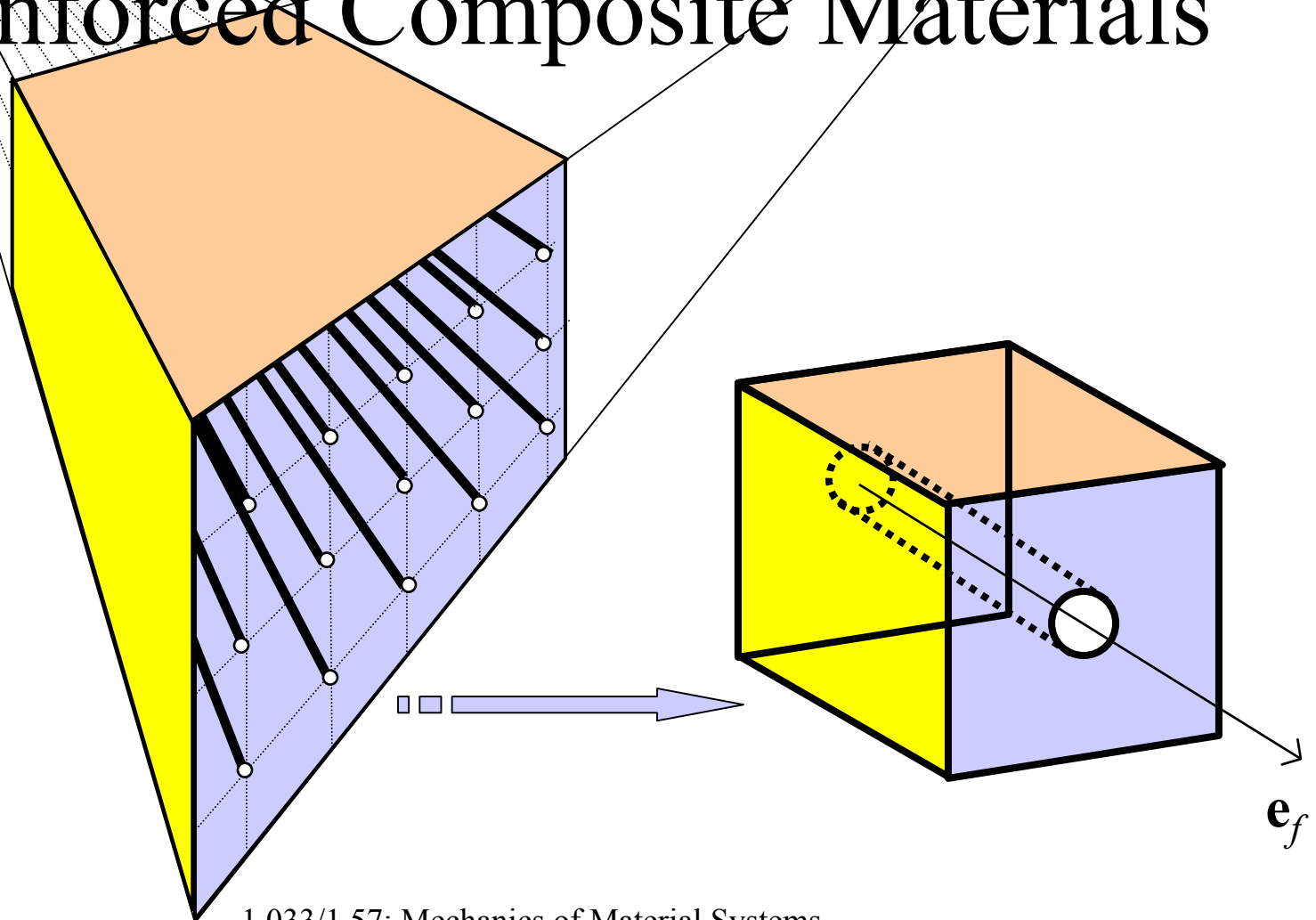


(b)

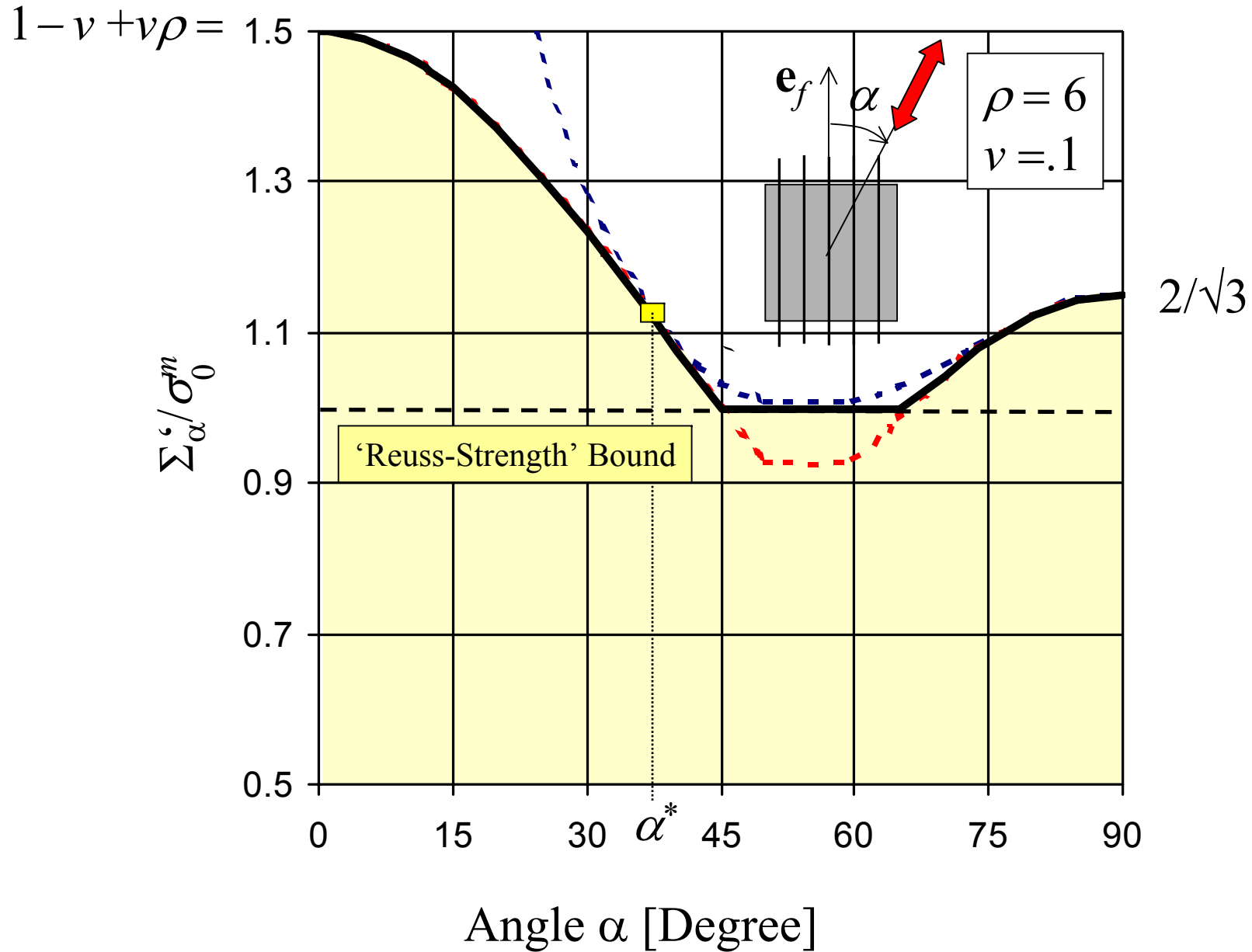
# Plastic Hinge



# Strength Domain of Fiber Reinforced Composite Materials

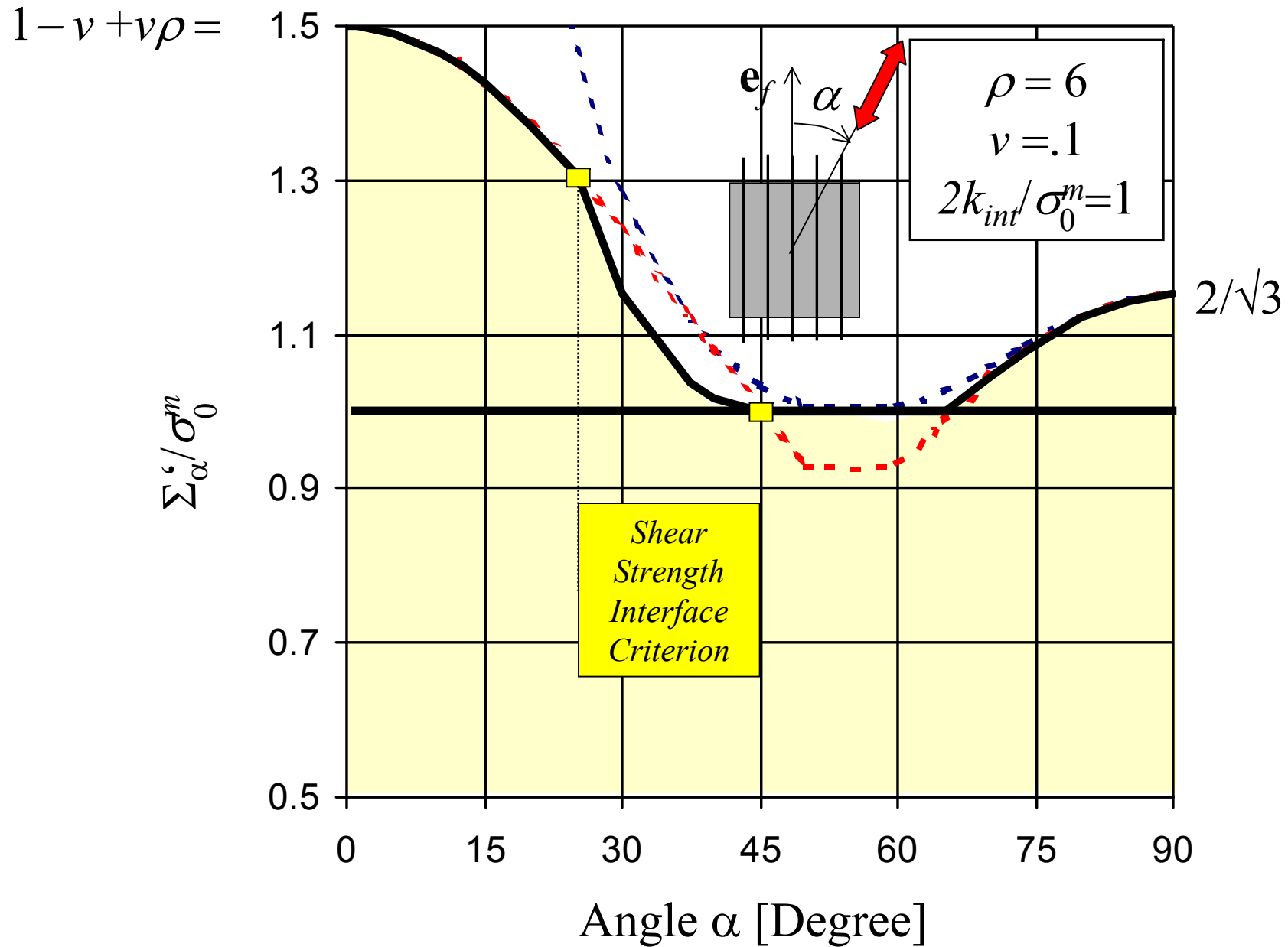


# Improved Lower Bound for 'Bulk' Collapse



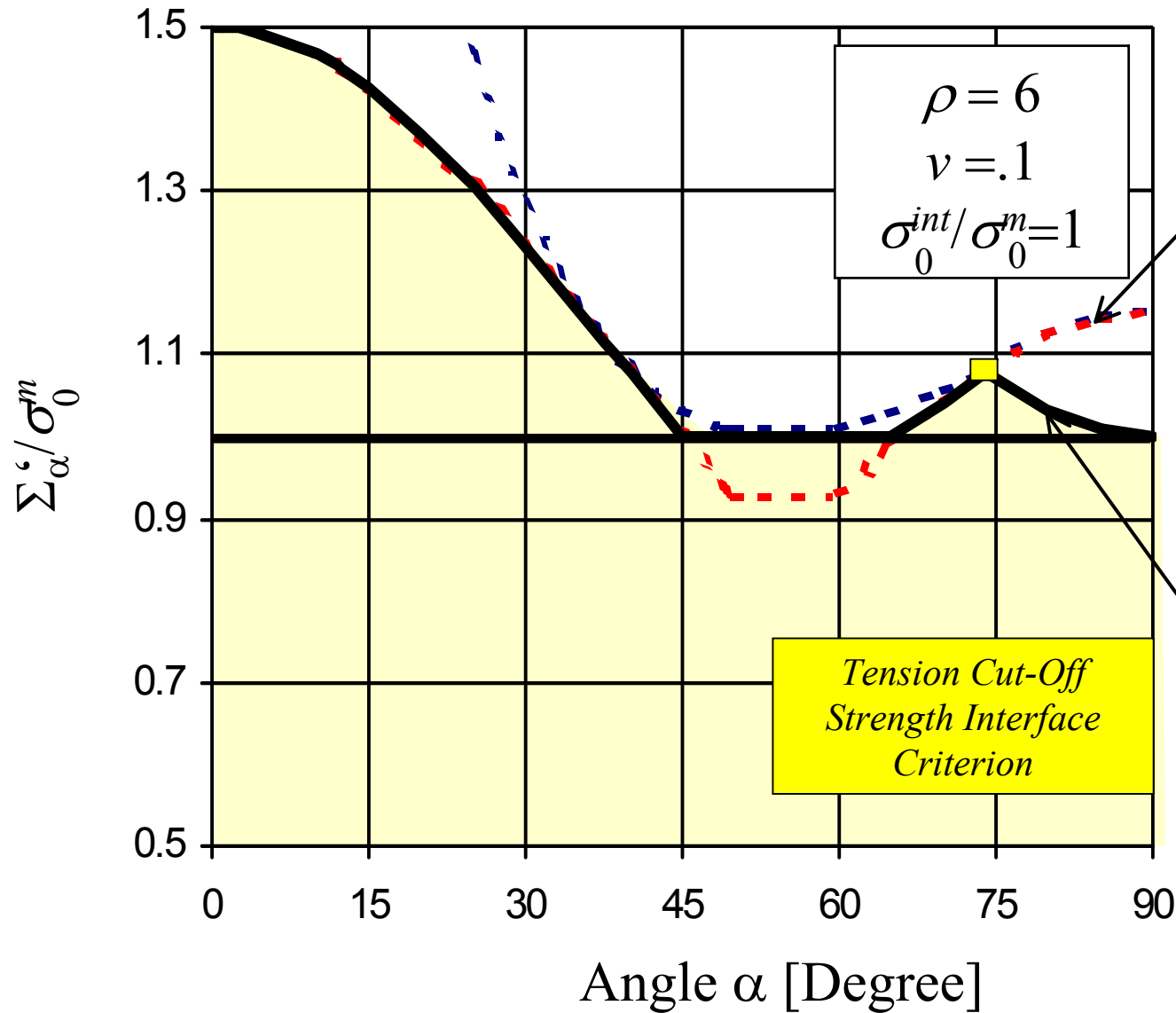


# Effect of Debonding Strength Interface Criterion



# Effect of Delamination Interface Strength Criterion

$$1 - \nu + \nu\rho =$$



# Section Strength of Combined Bending and Axial Force

