1.020 Ecology II: Engineering for Sustainability
Spring 2008

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Civil and Environmental Engineering

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Practice Questions and Solutions for Exam 2 on May 7, 2008 9:30-11am
Exam Review Friday May 2nd, 2008 9-11am, Monday May 5th, 2008, 7-9pm

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1. Open Systems

a) A stream of 5 m3/s of cooling water at 15 °C flows through a power plant and accepts 10 MW of heat before leaving. What is the final temperature of the cooling water if this is happening at steady state? Use the following values: c_p of water = 4200 J kg^{-1} K^{-1}; density = 1000 kg m^{-3}. Assume the water inside the power plant is well mixed.

Solution: Open system energy conservation:

$$m_{powerplant} c_v \frac{dT_{out}}{dt} = \dot{m}h_{in} - \dot{m}h_{out} + \dot{Q}$$

$\frac{dT_{out}}{dt} = 0;$ $\quad \dot{m}h_{in} = \rho q c_p T_{in}$ $\quad \dot{m}h_{out} = \rho q c_p T_{out}$

$$\rightarrow \quad 0 = \rho q c_p T_{in} - \rho q c_p T_{out} + \dot{Q}$$

$$0 = \rho q c_p (T_{in} - T_{out}) + \dot{Q}$$

Solving for T_{out}, $T_{out} = T_{in} + \frac{\dot{Q}}{\rho q c_p} = 19.8\,°\text{C}$.

b) A room initially at 20 degrees C has constant ventilation from an outside breeze that enters and leaves on opposite sides of the room. The walls and windows are good insulators of conduction and convection. The walls do not transmit the incoming solar radiation into the room, however it can emit radiation from the room outward to the outside. The windows initially have their blinds shut, blocking off solar radiation. In the middle of the day, you open the blinds. Assume that the solar radiation received is uniform on all surfaces of the room. Write a differential equation for the unknown temperature of the room *T*. Calculate the initial rate of increase in the temperature of the room (degrees C s^{-1}) given the following:

Constant volume specific heat of air: c_v = 0.716 joule gm^{-1} °C^{-1} (to compute internal energy)
Constant pressure specific heat of air c_p = 1.0 joule gm^{-1} °C^{-1} (to compute enthalpy)
Incoming solar radiation S_g= 1000 W m^{-2}
Stefan Boltzmann constant $\sigma = 5.7 \times 10^{-8}$ W m^{-2} K^{-4}
Sky temperature T_{sky} = -5 °C
Emissivity of sky ε_{sky} =1

Emissivity of windows ε_{window} = 0.8
Emissivity of walls ε_{wall} = 0.05
Flow of draft through room = 0.3 m^3 s^{-1}
Density of air: 1100 gm m^{-3}
Volume of room: 300 m^3
Surface area of room (excluding windows) = 170 m^2
Surface area of windows = 15 m^2
Outside air temperature: 20 °C

Solution: Open system energy conservation:

$$m_{room}c_v \frac{dT}{dt} = \dot{m}h_{in} - \dot{m}h_{out} + \dot{Q} \quad \text{or} \quad \rho_{air}V_{room}c_v \frac{dT}{dt} = \dot{m}h_{in} - \dot{m}h_{out} + \dot{Q}$$

Enthalpy terms (energy from flow in/out of system) $\dot{m}h_{in} = \rho q c_p T_{in}$; $\dot{m}h_{out} = \rho q c_p T_{out}$

$\dot{Q}$ = radiative heat terms (J/s)

$$= S_g A_{windows} + (A_{window} + A_{wall})\sigma\varepsilon_{sky}T_{sky}^4 - A_{window}\sigma\varepsilon_{window}T^4 - A_{wall}\sigma\varepsilon_{wall}T^4$$

Therefore the differential equation for T is

$$\frac{dT}{dt} = \frac{1}{\rho_{air}V_{room}c_v}\left(\rho q c_p T_{in} - \rho q c_p T + S_g A_{windows} + (A_{window} + A_{wall})\sigma\varepsilon_{sky}T_{sky}^4 - A_{window}\sigma\varepsilon_{window}T^4 - A_{wall}\sigma\varepsilon_{wall}T^4\right)$$

At t= 0, $T = T_{in} = 293K$

$$\frac{dT}{dt} = \frac{1}{1200\frac{g}{m^3}300m^3 0.716\frac{J}{gK}}\begin{pmatrix} 1000\frac{w}{m^2}15m^2 + (15+170)m^2 5.7\times10^{-8}\frac{W}{m^2K^4}\times1.0\times(268K)^4 \\ -15m^2 5.7\times10^{-8}\frac{W}{m^2K^4}\times0.8\times(293K)^4 - 170m^2 5.7\times10^{-8}\frac{W}{m^2K^4}\times0.05\times(293K)^4 \end{pmatrix}$$

=+0.236°Cs^{-1}

2. Transport/Diffusion

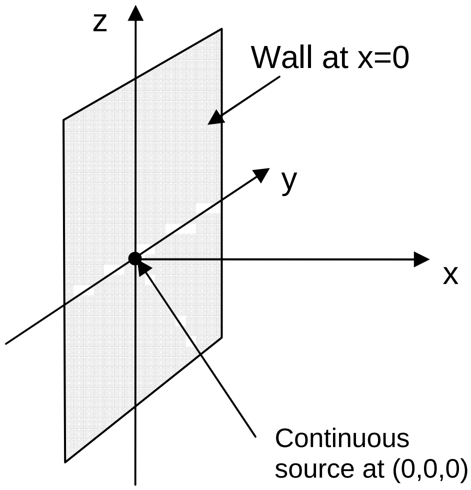
We start with the Langevin equation that describes a Random walk of a particle *i*:

$$x_{j,n+1}^i = x_{j,n}^i + V_{j,n}^i\Delta t + d_j\omega_{j,n}^i \qquad j = 1,2,3$$

Where $d_j = \sqrt{2D_j\Delta t}$ =Dispersion distance in direction *j* (m), D_j = Dispersion coefficient in direction *j* (m^2 sec^{-1}). The random walk model can be very useful, but to more accurately depict real life situations the model must account for certain physical or chemical behaviors. This question address two complications found in reality: the presence of boundaries and particle decay.

a) Write a MATLAB pseudocode that will model a continuous source at (0,0,0) located on a boundary (e.g. a wall). The wall extends infinitely on the *y-z* plane and is described by *x*=0. The particles are released at the origin and diffuses into the region x>0, and cannot cross the wall. There is a flow in the y direction of 15 m/s. Include in your code how you would account for the fact that particles are not able to cross the boundary. In the code use the following numbers:

$D_x = D_y = D_z = 25\,\mathrm{m^2 s^{-1}}$. [HINT: the function abs(x) in MATLAB gives the absolute value of x].



b) Say that the particles in the above problem represent a substance that undergoes first order decay, i.e. the concentration is in the form $C = C_0 \exp(\quad kt)$. As an approximation, we assume that if the particle persists for longer than three times the *e*-folding time, they will disappear. Add to the pseudocode in part a) to account for the described decay behavior.

[example code not included in MIT OpenCourseWare materials]

3. Economics

Suppose that a hybrid car costs $2500 more than and has a gas mileage 10 mi/gallon greater than a conventional alternative (28 mi/gallon). If gasoline is $3.5/gallon and you drive 10,000 miles per year, is the hybrid economically justified purchase, assuming an annual interest rate of 5% and a car lifetime of 10 years?

Solution:

Yearly fuel cost of hybrid car = $\frac{10000mi}{yr}\frac{gal}{38mi}\frac{3.5\$}{gal} = \$921.05 / year$

Yearly fuel cost of conventional car = $\frac{10000mi}{yr}\frac{gal}{28mi}\frac{3.5\$}{gal} = \$1250 / year$

Yearly fuel savings by hybrid vehicle = 1250-921.05 = $328.95/year

To translate the yearly savings to a present value, use the following:

$$CRF = \frac{1-(1+r)^{-N}}{r}$$

$$CRF = \frac{1-(1+0.05)^{-10}}{0.05} = 7.72$$

Present value = $CRF \times B_t = 7.72 \times \$328.95 = \$2539.49$

Since the present value of the saving is greater than the initial extra cost of the hybrid, buying the hybrid car is worthwhile.

4. Optimization – Resource allocation

Supplier A and B distributes natural gas to Destinations 1 2 and 3. Their maximum natural gas production are P_{maxA} and P_{maxB}. The costs of transporting the natural gas from each supplier to each destination are different due to the difference in distances. These are listed in the table below.

Supplier	Cost of distribution c($/ton) to Destination		
	1	2	3
A	c_{A1}	c_{A2}	c_{A3}
B	c_{B1}	c_{B2}	c_{B3}

The demand over one year of natural gas from the three destinations are as follows

Destination	1	2	3
Natural gas demand d (tons yr^{-1})	d_1	d_2	d_3

Supplier A and B would like to collectively minimize costs ($ yr^{-1}) of natural gas distribution (tons yr^{-1}) while meeting the demands of each destination (tons yr^{-1}).

Set up this optimization problem by defining the decision variables, objective functions, and constraints. Determine whether you should use **linprog** or **quadprog** (linear or quadratic programming) and provide all of the matrices (with the values in symbolic form) that you would enter into the appropriate MATLAB program.

Solution:

Decision variables x = $[P_{A1} \quad P_{A2} \quad P_{A3} \quad P_{B1} \quad P_{B2} \quad P_{B3}]$
Gas distribution from each supplier to demand (ton yr^{-1})

Objective function:

$$\underset{x}{Minimize} \quad \text{Cost}(x, \$\text{yr}^{-1}) = c_{A1}P_{A1} + c_{A2}P_{A2} + c_{A3}P_{A3} + c_{B1}P_{B1} + c_{B2}P_{B2} + c_{B3}P_{B3}$$

Constraints:

Supply:
$$P_{A1} + P_{A2} + P_{A3} \le P_{A\max}$$
$$P_{B1} + P_{B2} + P_{B3} \le P_{B\max}$$

Demand:
$$P_{A1} + P_{B1} \ge d_1$$
$$P_{A2} + P_{B2} \ge d_2$$
$$P_{A3} + P_{B3} \ge d_3$$

Non-negativity constraints (lower bound): $[P_{A1} \quad P_{A2} \quad P_{A3} \quad P_{B1} \quad P_{B2} \quad P_{B3}] \ge 0$

The objective function is linear with respect to the decision variables, so we will use **linprog** in MATLAB.
The syntax of linprog is

```
[X,FVAL,EXITFLAG,OUTPUT,LAMBDA]= LINPROG(f,A,b,Aeq,beq,LB,UB)
```

So in the example:

$$f = \begin{bmatrix} c_{A1} \\ c_{A2} \\ c_{A3} \\ c_{B1} \\ c_{B2} \\ c_{B3} \end{bmatrix}; A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 \end{bmatrix}; \; b = \begin{bmatrix} P_{A\max} \\ P_{B\max} \\ -d_1 \\ -d_2 \\ -d_3 \end{bmatrix}; A_{eq}=[]; b_{eq}=[]; \; lb = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; ub=[];$$

5. Multiple objective optimization

a) There are 3 farms, all growing rice.
Water is allocated by an irrigation district to each farm to maximize district income (over all 3 farms)
The maximum land available for cultivation is specified for each farm.

Yield is enhanced by fertilizer application.
Fertilizer cost is negligible.
The objectives of this problem are to 1) allocate agricultural water to maximize profits, and 2) minimizing the nitrogen runoff from the farms

$p =$ Rice price (\$ tonnes^{-1})
$L_i =$ Crop area for Farm *i* (ha)
$Y_i = Y_{0i} + \gamma_i F_i$ = Net yield Farm *i* (tonnes ha^{-1} season^{-1})
$Y_{0i} =$ Nominal yield Farm *i* (tonnes ha^{-1} season^{-1})
$\gamma_i =$ Fertilizer enhanced yield coefficient for Farm *i* (tonnes crop (kg fertilizer)$^{-1}$)
$F_i =$ Amount of fertilizer applied to Farm *i* (kg ha^{-1} season-1)
$N_i = \eta_i F_i L_{\max i}$ = Nitrogen runoff from Farm *i* (kg season^{-1})
$L_{\max i} =$ Maximum land area Farm *i*
[Note: As a simplification, above expression assumes fertilizer is applied to entire farm]
$\eta_i =$ Fraction of applied nitrogen that runs off Farm *i* (unitless)
$R = \sum_{i=1}^{3} N_i =$ Total nitrogen runoff (kg season^{-1})
$W = \sum_{i=1}^{3} W_i L_i =$ Total water used (MCM season^{-1})
$W_i =$ Unit water requirement Farm *i* (MCM ha^{-1} season^{-1})

Resource and environmental constraints:
Water: $W \le W_{avail}$
Land: $L_i \le L_{\max i}$ for each Farm *i*

i) Define the decision variables and the two objective functions
ii) Introduce the constraints such that the two objective functions are combined into one minimization problem.
iii) Sketch the Pareto frontier for the tradeoff of revenue and Nitrogen for this problem.

Solution

The two objective functions are
Maximize revenue:

$$\underset{x}{Maximize} \quad \text{Revenue(x) (\$yr}^{-1}) = p\left(Y_{o1}L_1 + \gamma_1 F_1 L_1 + Y_{o2}L_2 + \gamma_2 F_2 L_2 + Y_{o3}L_3 + \gamma_3 F_3 L_3\right)$$

Minimize nitrogen runoff.

$$\text{Minimize } Runoff = \sum_{i=1}^{3} N_i = \sum_{i=1}^{3} \eta_i F_i L_{\max i}$$

$$= -p\frac{1}{2}\begin{bmatrix} L_1 & L_2 & L_3 & F_1 & F_2 & F_3 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 & \gamma_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma_3 \\ \gamma_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_3 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ F_1 \\ F_2 \\ F_3 \end{bmatrix} + -p\begin{bmatrix} Y_{o1} & Y_{o2} & Y_{o3} & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

$$= \frac{1}{2}x^T Hx + f^T x$$

Based on the constraints

$$\begin{bmatrix} W_1 & W_2 & W_3 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \eta_1 L_{\max 1} & \eta_2 L_{\max 2} & \eta_3 L_{\max 3} \end{bmatrix}\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ F_1 \\ F_2 \\ F_3 \end{bmatrix} \le \begin{array}{cl} \begin{bmatrix} W_{avail} \\ L_{1\max} \\ L_{2\max} \\ L_{3\max} \\ R_{\max} \end{bmatrix} & \begin{array}{l} \text{water} \\ \text{land1} \\ \text{land2} \\ \text{land3} \\ \text{nitrogen runoff (2nd Objective function)} \end{array} \end{array}$$

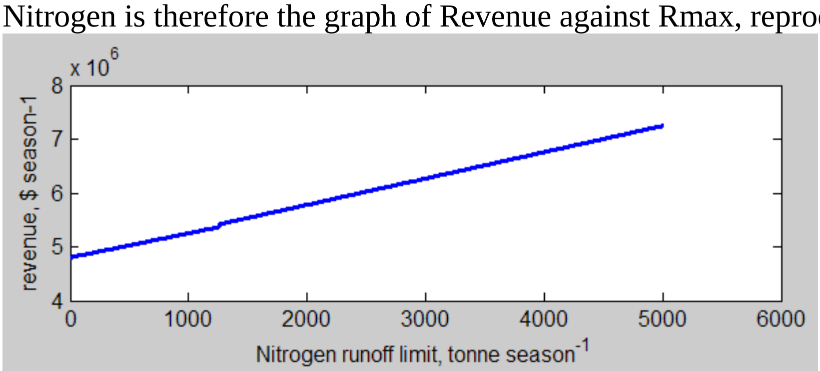
$$Ax \le b$$

And the lower bounds are

$$\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ F_1 \\ F_2 \\ F_3 \end{bmatrix} \ge \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

There are no equality constraints or upper bounds defined here, so the Aeq, beq and ub matrices are empty [].

This is now formulated in the same way as Problem Set 6. The tradeoff curve for revenue and Nitrogen is therefore the graph of Revenue against Rmax, reproduced from PS6 here.



6. Life Cycle Analysis

The process of making flat glass is complex and involves several materials, including silica sand, soda ash, limestone, dolomite and cullets (recycled glass fragments). The tables below provide a simplified list of input/outputs for some of the processes involved in the production of flat glass. Draw a process flow chart, construct technology and environmental matrices for a Life Cycle Analysis and calculate the total Antimony Emissions for a final total economic output of 20 kg of flat glass. (Note: antimony and its compounds are toxic in a similar way as arsenic).

Final product: 20 kg of flat glass

Silica Sand mining

Output	**Input**
Silica Sand: 1 kg	Silica Sand (resource):1.15 kg
Antimony: 1.7E-13 Kg	

Transportation 16t truck

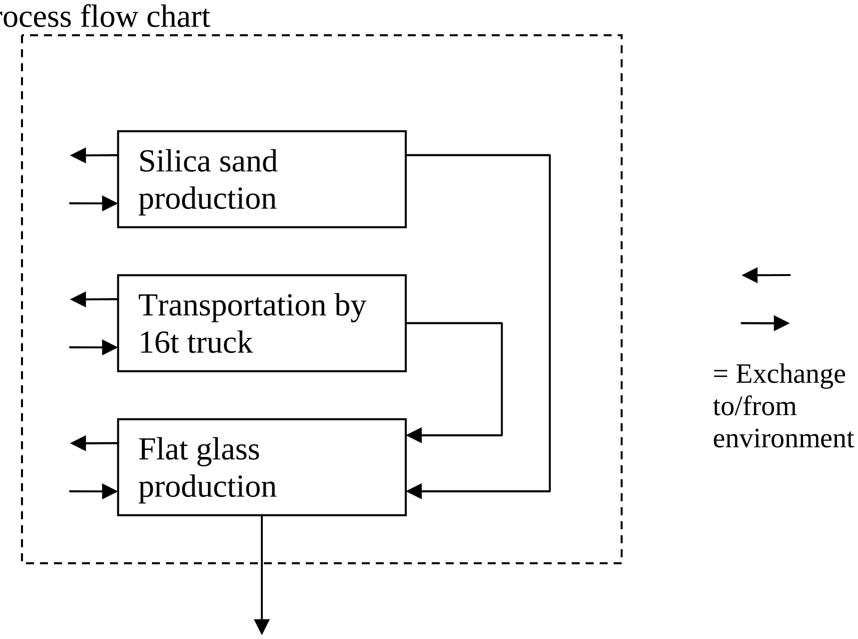
Output	**Input**
16t truck: 1 ton-km	Diesel fuel:0.21 kg
Antimony: 3.691E-9 Kg	

Flat Glass production

Output	**Input**
Flat Glass: 1 kg	Silica Sand: 0.51 kg
Antimony: 2.91E-10 Kg	Soda Ash (resource): 0.16 kg
	16t truck: 0.13 ton-km

Solution:

Process flow chart



Based on the Process Flow chart and inputs and output table,

The A matrix is (note: convention is to read down each column and account for the processes required on each row, and not vice versa)

A matrix	Flat Glass kg	Silica sand kg	Transportation 16t truck 1ton-km
Flat Glass 1kg	1	0	0
Silica sand kg	-0.51	1	0
Transportation 16t truck ton-km	-0.13	0	1

The economic matrix f is accounts for what we want out of the process; in this case it is 20 kg of Flat glass.

f matrix	Economic matrix
Flat Glass 1kg	20
Silica sand kg	0
Transportation 16t truck ton-km	0

In order to scale up the production from 1 kg to 20 kg of flat glass, we need to calculate a scaling matrix (s) from the economic matrix (f) and the technology matrix (A).

$$s = A^{-1} f = \begin{bmatrix} 1 & 0 & 0 \\ -0.51 & 1 & 0 \\ -0.13 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 20 \\ 10.2 \\ 2.6 \end{bmatrix}$$

The environmental matrix B consists of the resources and environmental variables

B matrix	**Flat Glass kg**	**Silica sand kg**	**Transportation 16t truck ton-km**
Silica sand (resource) kg	0	-1.15	0
Diesel fuel kg	0	0	-0.21
Soda ash (resource) kg	-0.16	0	0
Antimony kg	2.91E-10	1.7E-13	3.691E-9

Now the required matrices are complete for carrying out the LCA – the g matrix gives the net resources produced or taken to produce 20 kg of flat glass. This is calculated by

$$g = Bs = \begin{bmatrix} 0 & -1.15 & 0 \\ 0 & 0 & -0.21 \\ -0.16 & 0 & 0 \\ 2.91E-10 & 1.7E-13 & 3.691E-9 \end{bmatrix} \begin{bmatrix} 20 \\ 10.2 \\ 2.6 \end{bmatrix} = \begin{bmatrix} -11.73 \\ -0.546 \\ -3.2 \\ 1.54E-8 \end{bmatrix}$$

So the results can be tabulated as follows:

g matrix	**Resources produced for 20 kg Flat glass**
Silica sand (resource) kg	-11.73
Diesel fuel kg	-0.546
Soda ash (resource) kg	-3.2
Antimony kg	1.54E-8