1.010 Uncertainty in Engineering
Fall 2008

1.010 - Brief Notes # 2
Random Variables: Discrete Distributions

- **Discrete Distributions**

  - Probability Mass Function (PMF)

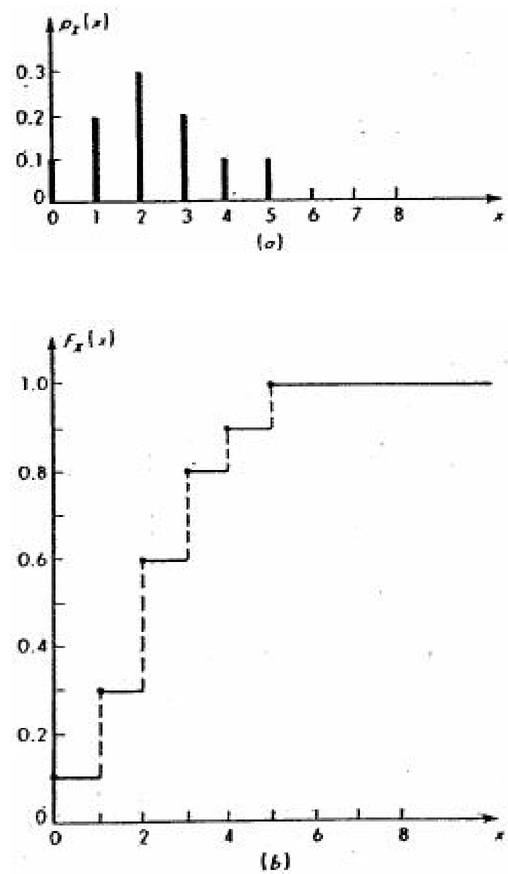
    $P_X(x) = P(X = x) = \sum_{\text{all O: } X(O)=x} P(O)$

  - Properties of PMFs
    1. $0 \le P_X(x) \le 1$
    2. $\sum_{\text{all } x} P_X(x) = 1$

  - Cumulative Distribution Function (CDF)

    $F_X(x) = P(X \le x) = \sum_{u \le x} P_X(u)$

  - Properties of CDFs
    1. $0 \le F_X(x) \le 1$
    2. $F_X(-\infty) = 0$
    3. $F_X(\infty) = 1$
    4. if $x_1 > x_2$, then $F_X(x_1) \ge F_X(x_2)$



Discrete distributions
(a) Probability Mass Function PMF
(b) Cumulative Distribution Function CDF

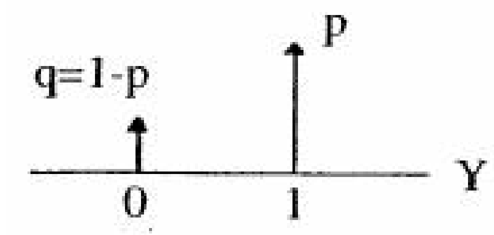
- **Examples of discrete probability distributions**

  - Bernoulli distribution

    $$Y = \begin{cases} 1, & \text{if an event of interest occurs (success)} \\ 0, & \text{if the event does not occur (failure)} \end{cases}$$

    Y is called a Bernoulli or indicator variable

    $$P_Y(y) = \begin{cases} p, & y = 1 \\ q = 1 - p, & y = 0 \end{cases}$$



  - Geometric distribution

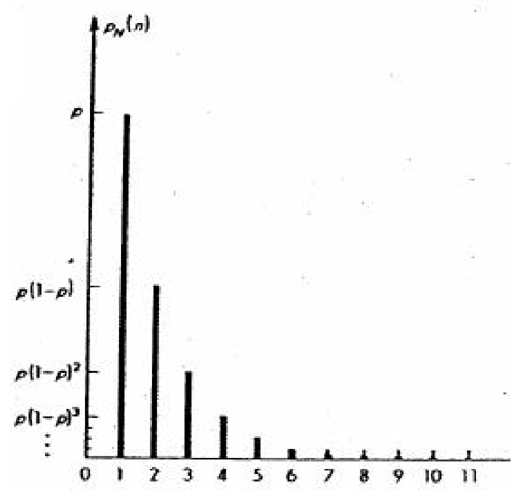
    Sequence of Bernoulli trials

    N = number of trials at which first success occurs

    $N = 1, 2, 3, \ldots$

    $P_N(n) = P(N = n) = (1-p)^{n-1}p$

    $F_N(n) = \sum_{i=1}^{n} P_N(i) = \sum_{i=1}^{n} (1-p)^{i-1}p = 1 - (1-p)^n$



Geometric Distribution

  - Binomial distribution

    Consider a sequence of Bernoulli trials

    Let M = number of successes in n trials

    $M = 1, 2, 3, \ldots, n$

    $P_M(m) = \dfrac{n!}{m!(n-m)!} p^m q^{n-m},$

    where $\dfrac{n!}{m!(n-m)!} = \binom{n}{m}$ = binomial coefficient

    where p and $q = 1 - p$ are the probabilities of success and failure in individual Bernoulli trials

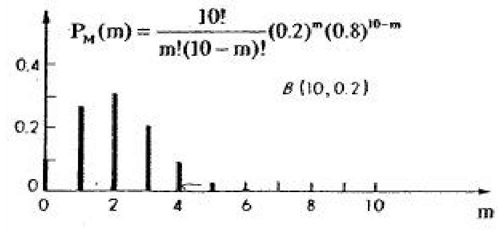
    In particular, the probability of no success is:

    $P_M(0) = q^n = (1-p)^n$

    $P_M(0) = 1 - pn$, if $pn << 1$

    and the probability of all successes is:

    $P_M(n) = p^n$



Binomial distribution B(n,p)

- Poisson distribution

  Assumptions:

  1. In a time interval of short duration Δ, the probability of one occurrence is $\lambda\Delta$, where

     $\lambda =$ occurrence rate (expected number of occurrences per unit time).

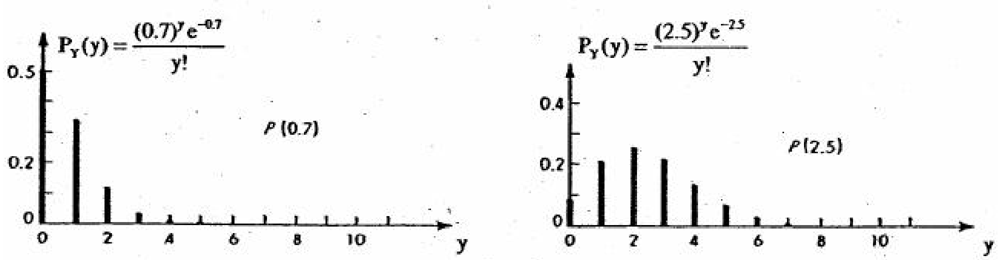
  2. The probability of two or more occurrences in Δ is negligible.
  3. The occurrences in non-overlapping intervals are independent.

  Under these conditions, the number of occurrences in each interval of duration Δ is either 0 or 1, with probability $p = \lambda\Delta$ of being 1. Let Y = no. of occurrences in [0, t], where $t = n\lambda$. Then Y has binomial distribution with probability mass function

  $P_Y(y) = \binom{n}{y} p^y q^{n-y}$, where $p = \lambda\Delta = \lambda\frac{t}{n}$

  As $n \to \infty$,

  $P_Y(y) = \frac{(\lambda t)^y e^{-\lambda t}}{y!}$ (Poisson PMF)



Poisson distribution P(λt)