

10.302
Fall 2004
FINAL EXAMINATION
December 14, 1004

GOOD LUCK!

Problem 1 (40 points)

Consider a small droplet ($d_p = 1$ mm) of n-butane that falls through still air at 325K. As the droplet evaporates, it will cool, but then reach a steady temperature. It is agreed that radiation and diffusion-induced convection may be neglected.

- A. What is the value of the steady temperature?
- B. What is the value of the convective heat transfer coefficient that characterizes heat transfer from the air to the droplet?
- C. Very approximately (seconds, minutes, or hours), how long will it take to reach this steady temperature? Justify.
- D. If diffusion-induced convection had been included in the analysis, would the steady state temperature have been higher or lower? Explain.

DATA

$$\begin{aligned} D_{\text{air}/c_4} &= 0.22 \text{ cm}^2/\text{s} \\ \rho_{\text{air}} &= 1.16 \text{ kg/m}^3 \\ v_{\text{air}} &= 1.6 \times 10^{-5} \text{ m}^2/\text{s} \\ c_{p, \text{air}} &= 1000 \text{ J/kg}\cdot\text{K} \\ \alpha_{\text{air}} &= 2.2 \times 10^{-5} \text{ m}^2/\text{s} \\ \Delta H_{\text{VAP}} &= 400,000 \frac{\text{J}}{\text{kg}} \\ \text{MW}_{c_4} &= 58 \\ \rho_{c_4} (\text{liq.}) &= 580 \text{ kg/m}^3 \\ c_{p, c_4} (\text{liq.}) &= 2000 \text{ J/kg}\cdot\text{K} \\ k_{c_4} (\text{liq.}) &= 0.15 \text{ w/m}\cdot\text{K} \end{aligned}$$

The vapor pressure curve for n-butane is attached.

Figure removed due to copyright restrictions. [Diagram of vapor pressure (mole fraction) versus temperature for n-butane.]

FIGURE 1

Problem 2 (60 points)

Many current models of automobiles have a readout in the passenger compartment that indicates ambient temperature. The temperature sensor for this instrument is usually located in the air intake manifold for the motor, downstream of the air filter. A sketch of the sensor, and the long circular duct in which it is located, is shown in Figure 2. A thermocouple is located inside a small diameter stainless steel sheath that protrudes through the wall of the duct.

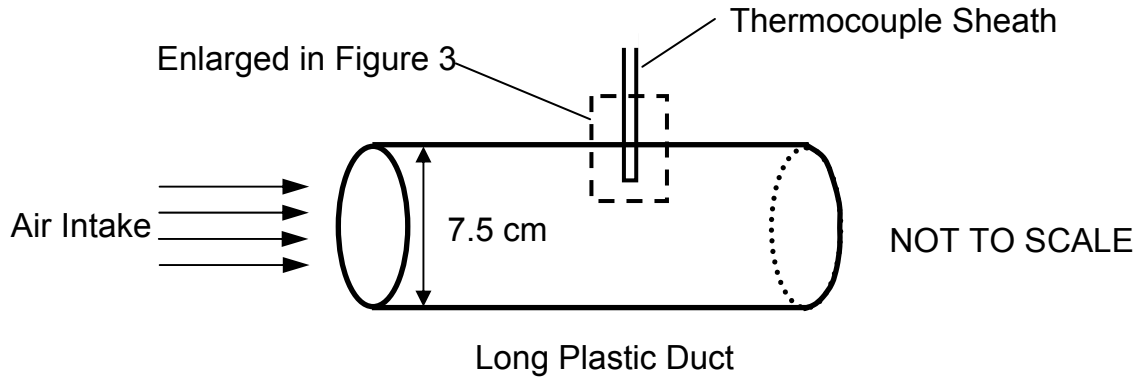


FIGURE 2

The duct is 7.5 cm in diameter and carries air at between 2000 and 8000 liters/min depending on engine speed. The driver of the car would like the thermocouple to indicate an accurate air temperature independent of the engine speed and temperature of the wall of the air duct. An expanded sketch of the thermocouple and its sheath is given in Figure 3.

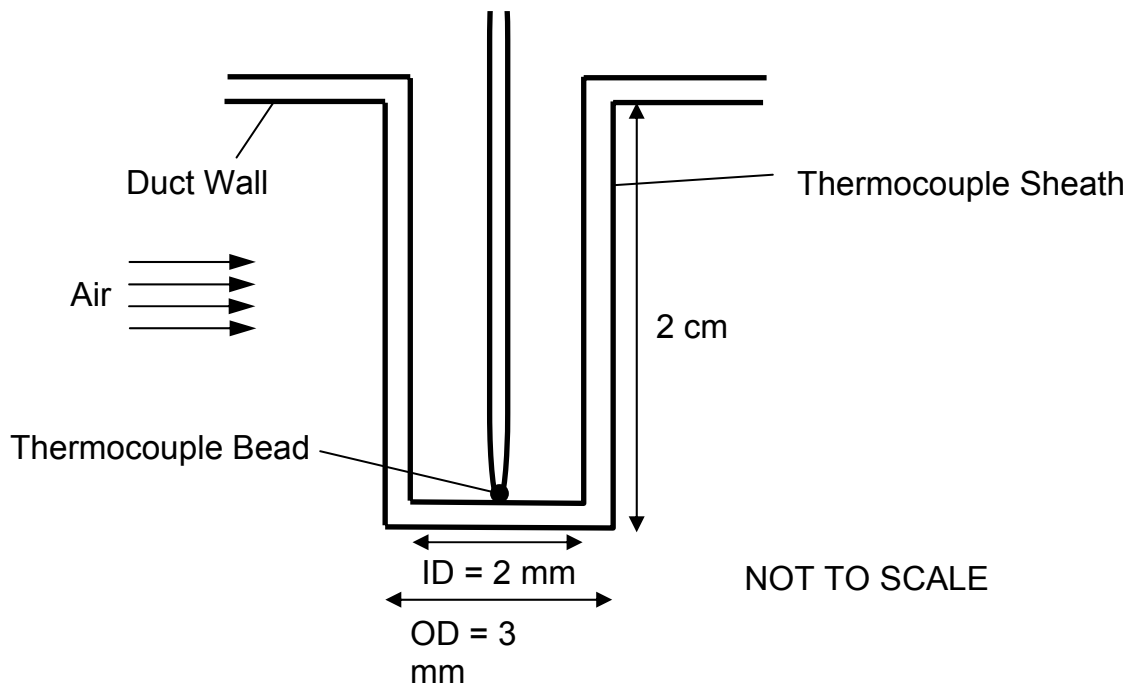


FIGURE 3

The thermocouple sheath is made of stainless steel and has an outside diameter of 3 mm, an inside diameter of 2 mm, and a length from duct wall to thermocouple bead of 2.0 cm. Conduction along the leads of the thermocouple and within the air space between the thermocouple wires and the walls of the sheath can be neglected.

Properties of Stainless Steel

Density ρ	8000 kg/m ³
Heat capacity c	0.480 kJ/kg K
Thermal conductivity k	15 W/m K
Thermal diffusivity α	4×10^{-6} m ² /s

Properties of Air

Density ρ	1.2 kg/m ³
Heat capacity c	1.0 kJ/kg K
Thermal conductivity k	0.026 W/m K
Thermal diffusivity α	2.2×10^{-5} m ² /s
Kinematic viscosity	1.6×10^{-5} m ² /s
Prandtl number	0.7

- A. If the airflow rate in the duct is 3000 liters/min what is the average heat transfer coefficient to the thermocouple sheath?
- B. If the true air temperature is 25 °C and the wall of the duct is 80 °C what temperature will the thermocouple read? The heat transfer coefficient along the entire length of the sheath, including the convective loss from the tip, may be assumed to be that found in part A. You may neglect radiation. If you did not obtain a numerical value in part A, assume that $h = 150 \text{ W/m}^2 \text{ K}$.
- C. Estimate an average radiative heat transfer coefficient between the thermocouple sheath and the walls of the duct. The duct may be assumed to be much larger than the thermocouple sheath. The air is at 25°C and the duct wall is at 80°C.

Emissivities

Stainless steel	0.2
Plastic duct wall	0.8

- D. Derive a differential equation for the temperature distribution along the length of the thermocouple sheath incorporating both convective and radiative heat transfer. The equation should account for the physical and thermal properties of the sheath; the convective and radiative heat transfer coefficients, and the temperatures of the duct wall and air. Specify appropriate boundary conditions for your equation.
- E. The convective heat transfer coefficient for the tip of the thermocouple sheath is likely to be three times larger than the coefficient along its length. Will this fact affect the temperature reading of the thermocouple by more than ½ °C?
- F. What changes in design would you recommend to reduce the difference between reading of the thermocouple and the true air temperature? Justify your recommendations.

Problem 3 (40 points)

The role of aerosols in human health and in the solar energy budget of the Earth has become increasingly apparent. One interesting problem of this sort is the interaction of anthropogenic pollution with natural forest products. For instance, the Smoky Mountains owe their name to a natural aerosol associated with pinenes and related species. These can interact with nitrates emitted from upwind urban sources. The PROPHET site in northern Michigan is dedicated to the study of this interaction. One of its facilities is a tower (30 m) that permits sampling in the air space just above the forest canopy. Unfortunately, it is usually necessary to run sampling lines to instrumentation located on the ground.

You have been asked to assess the loss of aerosol particles to the walls of the sampling line. In other words, since most atmospheric aerosol “particles” are actually liquid, if they contact the wall, they will simply “wet” the wall and remain there. It is agreed that, since the aerosol particles are small, they are subject to Brownian diffusion, for which

$$D = \frac{kT}{6\pi\mu a} \quad (1)$$

Thus, the behavior of the particles is exactly that which might be expected of a solute. For the base case, the sampling line will be 35 m long and 1 cm in I.D. The average velocity of the air will be 2.5 m/s. For a particle diameter ($2a$) of 20 nm, Eq. 1, when corrected for mean-free-path effects, gives $D = 1 \times 10^{-8} \text{ m}^2/\text{s}$.

- A. What is the value of h_m that describes the transport of particles to the wall?
- B. What fractional loss of particles would you expect?
- C. What fractional loss would you expect if the sampling velocity were reduced from 2.5 m/s to 1 m/s?
- D. What fractional loss would you expect if the sampling velocity were increased to 6 m/s?

Properties of Air

$$\begin{aligned} \rho &= 1.16 \text{ kg/m}^3 \\ \nu &= 1.6 \times 10^{-5} \text{ m}^2/\text{s} \\ c_p &= 1000 \text{ J/kg}\cdot\text{K} \\ \alpha &= 2.25 \times 10^{-5} \text{ m}^2/\text{s} \end{aligned}$$

Problem 4 (60 points)

A small aqueous waste stream, at a flow rate of 400 cc/min, is contaminated with an organic material at a concentration of 1 mg/cc. It has been suggested that most of this contaminant can be removed by passing the waste stream through a mass exchanger where the contaminant is transferred to a non-aqueous solvent stream. The mass exchanger is a small shell-and-tube device with many tiny plastic capillaries as the tubes. The aqueous stream flows through these tubes and the solvent stream flows through the shell side. The solvent stream enters the exchanger as a pure material free of all contaminant. The walls of the plastic capillaries allow the contaminant to diffuse but no water or solvent passes through the plastic.

Geometry of the Capillaries

Inside diameter	200 μm
Wall thickness	15 μm
Length	20 cm
Number	6500
Area for mass transfer (based on inside diameter)	0.817m ²

Properties of the Fluid Streams and Capillaries

Water Stream

Density	1000 kg/m ³
Viscosity	10 ⁻³ N s/m ²
Diffusivity of contaminant	2 x 10 ⁻⁹ m ² /s

Solvent Stream

Density	850 kg/m ³
Viscosity	8 x 10 ⁻⁴ N s/m ²
Diffusivity of contaminant	10 ⁻⁹ m ² /s
Partition coefficient of contaminant $S_{\text{solvent/water}}$	10

Capillary Tubes

Diffusivity	2 x 10 ⁻¹¹ m ² /s
Partition coefficient of contaminant $S_{\text{capillary/water}}$	10

- A. If the mass exchanger were counter current and infinitely long what would be the solvent flow rate necessary to reduce the contaminant in the aqueous stream to 0.1 mg/cc?
- B. The mass transfer coefficient on the outside (shell side) of the tubes is 7×10^{-5} m/s (based on flux in solvent-based units) at a solvent flow rate of

75cc/min. What is the overall mass transfer coefficient based on the inside area of the tubes? Please express the overall coefficient with flux based on aqueous phase concentration units.

- C. The mass exchanger is operated in counter current flow at a solvent flow rate of 75 cc/min and with the overall mass transfer coefficient found in part B. The total area for mass transfer, based on the inside diameter of the capillaries, is 0.817m^2 . What are the concentrations of contaminant in the aqueous and solvent streams leaving the exchanger? How did you select the mass capacity rate to be used in finding NTU? If you did not find an answer to part B use $1.5 \times 10^{-5} \text{ m/s}$ for the overall mass transfer coefficient based on flux in aqueous-phase concentration units.
- D. It is desired that the concentration of the contaminant in the effluent aqueous stream be 0.1 mg/ml. What flow rate of solvent is required for a counter current exchanger? You may assume that the Sherwood number on the shell side follows the relation

$$\text{Sh} = C \text{Re}^{0.5} \text{Sc}^{0.33}$$

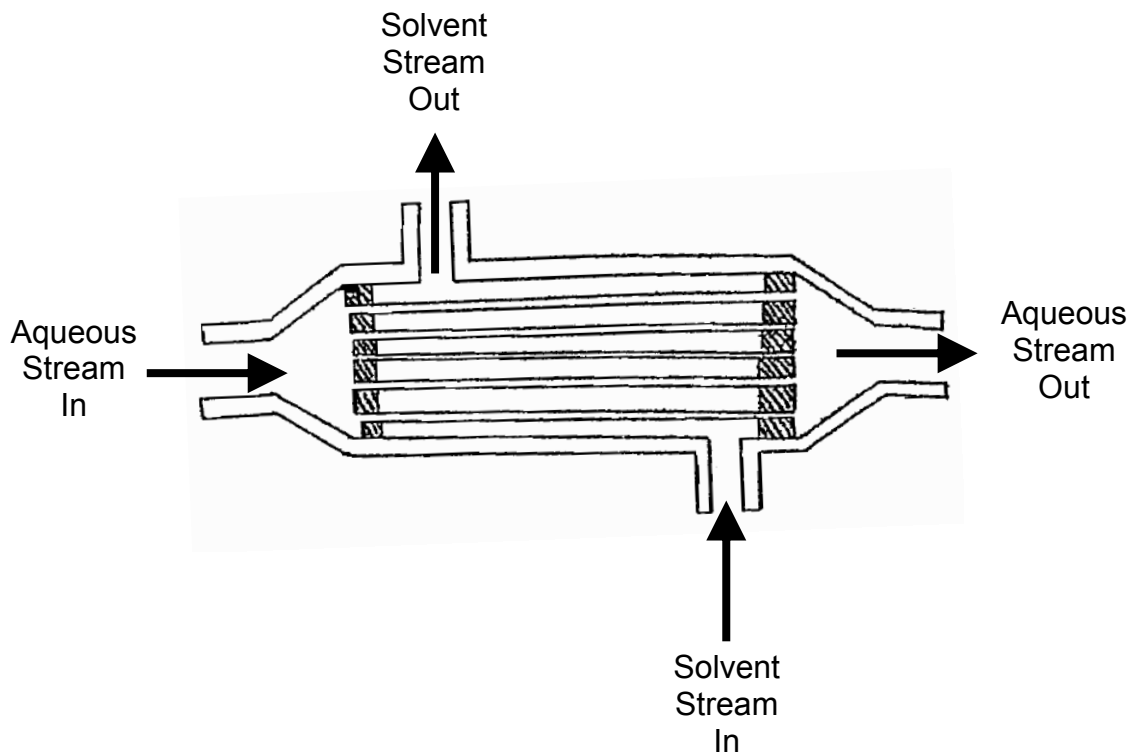


FIGURE 4