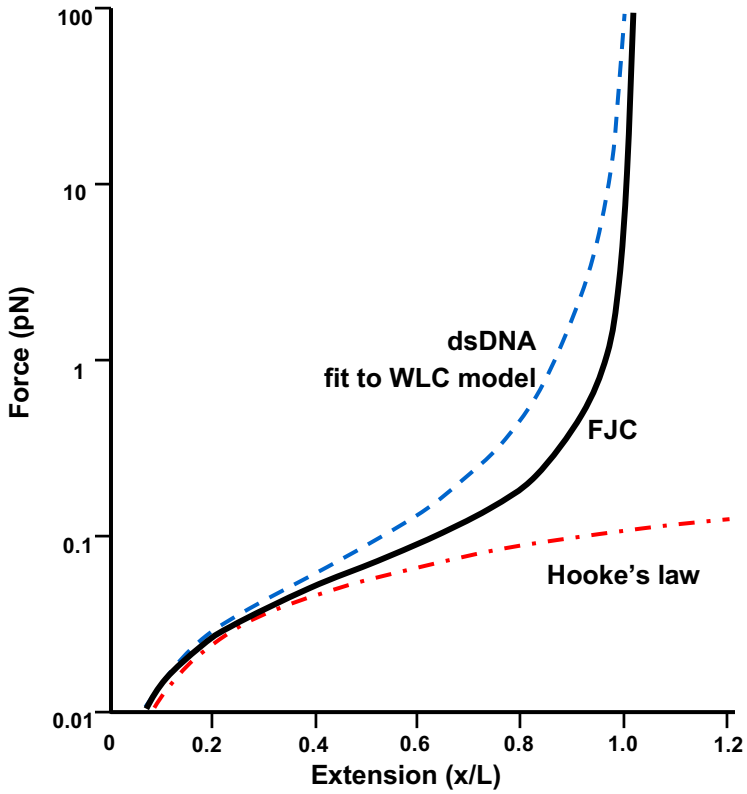
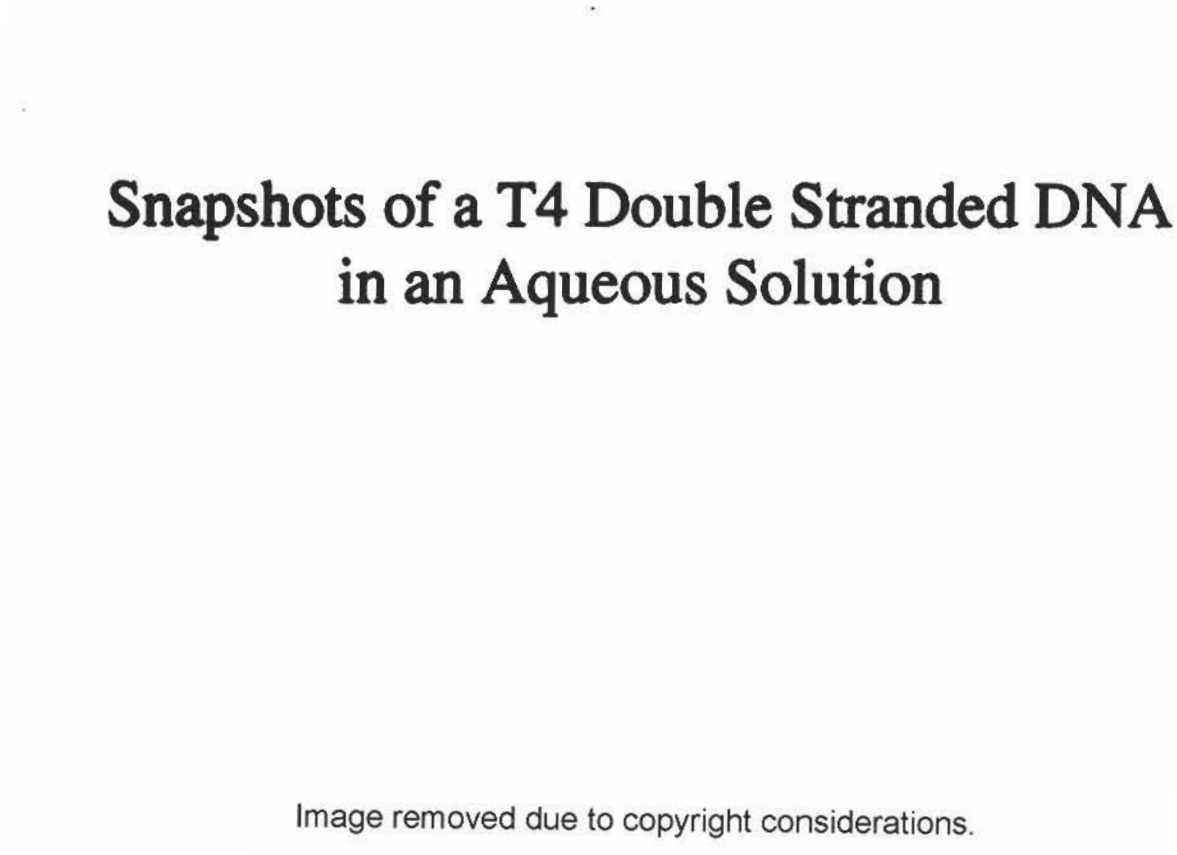


After Bustamante et al., *Current Opinion in Structural Biology*, 2001

Snapshots of a T4 Double Stranded DNA in an Aqueous Solution



P. Doyle 2003

Contour length = 64 μm

Elasticity of FJC at Arbitrary Forces

- Ensemble: constant $T, V, N, \underline{f}$ ← force
- Boltzmann Distribution:

$$P_j = \frac{1}{Q'} \exp\left(\frac{-U_j}{kT}\right) \exp\left(\frac{\underline{f}\cdot\underline{r}_j}{kT}\right)$$

- Useful Relationship:

$$Q' = \int_{\text{phase space}} \exp\left(\frac{-U_j}{kT}\right) \exp\left(\frac{\underline{f}\cdot\underline{r}_j}{kT}\right)$$

$$\frac{\partial Q'}{\partial \underline{f}} = \int_{\text{phase space}} \frac{\underline{r}_j}{kT} \exp\left(\frac{-U_j}{kT}\right) \exp\left(\frac{\underline{f}\cdot\underline{r}_j}{kT}\right)$$

$$\therefore \quad \boxed{\langle \underline{r} \rangle = \frac{kT}{Q'} \frac{\partial Q'}{\partial \underline{f}} = kT \frac{\partial}{\partial \underline{f}} \ln Q'}$$

- FJC: $Q' = \int_{\text{phase space}\ \{\underline{b}_i\}} \exp\left(\frac{\underline{f}\cdot\underline{r}_j}{kT}\right)$

$$\underline{r} = \sum_{i=1}^{N} \underline{b}_i \qquad \underline{f}\cdot\underline{b}_i = fb\cos\theta_i$$

$$Q' = \int_{\text{p.s.}} \exp\left[\frac{fb\cos\theta_1}{kT}\right] \exp\left[\frac{fb\cos\theta_2}{kT}\right] \cdots \exp\left[\frac{fb\cos\theta_N}{kT}\right]$$

decoupled

$$Q' = \left[\int_0^{2\pi} \int_0^{\pi} \exp\left[\frac{fb\cos\theta}{kT}\right] \sin\theta \, d\theta \, d\phi \right]^N$$

$$Q' = \left[\frac{2\pi \sinh\left(\frac{fb}{kT}\right)}{fb/kT} \right]^N$$

$$\langle \underline{r} \rangle = kT \frac{\partial}{\partial \underline{f}} \ln Q' = NkT \frac{\partial}{\partial \underline{f}} \ln \left[\frac{2\pi \sinh\left(\frac{fb}{kT}\right)}{fb/kT} \right]$$

$$\langle \underline{r} \rangle = \underbrace{Nb}_{\text{chain length}} \underbrace{\left[\coth\left(\frac{fb}{kT}\right) - \frac{kT}{fb} \right]}_{\text{Langevin function } \mathcal{L}\left(\frac{fb}{kT}\right)} \underbrace{\frac{\underline{f}}{f}}_{\text{unit vector in direction of } \underline{f}}$$

$$\coth(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

Limits

* Small fb/kT

$$\langle \underline{r} \rangle \approx \frac{1}{3} \frac{fb}{kT} Nb \left(\frac{\underline{f}}{f}\right) \iff$$

recall Gaussian chain:

$$\underline{r} = \frac{1}{3} Nb \left\langle \frac{\underline{f} b}{kT} \right\rangle$$

* large fb/kT

$$\langle \underline{r} \rangle \approx Nb \left[1 - \frac{kT}{fb} \right] \left(\frac{\underline{f}}{f}\right)$$