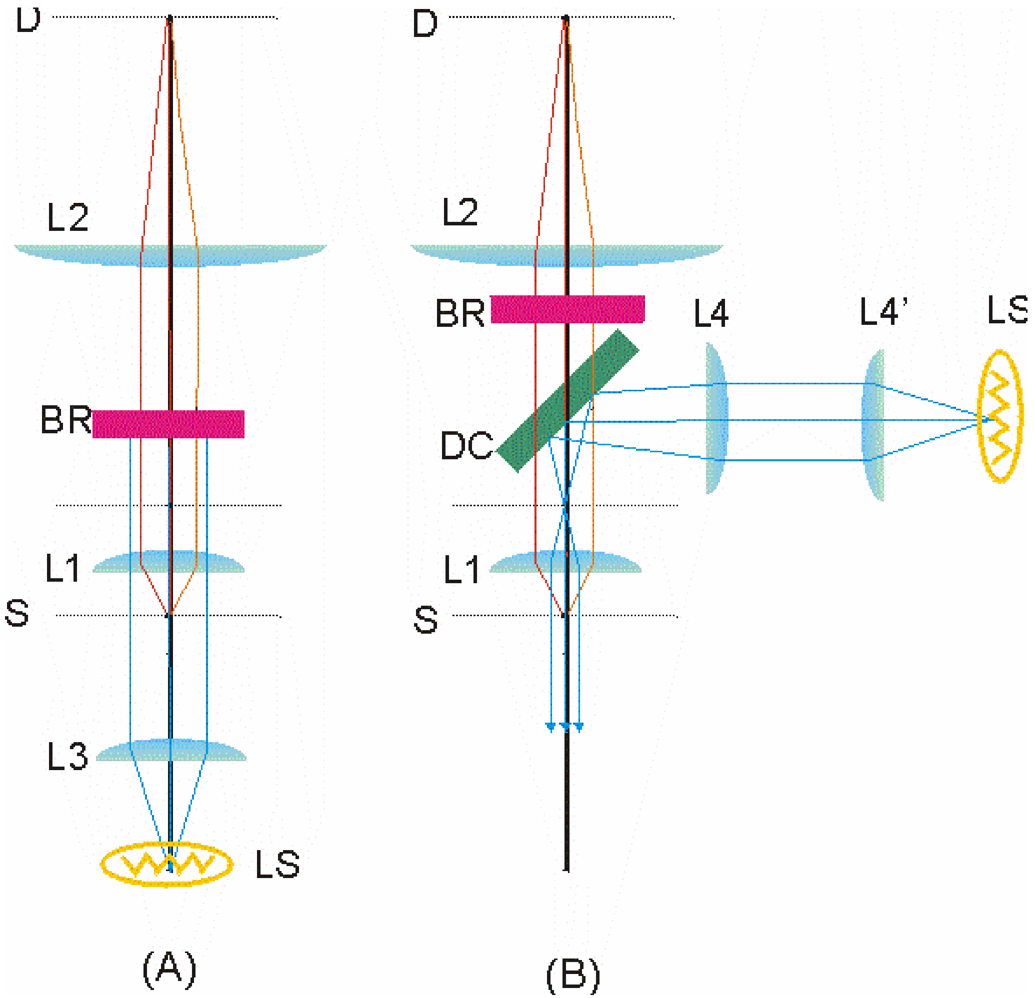
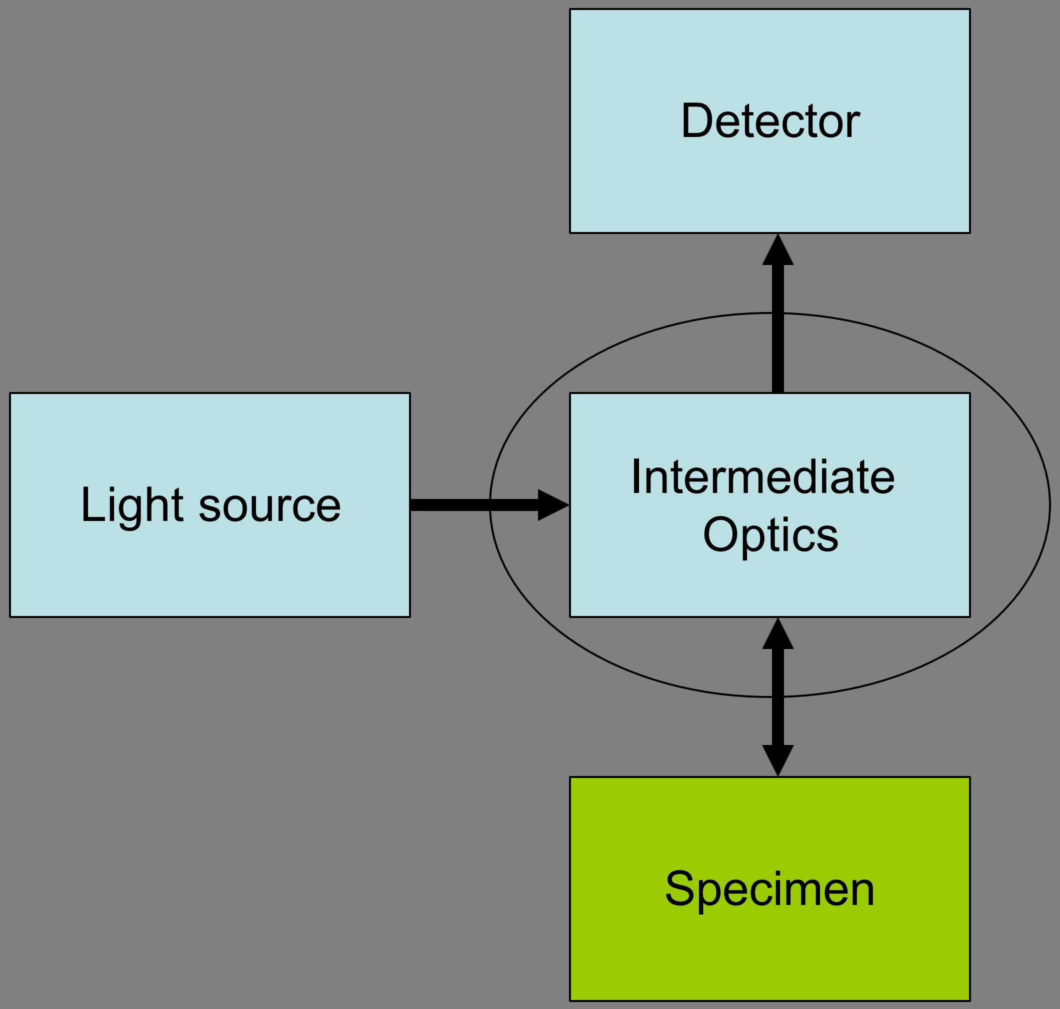
Optics and Microscopy I

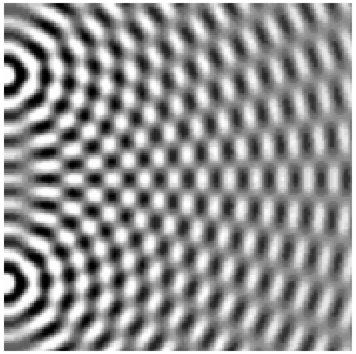


A typical biomedical optics experiment

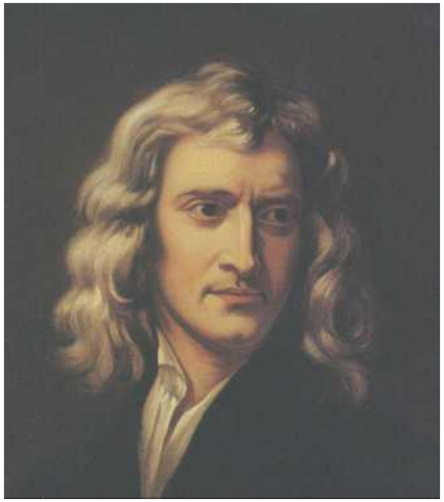


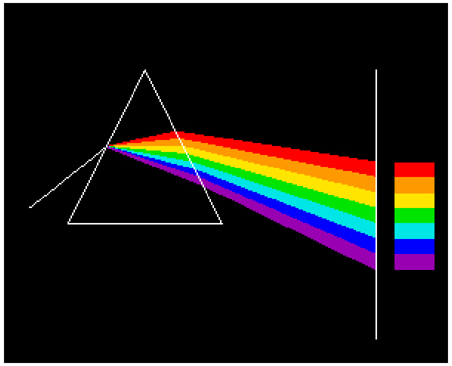
Wave and Particlel Nature of Light

Wave Nature of Light -- Huygen



Particle Nature of Light -- Newton





Physical Optics – Wave nature of light

Maxwell and His Equations

$$\nabla \cdot \vec{E} = 4pr$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$$

$$\nabla \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4p}{c}\vec{J}$$

Wave Equations

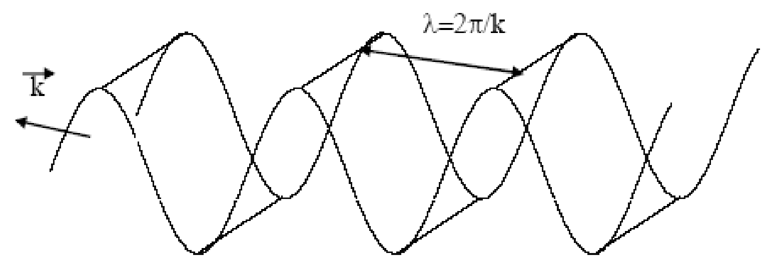
$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\nabla^2 \vec{B} - \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

Plane Wave Solution

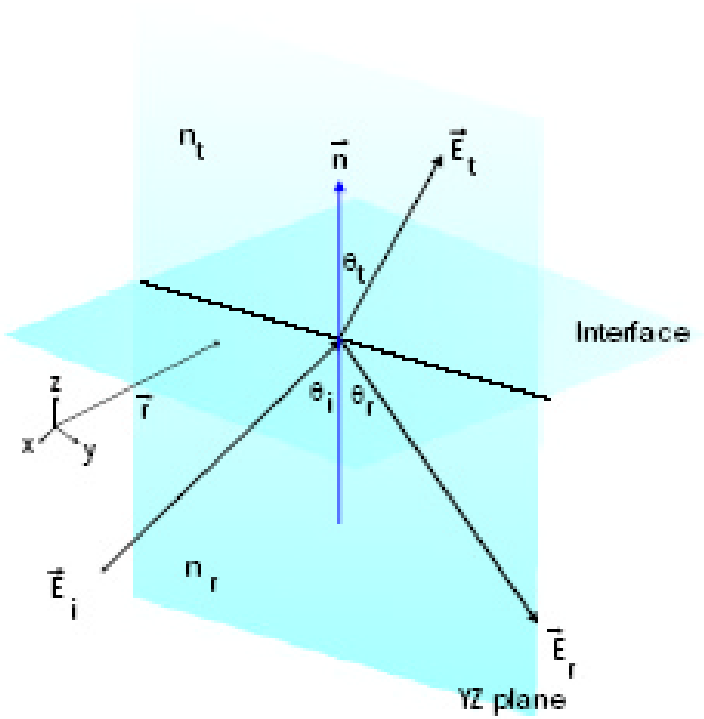
$$\vec{E}(x,t) = \vec{E}_0 \cos(kx - wt)$$

$$k = \frac{2p}{l} \quad w = 2pf \quad ck = w$$



Plane wave propagates like a “ray” of light

Reflection and Refraction of Light at Boundary



Reflection

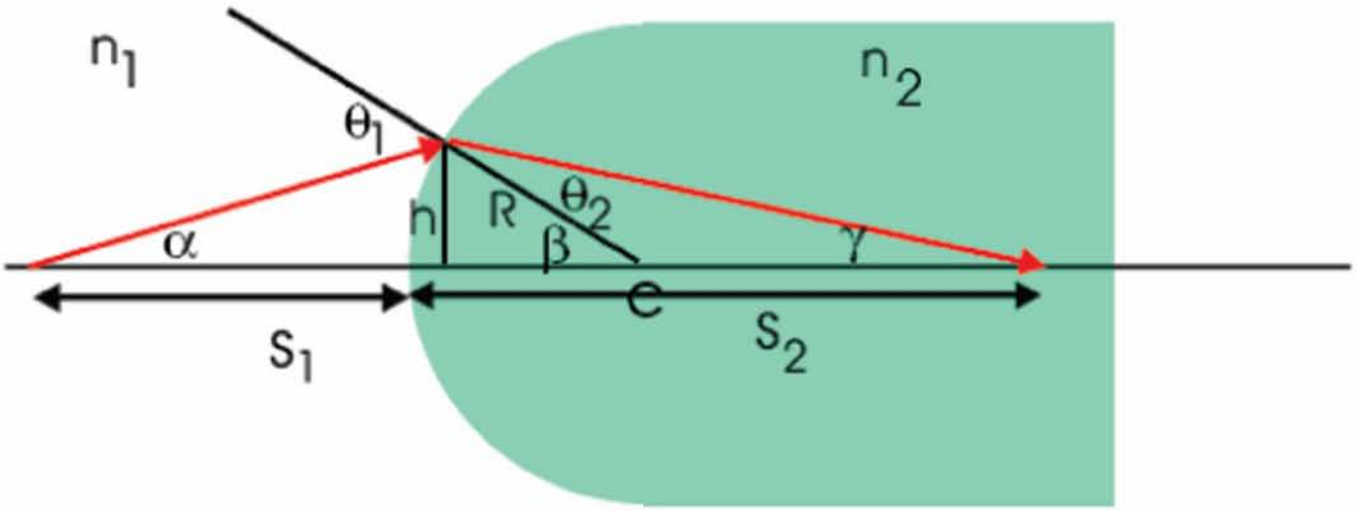
$$\sin q_i = \sin q_r$$

Refraction (Snell's Law)

$$n_i \sin q_i = n_t \sin q_t$$

Refraction at a spherical surface

Let's look at one of the simplest case of how light is transmitted through a spherical dielectric (glass)-air interface



We can trace a ray originating at a distance S_1 from the interface. How far from the interface (S_2) will the ray intersect the axis of the spherical interface?

This question can be settled by Snell's law and ray tracing:

From Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Also from geometry: $\theta_1 = \alpha + \beta$ and $\theta_2 = \beta - \gamma$.

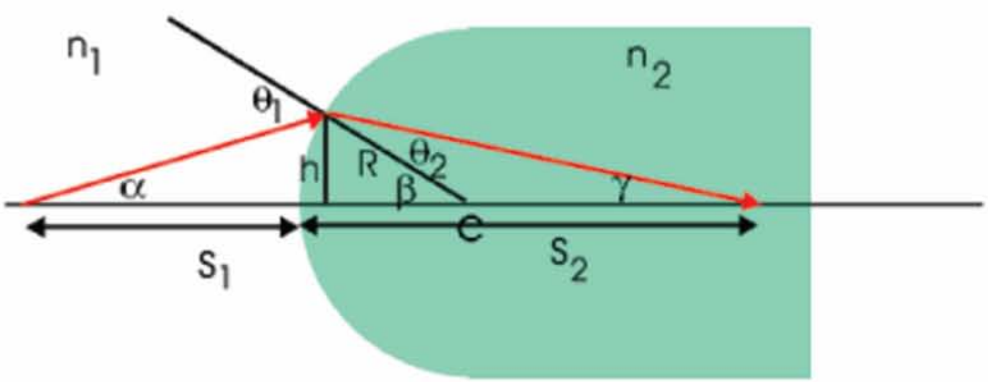
The solution is complicated unless all the angles are small. We will first consider this case. In this case, the sine of the sum of two small angles is the sum of the sines:

$$\sin\theta_1 = \sin(\frac{h}{S_0}) + \sin(\frac{h}{R}) = \frac{h}{S_0} + \frac{h}{R}$$

Rename S_1 to S_o
S_2 to S_i

$$\sin\theta_2 = \sin(\frac{h}{R}) - \sin(\frac{h}{S_i}) = \frac{h}{R} - \frac{h}{S_i}$$

From Snell's law:



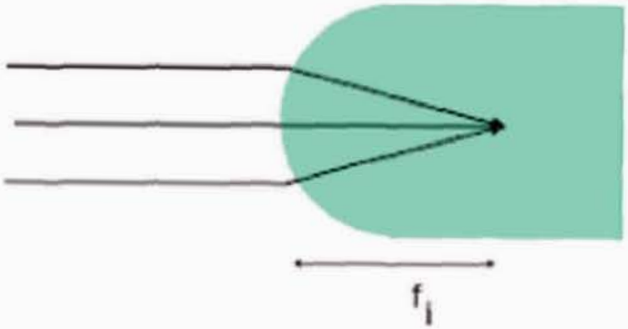
$$\frac{n_1}{S_0} + \frac{n_1}{R} = \frac{n_2}{R} - \frac{n_2}{S_i}$$

This simplies to:

$$\frac{n_1}{S_0} + \frac{n_2}{S_i} = \frac{n_2 - n_1}{R} \qquad (1)$$

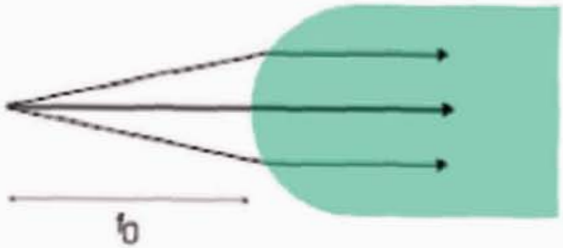
The assumption of small angle is called the paraxial approximation.

Let's look at two special situations: the conversion of spherical wave to plane wave.



When parallel (collimated) rays are converging at a convex surface, it will be focus at a distance, f_i, the image focus.

$$f_i = \frac{n_2}{n_2 - n_1} R$$



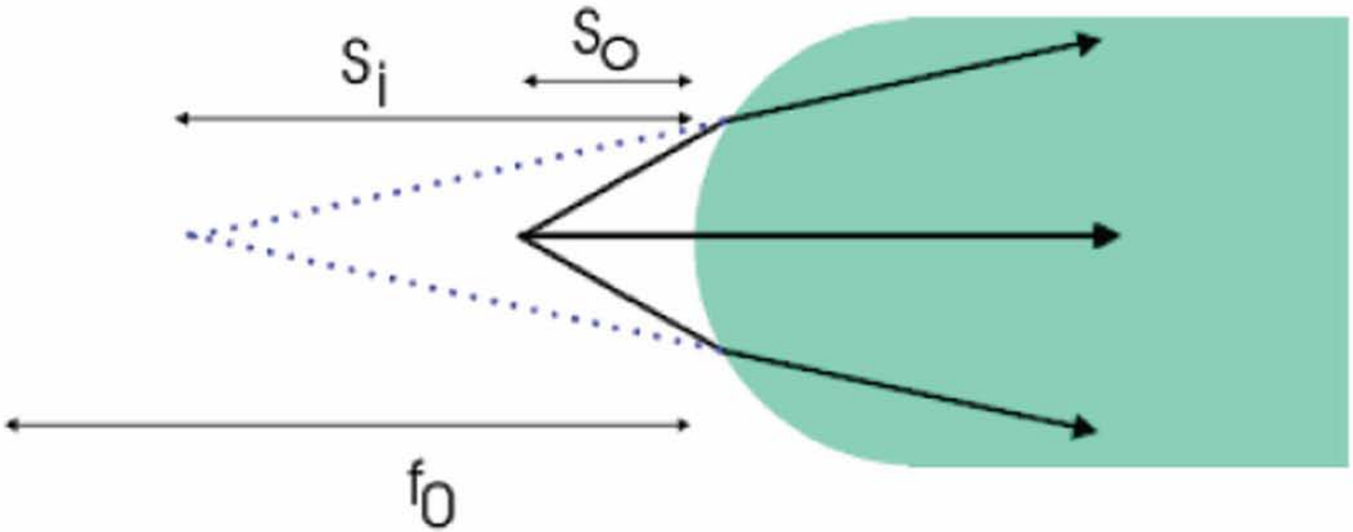
Conversely, when a spherical wave emerges from a point at a distance, f_o, from the lens, it will become collimated. This distance is called the object focus.

$$f_o = \frac{n_1}{n_2 - n_1} R$$

Derivation of the Lensmaker's Formular

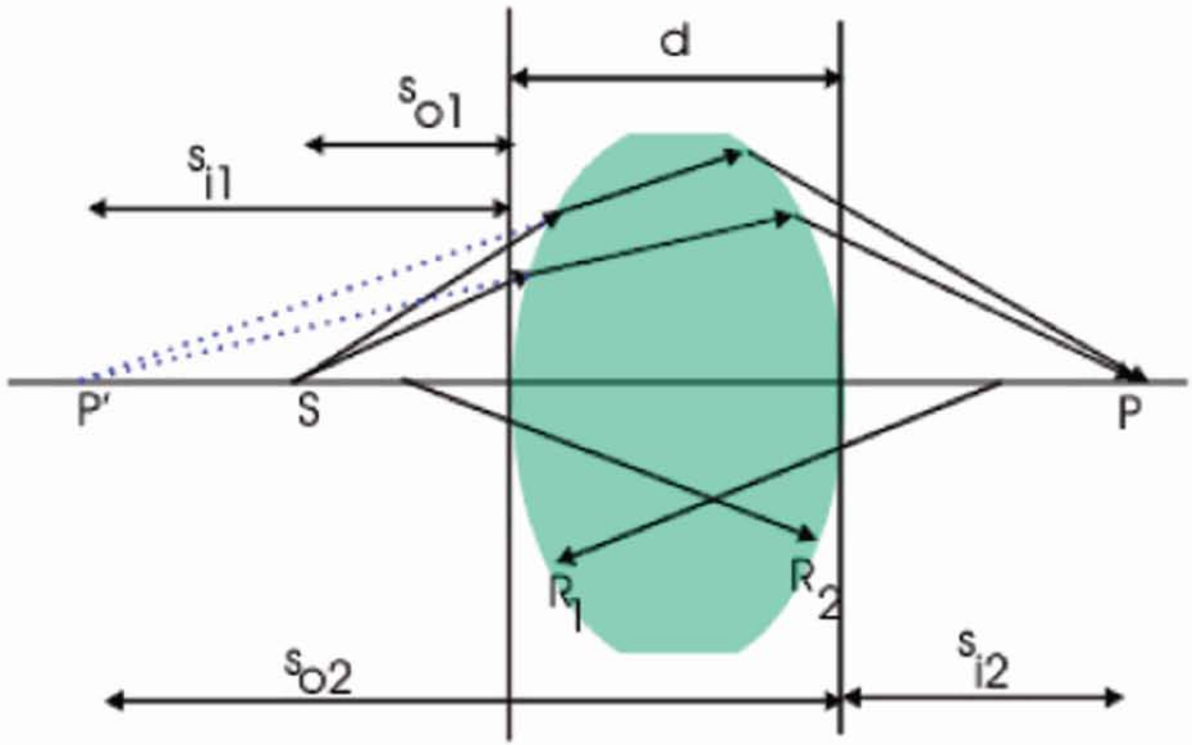
What happen to the focal distance S_i, when the point source of light is moved to a distance shorter than the object focus, P_o?

From equation **(1)** we see that S_i becomes negative. This correspond to the formation of a virtue image.



A lens is basically two spherical glass surfaced bond together back-to-back. Let's extend the virtual image forming case and place a 2nd interface to the right.

A combination of two dielectric interfaces:

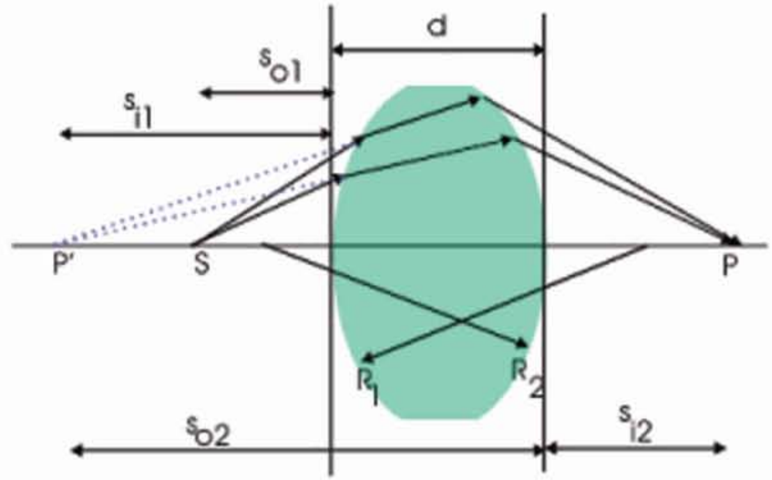


For the first interface, we form a virtual image at P'.

From **(1)** we have:

$$\frac{n}{s_{o1}}+\frac{n'}{s_{i1}}=\frac{(n-n')}{R_1} \qquad \textbf{(2)}$$

Note that S_{i1} is negative as it is virtual. For the 2nd interface, the incoming rays will appear to originate from P'. Therefore the object distance, So2, is



$$s_{o2} = d - s_{i1} = d + |s_{i1}|$$

and from (1) again:

$$\frac{n'}{d + s_{i1}} + \frac{n}{s_{i2}} = \frac{(n'-n)}{R_2} \qquad (3)$$

Adding (2) & (3), we got:

$$\frac{n}{s_{o1}} + \frac{n}{s_{i2}} = (n'-n)(\frac{1}{R_1} - \frac{1}{R2}) + \frac{n'd}{(s_{i1} - d)s_{i1}} \qquad (4)$$

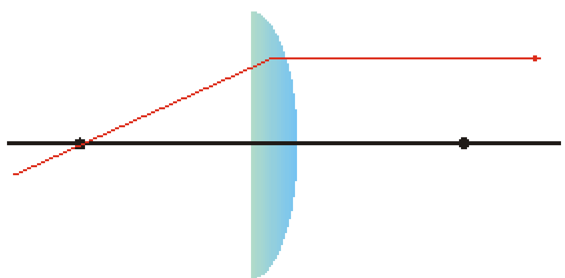
For the case of thin lens, the last term can be neglected. Further, if the lens is place in air, n=1, we get the familiar Lensmaker's equation.

$$\frac{1}{S_{o1}} + \frac{1}{S_{i2}} = (n'-1)(\frac{1}{R_1} - \frac{1}{R_2})$$

Simple Ray Tracing I

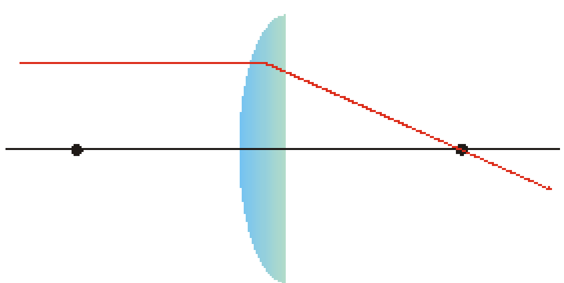
Ray Tracing is just based on the application of Snell's law to a curved (spherical) surface. We will focus on 4 simple rules of ray tracing.

Rays pass through the focal point becomes parallel to the optical axis.
Rays parallel to the optical axis are deflected through the focal point.



(1)

Rays originated from the focal point emerge parallel to the optical axis

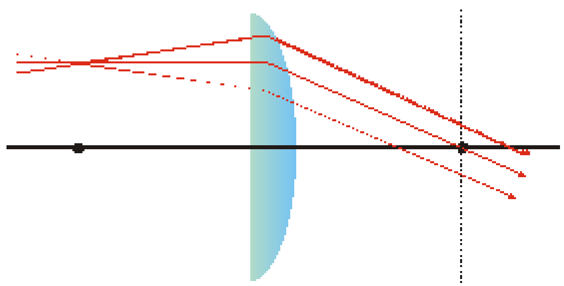


(2)

Rays parallel to the optical axis converges to the focal point

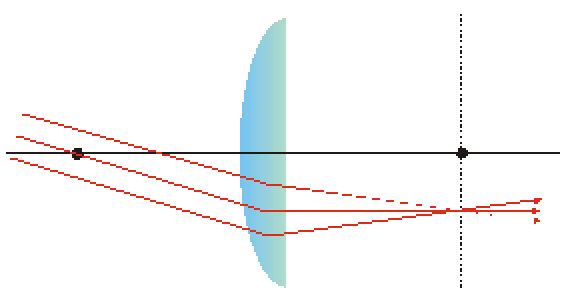
Simple Ray Tracing II

Rays originate from the focal plane becomes collimated.
Collimated rays converges at the focal plane.



(3)

Rays originated from the plane emerge collimated

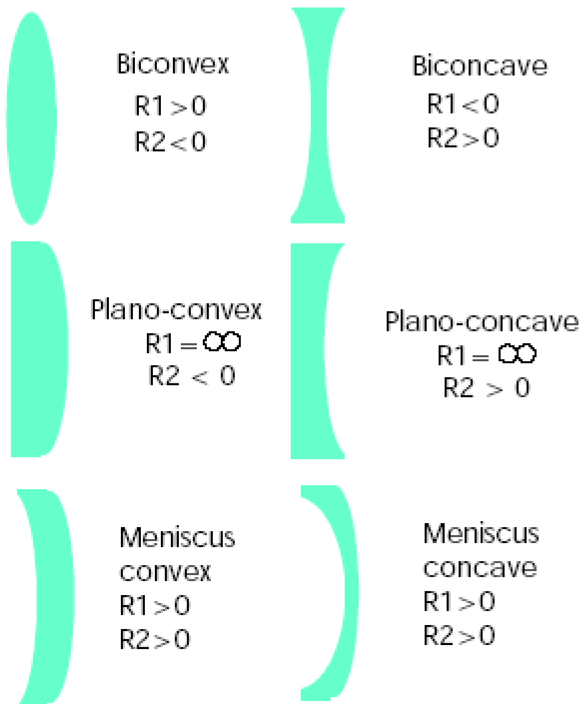


(4)

Collimated rays emerge focus at The focal plane

Optical elements I: Lens

We have been using lens throughout this lecture, it may be useful to pause for a moment to describe what are the typical terminology associated with lens.



Optical element II: mirrors, prism, apertures

These are common optical elements that is mostly self explanatory and I will not spend much time on these.

Mirrors: Mirrors has similar terminology as lens but only has one surfave.

Prisms: Prisms has a number of applications such as dispersing different color of light and directing light and image.

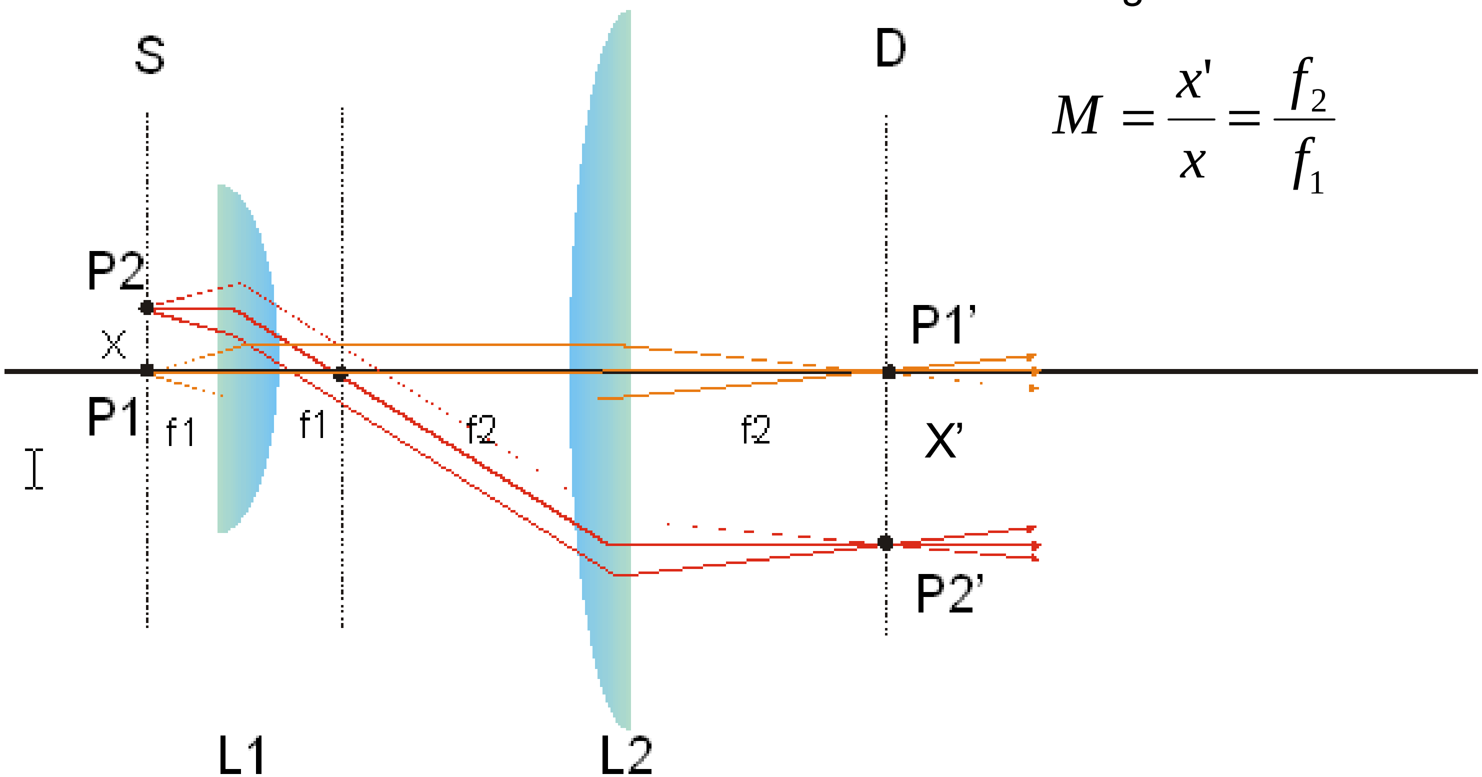
Apertures & Stops: As discussed before, aperture and stops serves to define optical path and to minimize aberration effect by eliminating non-paraxial rays.

Optical Microscopy I

Detection path of an optical microscope. Note that at the detector, the magnification is the ratio of the focal length of the objective and the tube lens.

Magnification

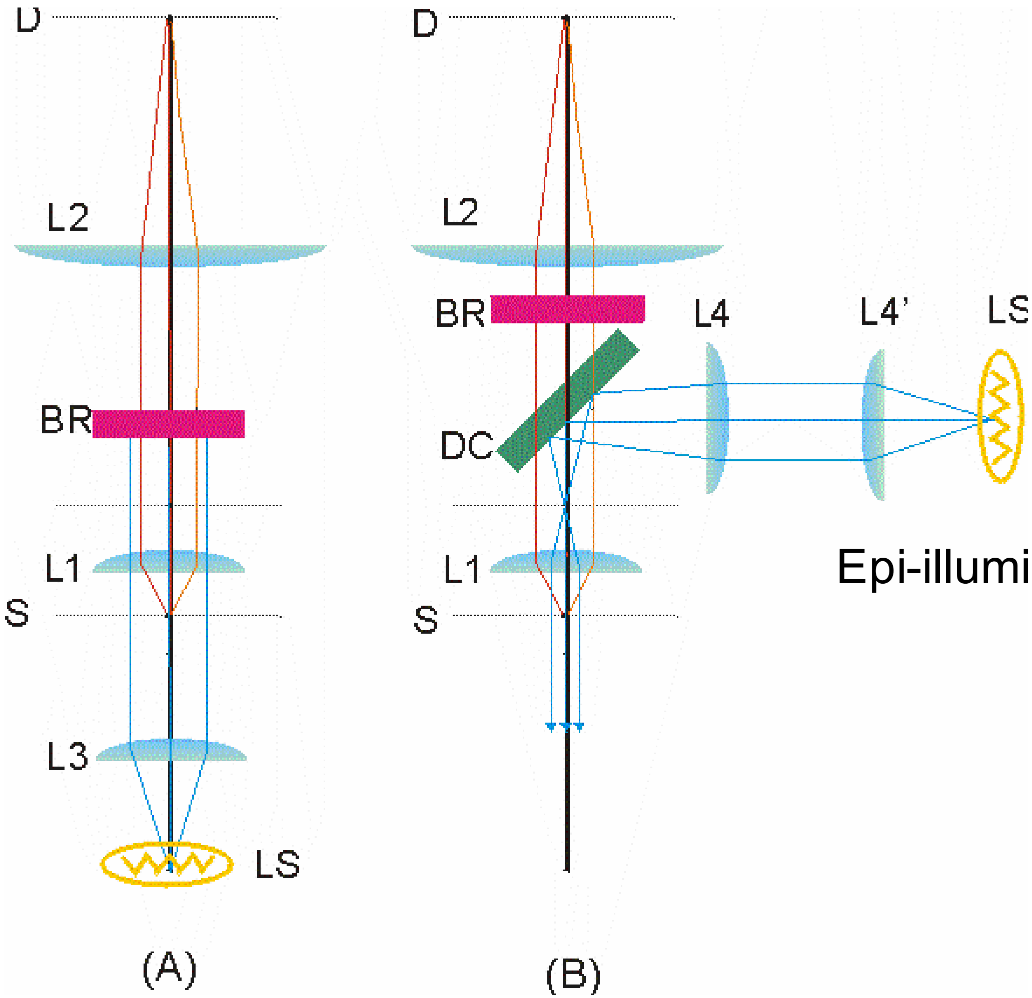
$$M = \frac{x'}{x} = \frac{f_2}{f_1}$$



Optical Microscopy III

Kohler illumination ensure that the structure of the light source (such as the filament of lamp) is not imaged at the specimen.

Trans-illumination



Epi-illumination

Interference I

Consider combining two plane waves:

$$\vec{E}_1(\vec{r},t) = \vec{E}_{01}\cos(\vec{k}_1\cdot\vec{r} - wt)$$

$$\vec{E}_2(\vec{r},t) = \vec{E}_{02}\cos(\vec{k}_2\cdot\vec{r} - wt)$$

$$\left|\vec{k}_1\right| = \left|\vec{k}_2\right| = k$$

The combined field is

$$\vec{E}(\vec{r},t) = \vec{E}_{01}\cos(\vec{k}_1\cdot\vec{r} - wt) + \vec{E}_{02}\cos(\vec{k}_2\cdot\vec{r} - wt)$$

The brightness or intensity is the “mean square” of the field

$$I = \frac{1}{T}\int_0^T \vec{E}(\vec{r},t)\cdot\vec{E}^*(\vec{r},t)dt \equiv \left\langle E^2\right\rangle$$

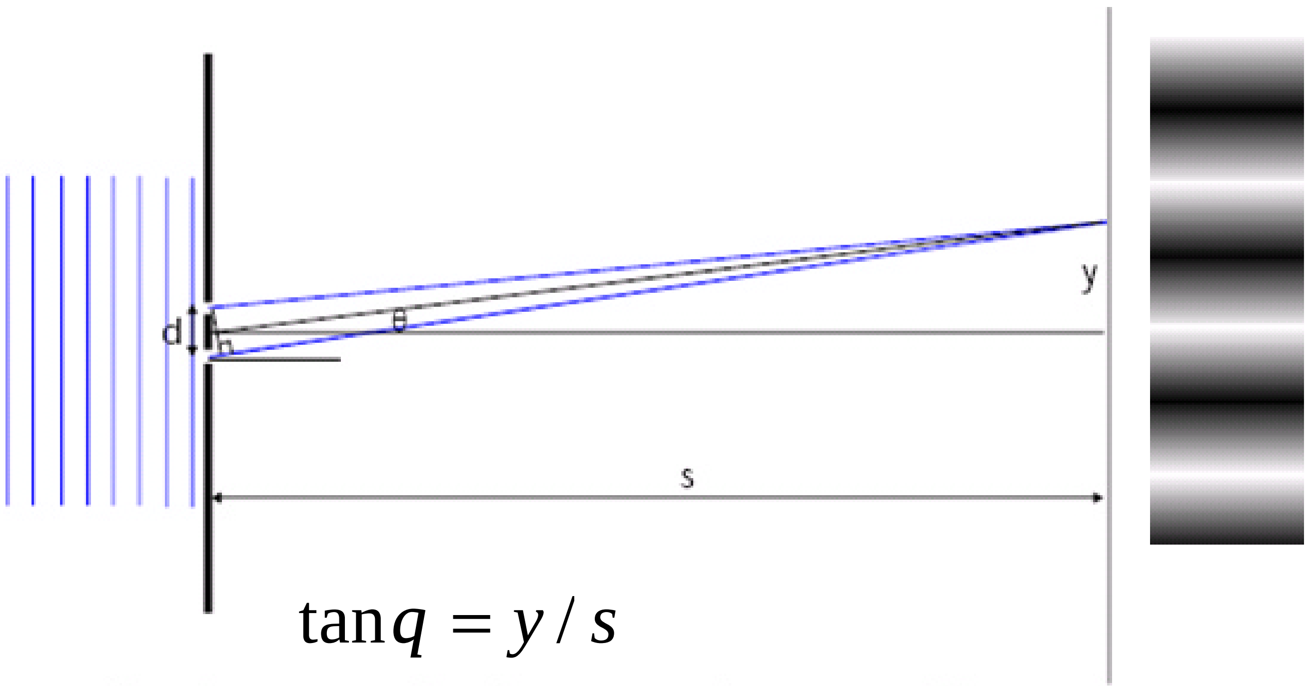
$$= \frac{E_{01}^2}{2} + \frac{E_{02}^2}{2} + 2\sqrt{\frac{E_{01}^2}{2}\frac{E_{02}^2}{2}}\cos[(\vec{k}_1 - \vec{k}_2)\cdot\vec{r}]$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos d$$

δ is a phase factor measuring the path length difference of the two beams at $\vec{r}$ multiplied by k

Interference

Consider two thin slits separated by d. What is the intensity of light at a screen distance s away at a position y?



Constructive

$$d = 0, 2p, 4p, \cdots$$

$$d \sin q = m l$$

Destructive

$$d = p, 3p, 5p \cdots$$

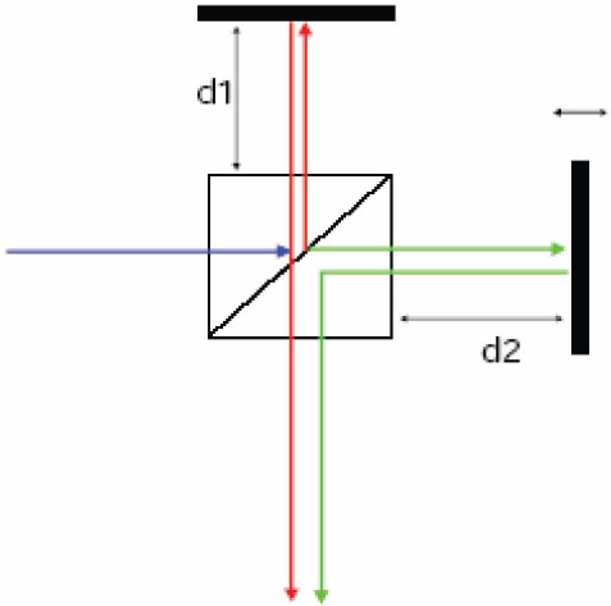
$$d \sin q = (m + \frac{1}{2}) l$$

$$\tan q = y / s$$

$$d = kd \sin q = \frac{2pd \sin q}{l}$$

What happen if we have many slits with separation d?
Implications in the photonic crystal experiment?

Optical application of interference – Michaelson Interferometer



One of the most common use of interference is in the construction of interferometers (device that generate interference). They are a class of instrument that has provide some of the most precise measurement of distance and the wavelength of light.

Let's consider what is the interference effect of the red & green light rays:

$$I = (E_R + E_G)^2 = 2I[1 + \cos(\frac{2\pi}{\lambda} 2(d_1 - d_2))] =$$

If we keep one mirror constant, we will see intensity variation with the travel of the second mirror as:

$$d_1 - d_2 = \frac{(n-1)\lambda}{2}$$; n=1,2,3… maxima

$$d_1 - d_2 = \frac{(2n+1)\lambda}{4}$$; n=0,1,2… minima