16.901: Sample Homework # 2

In this homework, we will consider numerical solutions to the one-dimensional diffusion equation,

$$\frac{\partial U}{\partial t} = \nu \frac{\partial^2 U}{\partial x^2},$$

where ν is a positive constant. Specifically, consider a forward Euler time integration and a 2nd-order centered-difference approximation in space,

$$\frac{U_j^{n+1} - U_j^n}{\Delta t} = \nu \frac{U_{j+1}^n - 2U_j^n + U_{j-1}^n}{\Delta x^2}.$$

1. Perform a semi-discrete Fourier analysis of this discretization and determine the eigenvalues $\lambda_m(\beta_m)$.

2. What is the largest value of $\nu \Delta t / \Delta x^2$ for which the discretization is stable when integrated with forward Euler?