16.901: Sample Homework # 1

1. Consider the convection-diffusion equation,

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \nu\frac{\partial^2 T}{\partial x^2},$$

where ν is a constant. Assuming a wave of the form, $T(x,t) = \hat{T}_k(t)\exp(ikx)$, determine the solution for $\hat{T}_k(t)$ given the initial condition $\hat{T}_k(0) = 1$. Using this solution, show that the amplitude of $\hat{T}_k(t)$ decays as $t \to \infty$ only if $\nu > 0$.

2. Consider the fourth-order equation:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \sigma\frac{\partial^4 T}{\partial x^4},$$

where σ is a constant. Assuming a wave of the form, $T(x,t) = \hat{T}_k(t)\exp(ikx)$, determine the solution for $\hat{T}_k(t)$ given the initial condition $\hat{T}_k(0) = 1$. Using this solution, for what values of σ will the amplitude of $\hat{T}_k(t)$ decay as $t \to \infty$?