16.901: Computational Methods in Aerospace Engineering
Final Exam Material: Spring 2005

The format of the exam is the same as for the first exam. I will give you a written form of the questions that will be the basis of the exam during the first 30 minutes of your scheduled time. Then, the oral exam will occur during the last 30 minutes. A few specific items are:

- There are no notes or other resources allowed during any part of this exam.

The final exam will be based on the second and third projects. I will ask questions to assess your understanding of not only the portions of the projects you did, but also the methods in general including the portions that were provided to you. A couple of specific comments regarding the material that may be assessed in the final exam:

- Questions will be included on the eigenvalue stability analysis of PDE's that you performed in Project #2. As a result, some assessment will be performed based on the ODE and Finite-Difference portion of the course.
- Questions will not be asked on the use of rank correlation coefficient to screen input sensitivities.
- Questions will not be asked on bootstrapping, Latin Hypercube Sampling, and response surface methods.

The following is a complete list of skills/abilities that you could be assessed on during the final exam.

Integration Methods for ODE's

1. (a) Describe the Adams-Bashforth, Adams-Moulton, and Backwards Differentation families of multi-step methods, (b) Describe the form of the Runge-Kutta family of multi-stage methods, and (c) Explain the relative computational costs of multi-step versus mult-stage methods.
2. (a) Explain the concept of stiffness of a system of equations, and (b) describe how it impacts the choice of numerical method for solving the equations.
3. Explain the differences and relative advantages between explicit and implicit methods to integrate systems of ordinary differential equations.
4. (a) Define eigenvalue stability, and (b) determine the stability boundary for a multi-step or multi-stage method applied to a linear system of ODE's.
5. Recommend an appropriate ODE integration method based on the features of the problem being solved.

Finite Difference and Finite Volume Methods for PDE's

1. Perform an eigenvalue stability analysis of a finite difference approximation of a PDE using either Von Neumann analysis or matrix-based analysis.

Finite Element Methods for PDE's

1. (a) Describe how the Method of Weighted Residuals (MWR) can be used to calculate an approximate solution to a PDE, (b) Describe the differences between MWR, the collocation method, and the least-squares method for approximating a PDE, and (c) Describe what a Galerkin MWR is.
2. (a) Motivate and describe the choice of approximate solutions (i.e. the test functions or interpolants on a discretized domain) used in the Finite Element Method, and (b) Give examples of a basis for the approximate solutions in particular including a nodal basis for at least linear and quadratic solutions.

3. (a) Describe how integrals are performed using a reference element, (b) Explain how Gaussian quadrature rules are derived, and (c) Describe how Gaussian quadrature is used to approximate an integral in the reference element.

4. Explain how Dirichlet, Neumann, and Robin boundary conditions are implemented for Laplace's equation discretized by FEM.

5. (a) Describe how the FEM discretization results in a system of discrete equations and, for linear problems, gives rises to the stiffness matrix, and (b) Describe the meaning of the entries (rows and columns) of the stiffness matrix and of the right-hand side vector for linear problems.

Probabilistic Methods

1. Describe how Monte Carlo sampling from multivariable, uniform distributions works.

2. Describe how to modify Monte Carlo sampling from uniform distributions to general distributions.

3. (a) Describe what an unbiased estimator is, (b) State unbiased estimators for mean, variance, and probability, and (c) State the distributions of these unbiased estimators.

4. (a) Define standard error, (b) Give standard errors for mean and probability, (c) Place confidence intervals for estimates of the mean and probability, and (d) Demonstrate the dependence of Monte Carlo convergence on the number of random inputs and the number of samples using the above error estimates.