

16.901: Homework # 9
Due Date: April 1, 2pm

In the lecture notes on the method of weighted residuals, we solved a one-dimensional heat diffusion problem using a combination of a quadratic and cubic polynomial. In this homework, you are again to apply the method of weighted residuals to the same problem as in the lecture notes, but we will use a different set of functions. As before, we will use two terms,

$$\tilde{T}(x) = 100 + \sum_{i=1}^2 a_i \phi_i(x),$$

This time, we will use sinusoidal functions, specifically,

$$\begin{aligned}\phi_1(x) &= \cos\left(\frac{\pi}{2}x\right), \\ \phi_2(x) &= \sin(\pi x).\end{aligned}$$

Note that these functions satisfy the boundary conditions that $\phi_i(\pm 1) = 0$.

1. Using the (Galerkin) method of weighted residuals, determine the coefficients a_1 and a_2 .

2. Plot the approximate solution and the exact solution on the same graph. Also, plot the error and the residual. Compare the quality of this sinusoidal approximation to the polynomial approximation (using the method of weighted residuals) applied in the lecture notes.