16.901: Homework # 7
Solution

In this homework, you will investigate the convergence of the finite volume method applied to one-dimensional convection. Specifically, you will use the Matlab script, **convect1d.m**

1. For the initial condition given (i.e. $U(x, 0) = U_0(x) = \exp(-x^2)$) and using $u = 1$, run the script using a fixed CFL but vary the mesh size. Note: the timestep and the mesh size are related to the CFL number by,

$$\Delta t = \text{CFL}\frac{\Delta x}{|u|}.$$

   As a result, as different spatial meshes are used (i.e. Δx varies) then Δt will vary. Specifically, use CFL = 0.5 and meshes with the number of control volumes Nx = 120, 240, and 480, and 960. For each mesh, determine the error in the solution at time $t = 1$ and location $x = 1$ (recall from Lecture that the exact solution is that the distribution of $U(x, 0)$ simply convects a distance ut). Include a table of the error for all Nx. What is the rate of convergence of the error with Δx for CFL = 0.5?

   Solution: The error is given in Table 1. The results show that the error is decreasing in magnitude by a factor of 2 for every factor of 2 increase in the number of cells. Thus, the algorithm is converging at a first-order rate, i.e. the error is $O(\Delta x)$.

Nx	Error
120	-0.032794
240	-0.016530
480	-0.008299
960	-0.004158

Table 1: Error for CFL = 0.5 simulations of convection equation.

2. Now, perform the same simulations but let CFL = 1.5 and determine the error as before (include a table of the error for all Nx) You should see some suspicious results for the Nx = 480 case, and some really suspicious results for the Nx = 960 case. For the smaller Nx values, what does the convergence rate appear to be? At a CFL = 1.5, do you think this finite volume method is convergent?

   Solution: For CFL = 1.5, the errors are shown in Table 2. The solution appears to be converging as $O(\Delta x)$ for smaller Nx as the error is decreasing by a factor of 2. However, the Nx=480 solution has oscillations which are present, though not near the point of interest ($x = 1$) thus it does not show up in the error. However, a plot of this simulation is shown in Figure 1 which clearly depicts the oscillations. For the even finer mesh (Nx = 960), the instability is present throughout the domain and the error is huge. Clearly, the method is not convergent as $\Delta x \to 0$.

Nx	Error
120	0.033019
240	0.016826
480	0.008440
960	1.1365E+08

Table 2: Error for CFL = 1.5 simulations of convection equation.

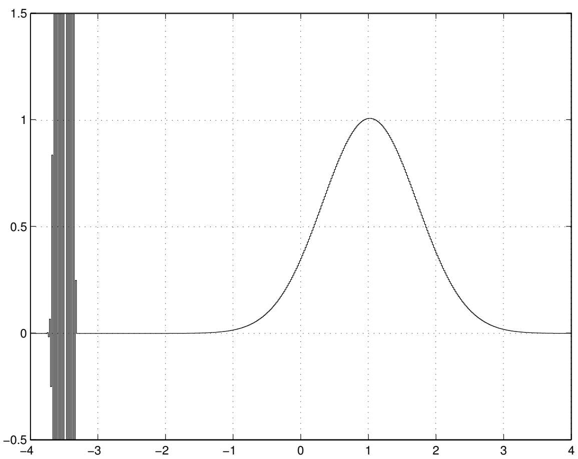

Figure 1: Solution of convection equation for Nx = 480, CFL = 1.5 at $t = 1$.