16.901: Homework # 7
Due Date: February 28, 2pm

In this homework, you will investigate the convergence of the finite volume method applied to one-dimensional convection. Specifically, you will use the Matlab script, **convect1d.m**

1. For the initial condition given (i.e. $U(x,0) = U_0(x) = \exp(-x^2)$) and using $u = 1$, run the script using a fixed CFL but vary the mesh size. Note: the timestep and the mesh size are related to the CFL number by,

$$\Delta t = CFL\frac{\Delta x}{|u|}.$$

   As a result, as different spatial meshes are used (i.e. Δx varies) then Δt will vary. Specifically, use CFL = 0.5 and meshes with the number of control volumes Nx = 120, 240, and 480, and 960. For each mesh, determine the error in the solution at time $t = 1$ and location $x = 1$ (recall from Lecture that the exact solution is that the distribution of $U(x,0)$ simply convects a distance ut). Include a table of the error for all Nx. What is the rate of convergence of the error with Δx for CFL = 0.5?

2. Now, perform the same simulations but let CFL = 1.5 and determine the error as before (include a table of the error for all Nx) You should see some suspicious results for the Nx = 480 case, and some really suspicious results for the Nx = 960 case. For the smaller Nx values, what does the convergence rate appear to be? At a CFL = 1.5, do you think this finite volume method is convergent?