16.901 Homework #3
Due Date: March 10, 2pm

Problem #1: Analysis of Simpson's Method

Simpson's method for integration of ODE's is given by,

$$\frac{v^{n+1} - v^{n-1}}{2\Delta t} = \frac{1}{6}\left[f(v^{n+1}, t^{n+1}) + 4f(v^n, t^n) + f(v^{n-1}, t^{n-1})\right].$$

1. Is the method explicit or implicit?
2. Using a Taylor series analysis, calculate the local truncation error for the method. Specifically, what is the order of accuracy?
3. Is the method convergent? Why or why not?
4. Plot the eigenvalue-stability region for this integration method. For purely real, negative eigenvalues, what is the largest timestep that can be taken while remaining stable for this method?

Problem #2: Accuracy of Derivative Approximations

In the following questions, we will look at approximations of 1st and 2nd derivatives, e.g. $\partial T/\partial x$ or $\partial^2 T/\partial x^2$. We will assume that the x-axis has been divided into a set of points equally-spaced a distance of Δx apart. The value of T at the j-th node is denoted with a subscript as T_j and is located at $x_j = j\Delta x$.

1. Perform a truncation error analysis and determine the order of accuracy p of the one-sided approximation,
$$\frac{T_j - T_{j-1}}{\Delta x} = \left.\frac{\partial T}{\partial x}\right|_j + O(\Delta x^p).$$
2. Develop the most accurate one-sided approximation for the first-derivative of the form,
$$\frac{aT_j + bT_{j-1} + cT_{j-2}}{2\Delta x} = \left.\frac{\partial T}{\partial x}\right|_j + O(\Delta x^p).$$
Specifically, determine the constants a, b, and c for the highest order of accuracy. What is the accuracy p for this approximation?
3. Perform a truncation error analysis and determine the order of accuracy p of the approximation,
$$\frac{T_{j+1} - 2T_j + T_{j-1}}{\Delta x^2} = \left.\frac{\partial^2 T}{\partial x^2}\right|_j + O(\Delta x^p).$$
4. Develop the most accurate one-sided approximation for the second-derivative of the form,
$$\frac{aT_j + bT_{j-1} + cT_{j-2}}{\Delta x^2} = \left.\frac{\partial^2 T}{\partial x^2}\right|_j + O(\Delta x^p).$$
Specifically, determine the constants a, b, and c for the highest order of accuracy. What is the accuracy p for this approximation?