16.901: Homework # 13
Solution

This homework builds upon the Monte Carlo method for the 1-D blade heat transfer problem that you developed in Homework #12 in which the thickness of the thermal barrier coating (TBC) is assumed to have a triangular distribution. The goal is to implement an error estimate for the mean value of T_{mh} and use it as a means to terminate the Monte Carlo simulation (i.e. set the sample size).

1. Using an error estimate, modify the Monte Carlo method so that the mean value of T_{mh} is calculated within an arbitrary accuracy at 99% confidence level. Attach a hard copy of your modified Matlab script.

   Solution: A 99% confidence interval occurs at ±3 standard errors from true mean,

   $$P\left\{-3\frac{\sigma_{T_{mh}}}{\sqrt{N}} \leq \overline{T_{mh}} - \mu_{T_{mh}} \leq 3\frac{\sigma_{T_{mh}}}{\sqrt{N}}\right\} \approx 0.99.$$

   Thus, if the desired error is ϵ, this requires that,

   $$3\frac{\sigma_{T_{mh}}}{\sqrt{N}} \leq \epsilon.$$

   As discussed in the notes, in practice we do not know $\sigma_{T_{mh}}$, so we use the sample variance as estimated by,

   $$s^2_{T_{mh}} = \frac{1}{N-1}\sum_{i=1}^{N}(T_{mhi} - \overline{T_{mh}})^2.$$

   The Monte Carlo method is modified to run until,

   $$3\frac{s_{T_{mh}}}{\sqrt{N}} \leq \epsilon.$$

   Note, since when N is small the estimate of the variance can potentially be small, we force $N > 25$ before allowing the Monte Carlo simulation to terminate. Also, in calculating the standard deviation of the sample, we use the following relationship (you might try proving this):

   $$s^2_y = \frac{N}{N-1}\left(\overline{y^2} - \overline{y}^2\right).$$

2. Assume that $L_{TBC\min} = 0.00025m$, $L_{TBCmpp} = 0.0003m$, and $L_{TBC\max} = 0.00075m$. Using the modified script, what sample size was required to estimate T_{mh} within 10K, 1K, and 0.1K? For each of these three Monte Carlo simulations, state the 99% confidence ranges (i.e. there will be three confidence ranges, one for 10K, one for 1K, and one for 0.1K accuracy).

   Solution: Running the Monte Carlo simulation for the three tolerance levels produces:

   $\epsilon = 10\,K$: $N = 96$ with 99% confidence interval, $1130.2\,K \leq \mu_{T_{mh}} \leq 1150.0\,K$

   $\epsilon = 1\,K$: $N = 8,728$ with 99% confidence interval, $1140.5\,K \leq \mu_{T_{mh}} \leq 1142.5\,K$

   $\epsilon = 0.1\,K$: $N = 862,421$ with 99% confidence interval, $1141.7\,K \leq \mu_{T_{mh}} \leq 1141.9\,K$