16.901: Homework # 12
Solution

In this homework, you will implement a Monte Carlo method for the 1-D blade heat transfer problem being discussed in class in which the thickness of the thermal barrier coating (TBC) is assumed to have a triangular distribution. Three sample codes are distributed with this homework:

- **bladeLtbc_tri.m**: The main script which should only need to be modified to change the most-probable value of the TBC thickness.
- **blade1D.m**: The analysis function which solves the 1-D heat transfer problem and is called by **bladeLtbc_tri.m**. You do not need to modify this script.
- **trirnd.m**: The function you are to write to generate a random number from a triangular distribution.

1. Implement the function **trirnd.m** to generate a random number from a triangular distribution in which $x_{\min}$ is the minimum value of x, x_{mpp} is the most-probable value of x, and $x_{\max}$ is the maximum value of x. Turn in a hard copy of your completed routine.

   Solution: As derived in the course notes, the cumulative distribution function (CDF) of a triangular distribution is,

$$F(x) = \frac{x_{mpp} - x_{\min}}{x_{\max} - x_{\min}} \left(\frac{x - x_{\min}}{x_{mpp} - x_{\min}} \right)^2, \tag{1}$$

   for $x_{\min} < x < x_{mpp}$, and

$$F(x) = 1 - \frac{x_{\max} - x_{mpp}}{x_{\max} - x_{\min}} \left(\frac{x_{\max} - x}{x_{\max} - x_{mpp}} \right)^2, \tag{2}$$

   for $x_{mpp} < x < x_{\max}$. Given a percentile drawn from a uniform random distribution, $u = F(x_u)$, the value x_u can be found by inverting the previous relationships for $F(x)$. Specifically, note that,

$$F(x_{mpp}) = \frac{x_{mpp} - x_{\min}}{x_{\max} - x_{\min}}.$$

   Thus, if $u < F(x_{mpp})$ then $x_{\min} < x < x_{mpp}$, we invert Equation (1) to find,

$$x_u = x_{\min} + \sqrt{u\,(x_{\max} - x_{\min})\,(x_{mpp} - x_{\min})}.$$

   Otherwise, if $u > F(x_{mpp})$ then $x_{mpp} < x < x_{\max}$, we invert Equation (2) to find,

$$x_u = x_{\max} - \sqrt{(1-u)\,(x_{\max} - x_{\min})\,(x_{\max} - x_{mpp})}.$$

   These relationships are implemented in **trirnd.m** which is available on the website.

2. Assume that $L_{TBC\min} = 0.00025m$ and $L_{TBC\max} = 0.00075m$. Run three different triangular distributions, specifically, with $L_{TBC\,mpp} = 0.0003m$, $0.0005m$, and $0.0007m$. What are the mean values and standard deviations of T_{mh} for the three results using a 1000 sample Monte Carlo? Also, include hard copies of the distributions of T_{mh} for the three cases.

   Solution: The mean and standard deviation for the three cases using a 1000 sample Monte Carlo are given in Table 1. The distributions of both L_{TBC} and T_{mh} are shown in Figures 1-3.

$L_{TBC\,mpp}$	$\mu_{T_{mh}}$	$\sigma_{T_{mh}}$
0.0003 m	1142 K	40.0 K
0.0005 m	1123 K	27.7 K
0.0007 m	1105 K	29.1 K

Table 1: Mean and standard deviation of T_{mh} for varying $L_{TBC\,mpp}$.

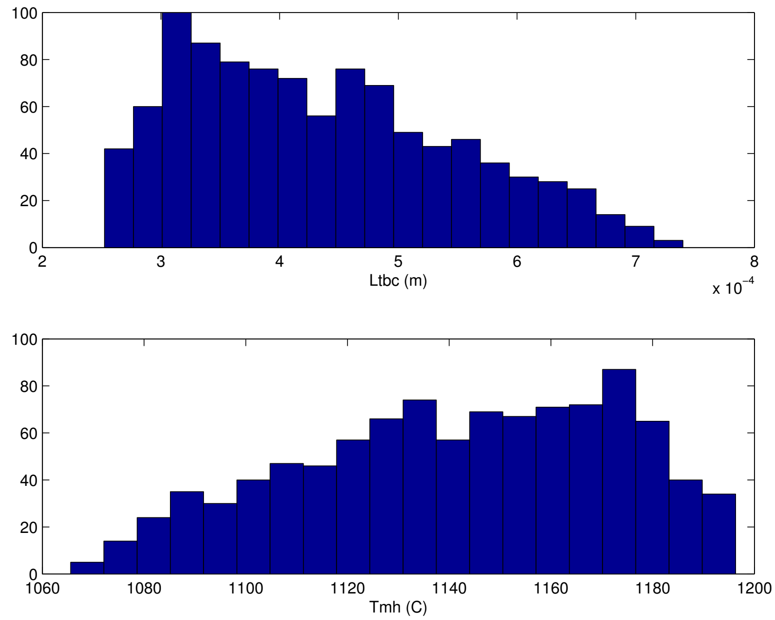

Figure 1: Histograms of L_{TBC} and T_{mh} for $L_{TBC\,mpp} = 0.0003$ m.

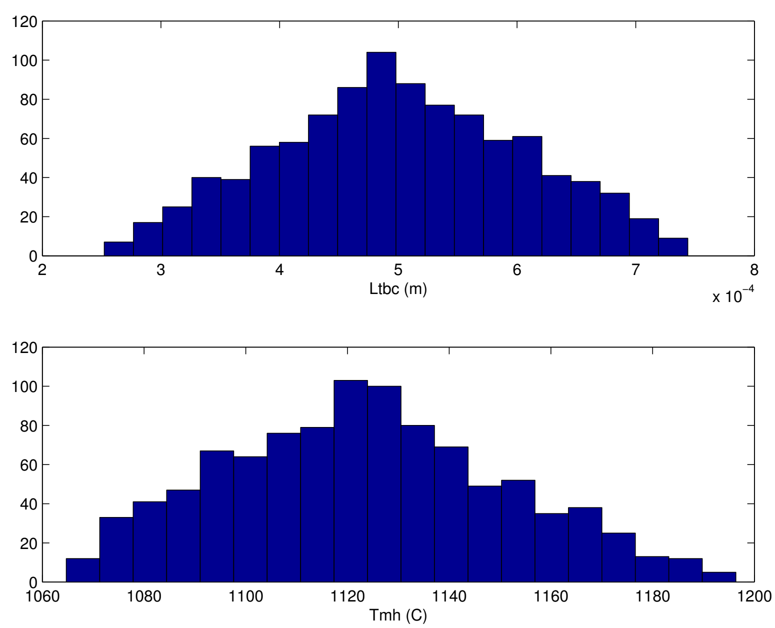

Figure 2: Histograms of L_{TBC} and T_{mh} for $L_{TBC\,mpp} = 0.0005$ m.

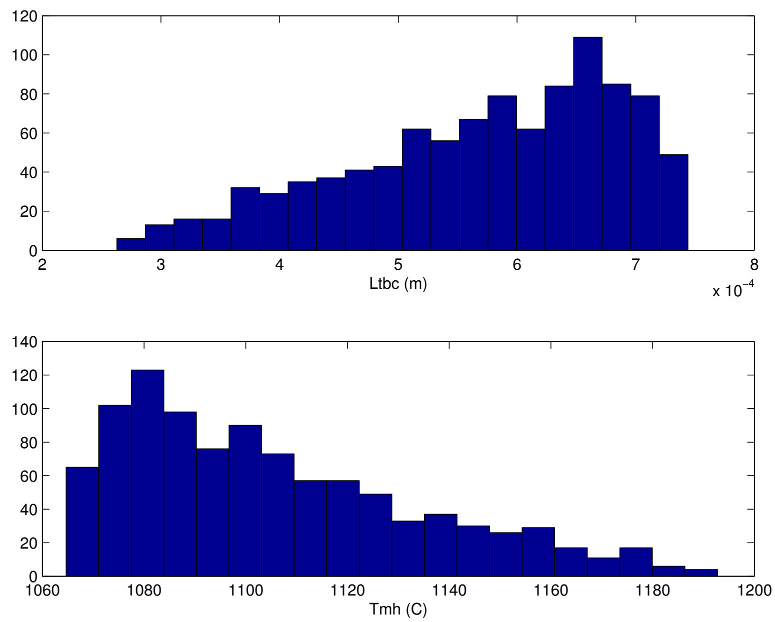

Figure 3: Histograms of L_{TBC} and T_{mh} for $L_{TBC\,mpp} = 0.0007$ m.