Analysis of Variance
ANOVA

Proposed Schedule Changes

- Switch lecture
- No quiz
  - Informal (ungraded) presentation of term project ideas
- Read Phadke ch. 7 -- Construction Orthogonal Arrays
  - Quiz on ANOVA
  - Noise experiment due

Learning Objectives

- Introduce hypothesis testing
- Introduce ANOVA in statistic practice
- Introduce ANOVA as practiced in RD
- Compare to ANOM
- Get some practice applying ANOVA in RD
- Discuss / compare / contrast

Hypothesis Testing

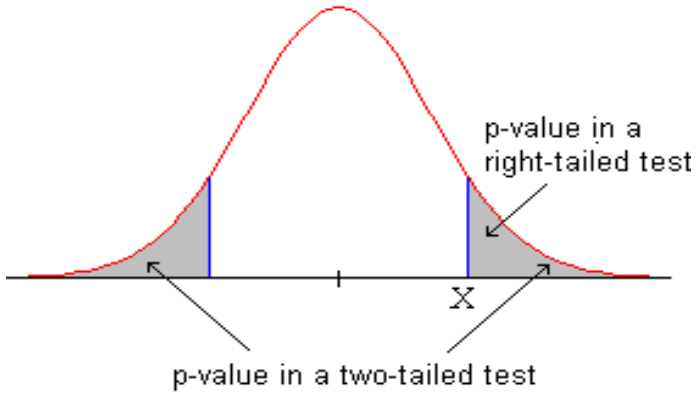
A technique that uses *sample* data from a population to come to reasonable conclusions with a certain degree of confidence

Hypothesis Testing Terms

- **Null Hypothesis** (H_o) -- The hypothesis to be tested (accept/reject)
- **Test statistic** -- A function of the parameters of the experiment on which you base the test
- **Critical region** -- The set of values of the test statistic that lead to rejection of H_o

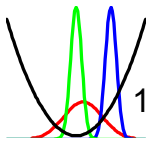
Hypothesis Testing Terms (cont.)

- **Level of significance** (α) -- A measure of confidence that can be placed in a result not merely being a matter of chance
- **p value** -- The smallest level of significance at which you would reject H_o



Comparing the Variance of Two Samples

- Null Hypothesis -- H_o: $\frac{\sigma_1}{\sigma_2} = r$
- Test Statistic -- $F = \frac{1}{r^2} \cdot \frac{\mathrm{Var}(X1)}{\mathrm{Var}(X2)}$
- Acceptance criteria -- $\left| pF(F, d1, d2) - 0.5 \right| < \frac{1-\alpha}{2}$
- Assumes independence & normal dist.

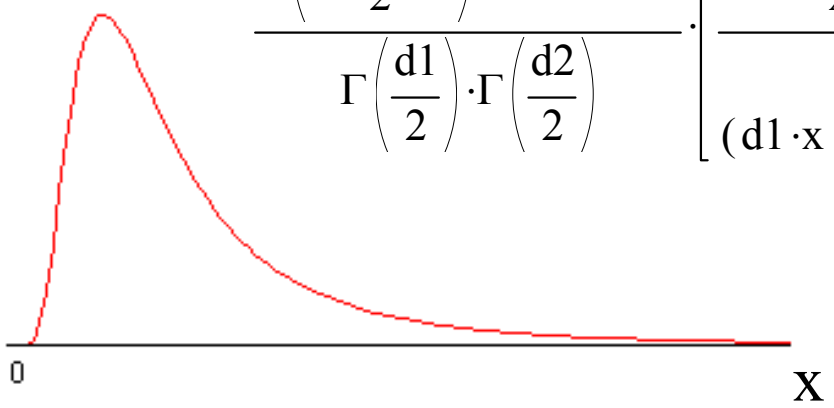


F Distribution

- Three arguments
  - d1 (numerator DOF)
  - d2 (denominator DOF)
  - x (cutoff)

$$\frac{\Gamma\left(\frac{d1+d2}{2}\right)\cdot d1^{\frac{d1}{2}}\cdot d2^{\frac{d2}{2}}}{\Gamma\left(\frac{d1}{2}\right)\cdot\Gamma\left(\frac{d2}{2}\right)}\cdot\left[\frac{x^{\frac{d1}{2}-1}}{(d1\cdot x+d2)^{\frac{d1+d2}{2}}}\right] \quad \text{for } x>0$$

F(x,d1,d2)



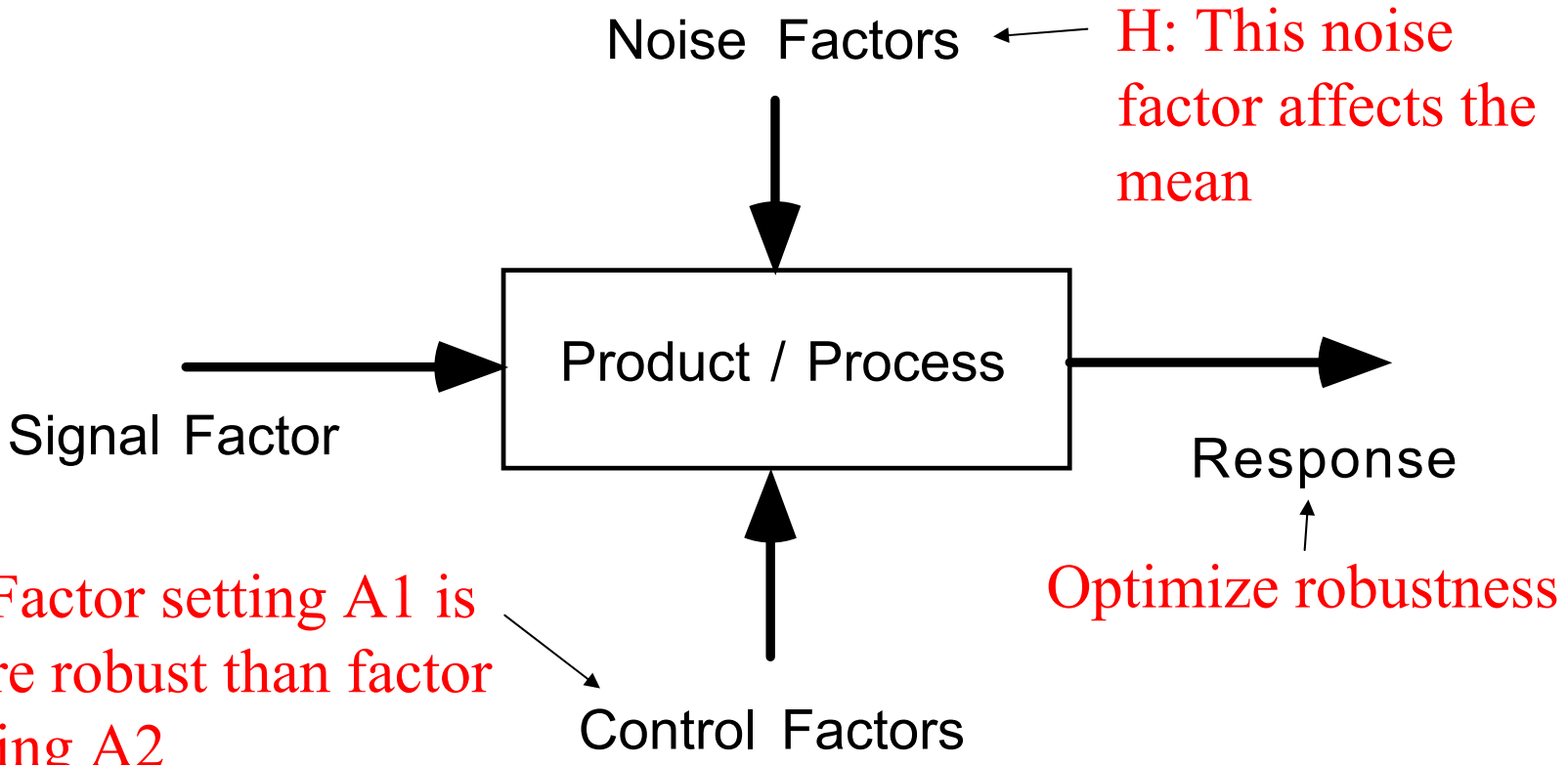
Rolling Dice

- Population 1 -- Roll one die
- Population 2 -- Roll two die
- Go to excel sheet “dice_f_test.xls”

One-way ANOVA

- Null Hypothesis -- H_o: $\mu_1 = \mu_2 = \mu_3 = \cdots$
- Test Statistic -- $F := \dfrac{\left(\dfrac{SSB}{dfB}\right)}{\left(\dfrac{SSW}{dfW}\right)}$
- Acceptance criteria -- $pF(F, dfB, dfW) < (1 - \alpha)$
- Assumes independence & normal dist.

ANOVA & Robust Design

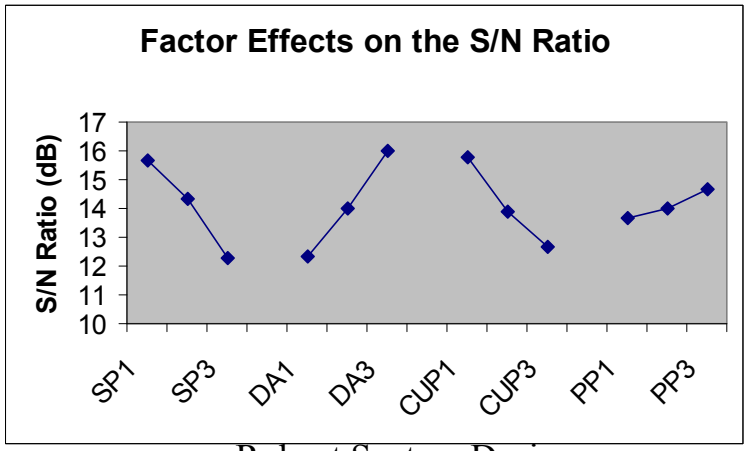


ANOVA and the Noise Experiment

- Did the noise factors we experimented with really have an effect on mean?
- Switch to Excel sheet "catapult_L4_static_anova.xls"

Why Test This Hypothesis?

- Factor setting PP3 is more robust than setting PP1
- Phadke -- "In Robust Design, we are not concerned with such probability statements, we use the F ratio for only qualitative understanding of the relative factor effects"



Analysis of Variance (ANOVA)

- ANOVA helps to resolve the relative magnitude of the factor effects compared to the error variance
- Are the factor effects real or just noise?
- I will cover it in Lecture 7.
- You may want to try the Mathcad "resource center" under the help menu

Additive Model

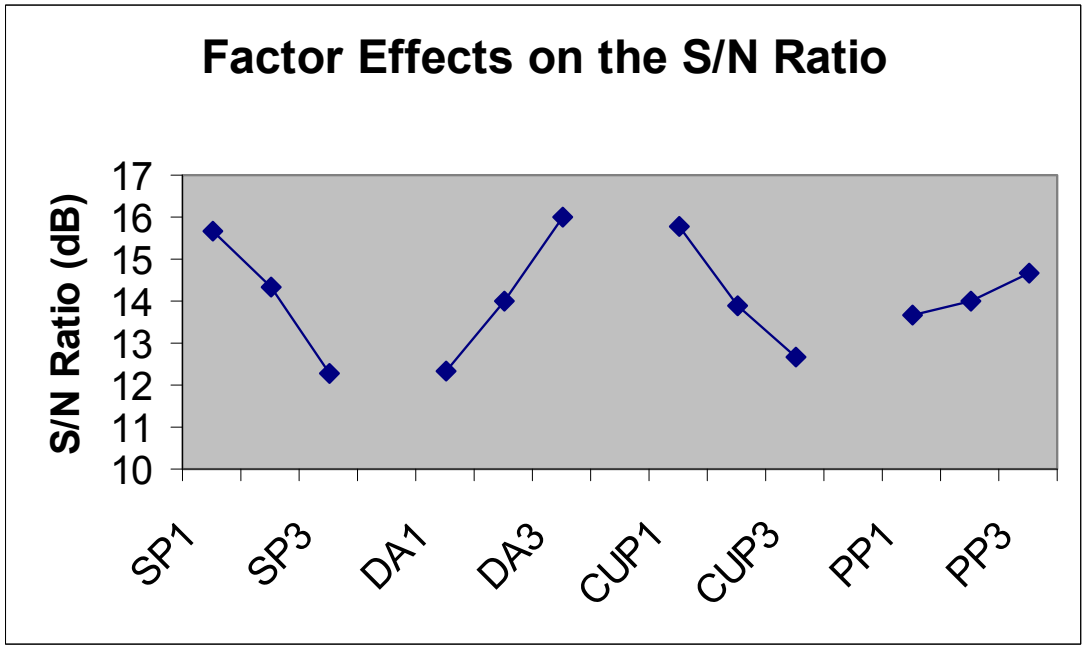
- Assume each parameter affects the response independently of the others

$$\eta(A_i, B_j, C_k, D_i) = \mu + a_i + b_j + c_k + d_i + e$$

A: Stop Pin	B: Draw Angle	C: Cup Position	D: Post Pin	Mean Distance	Std Deviation	Variance	S/N Ratio
A1	B1	C1	D1	16.9	3.0	8.8	15.1
A1	B2	C2	D2	46.6	8.1	65.7	15.2
A1	B3	C3	D3	91.9	13.4	178.5	16.7
A2	B1	C2	D3	25.8	5.8	34.1	12.9
A2	B2	C3	D1	49.2	11.9	141.6	12.3
A2	B3	C1	D2	67.2	8.7	75.2	17.8
A3	B1	C3	D2	18.1	6.4	41.5	9.0
A3	B2	C1	D3	45.9	8.7	76.3	14.4
A3	B3	C2	D1	53.0	11.2	125.1	13.5
			GRAND MEANS	46.1	8.6	83.0	14.1

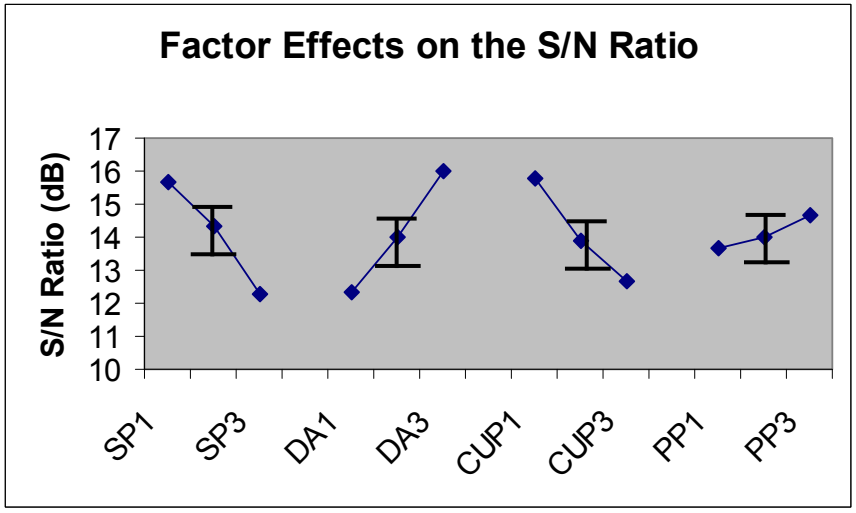
Analysis of Means (ANOM)

- Analyze the data to discover m_{A1} , a_i ...

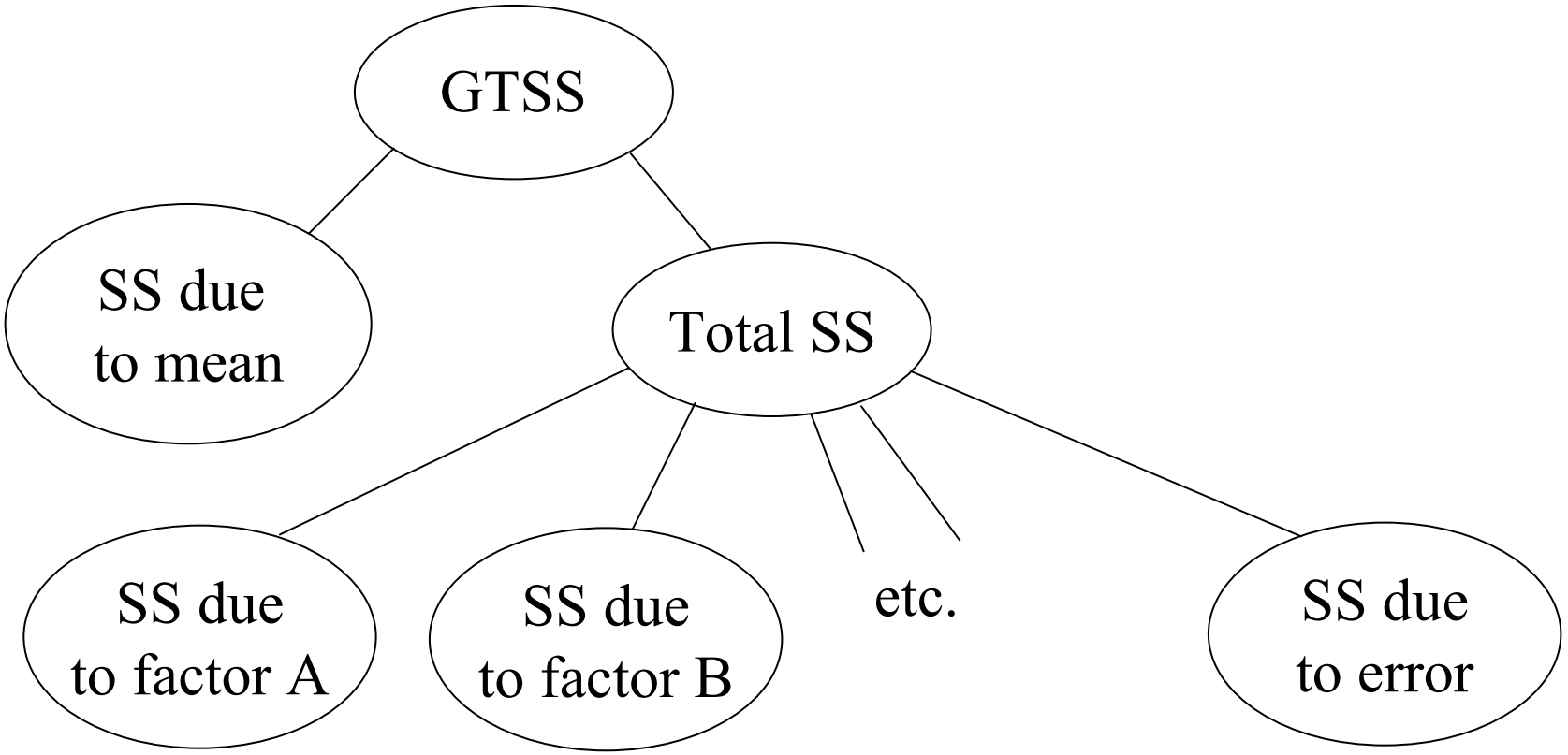


Analysis of Variance (ANOVA)

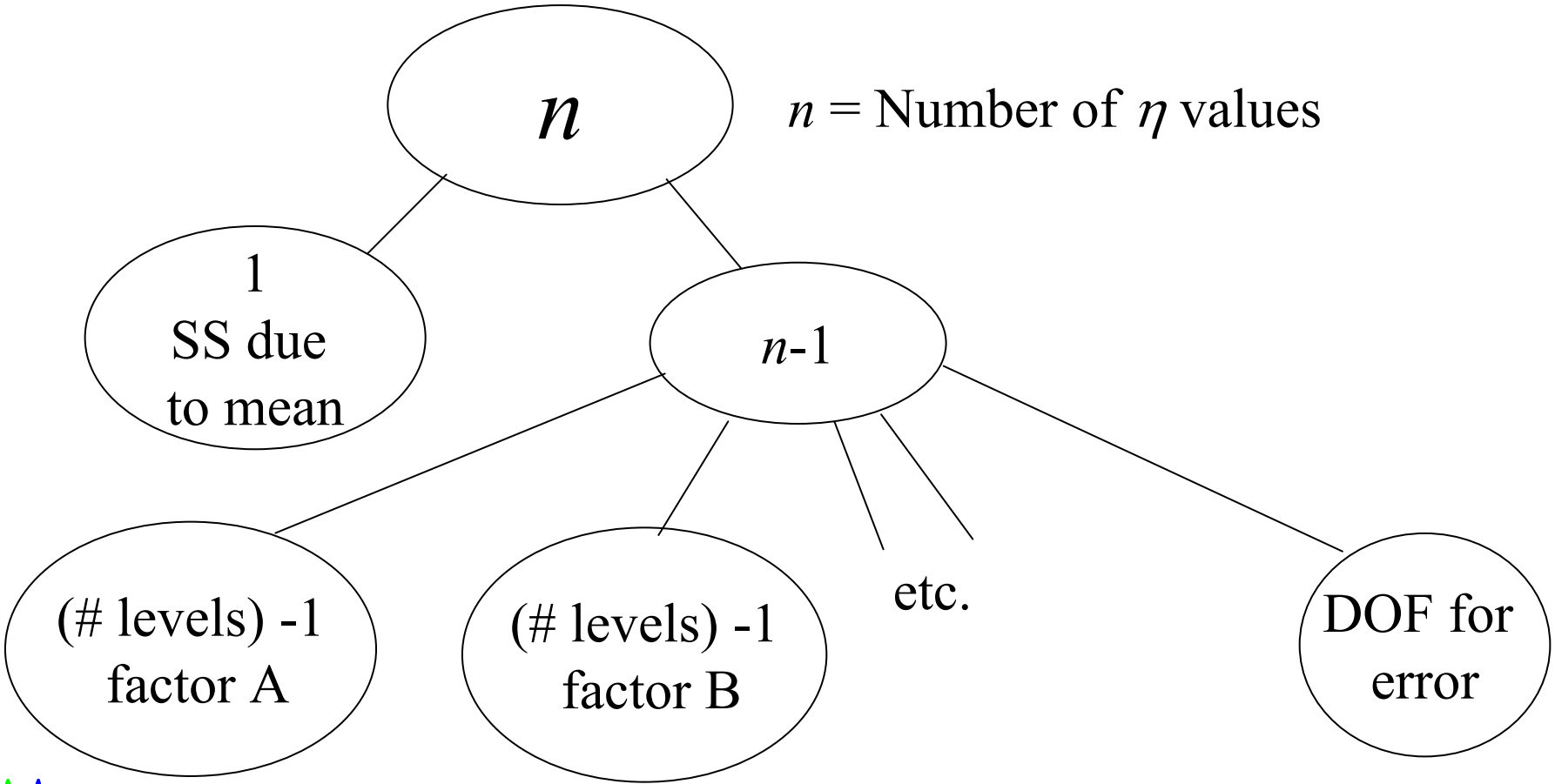
- Analyze data to understand the relative contribution of control factors compared to "error variance"



Breakdown of Sum Squares



Breakdown of DOF



Computation of Sum of Squares

- Grand total sum of squares $GTSS = \sum_{i=1}^{n} \eta_i^2$
- Sum of squares due to mean $= n\mu^2$
- Total sum of squares $= \sum_{i=1}^{n} (\eta_i - \mu)^2$
- Sum of squares due to a factor

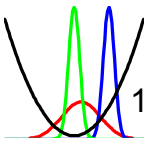
  $= replication\# \left[(m_{A1} - \mu)^2 + (m_{A2} - \mu)^2 + (m_{A3} - \mu)^2 \right]$
- Sum of squares due to error
  - Zero with no replicates
  - Estimated by "pooling"

Pooling

- Provides an estimate of error without empty columns or replicates
- Procedure
  - Select the bottom half of the factors (in terms of contribution to Total SS)

F-statistic

- $\text{Error variance} = \dfrac{\text{sum of squares due to error}}{\text{degrees of freedom for error}}$
- $F = \dfrac{\text{mean square for factor}}{\text{Error variance}}$
- $\text{mean square for factor} = \dfrac{\text{SS for factor}}{\text{DOF for factor}}$
  - F=1 Factor effect is on par with the error
  - F=2 The factor effect is marginal
  - F>4 The factor effect is substantial



Confidence Intervals for Factor Effects

- Phadke
  - Variance in a_i is error variance / replication #
  - 95% confidence interval for factor effects is two standard deviations in a_i
- How does one interpret this value?

Example
Catapult Experiment

- Switch to Excel "Catapult_L9_2.xls"

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			GRAND MEANS	46.1	8.6	83.0	14.1

Homework

- Grades are exceptionally high
- Some are spending vast amounts of time
- This represents 20% of the final grade

Mean	94.2	101.0	96.5	95.4
Standard deviation	2.7	4.4	6.3	1.5
Maximum	98	109	101	97.3

Quizzes

- Some consistently score high
- Others struggling, but learning
- Remember, this is only 10%

	Quiz #1	Quiz #2	Quiz #3	Quiz #4
Mean	74.3	83.4	77.5	82.8
Standard deviation	19.3	10.4	22.3	17.6
Maximum	100	100	100	110

Next Steps

- Hand in homework #5
- Homework #7 due on Lecture 10.
- Next session tomorrow
  - Present your ideas for a term project
- Following session
  - Quiz on ANOVA
  - Homework #6 (Noise Exp.) due
  - Constructing orthogonal arrays (read ch. 7)