Attitude Determination and Control (ADCS)

16.684 Space Systems Product Development
Spring 2001

Olivier L. de Weck

Department of Aeronautics and Astronautics
Massachusetts Institute of Technology

ADCS Motivation

- Motivation
  - In order to point and slew optical systems, spacecraft attitude control provides coarse pointing while optics control provides fine pointing
- Spacecraft Control
  - Spacecraft Stabilization
    - Spin Stabilization
    - Gravity Gradient
    - Three-Axis Control
    - Formation Flight
  - Actuators
    - Reaction Wheel Assemblies (RWAs)
    - Control Moment Gyros (CMGs)
    - Magnetic Torque Rods
    - Thrusters
  - Sensors: GPS, star trackers, limb sensors, rate gyros, inertial measurement units
  - Control Laws
- Spacecraft Slew Maneuvers
  - Euler Angles
  - Quaternions

Key Question:
What are the pointing requirements for satellite ?

NEED expendable propellant:

- **On-board fuel often determines life**
- **Failing gyros are critical (e.g. HST)**

Outline

- Definitions and Terminology
- Coordinate Systems and Mathematical Attitude Representations
- Rigid Body Dynamics
- Disturbance Torques in Space
- Passive Attitude Control Schemes
- Actuators
- Sensors
- Active Attitude Control Concepts
- ADCS Performance and Stability Measures
- Estimation and Filtering in Attitude Determination
- Maneuvers
- Other System Consideration, Control/Structure interaction
- Technological Trends and Advanced Concepts

Opening Remarks

- Nearly all ADCS Design and Performance can be viewed in terms of RIGID BODY dynamics
- Typically a Major spacecraft system
- For large, light-weight structures with low fundamental frequencies the flexibility needs to be taken into account
- ADCS requirements often drive overall S/C design
- Components are cumbersome, massive and power-consuming
- Field-of-View requirements and specific orientation are key
- Design, analysis and testing are typically the most challenging of all subsystems with the exception of payload design
- Need a true "systems orientation" to be successful at designing and implementing an ADCS

Terminology

ATTITUDE : Orientation of a defined spacecraft body coordinate system with respect to a defined external frame (GCI,HCI)

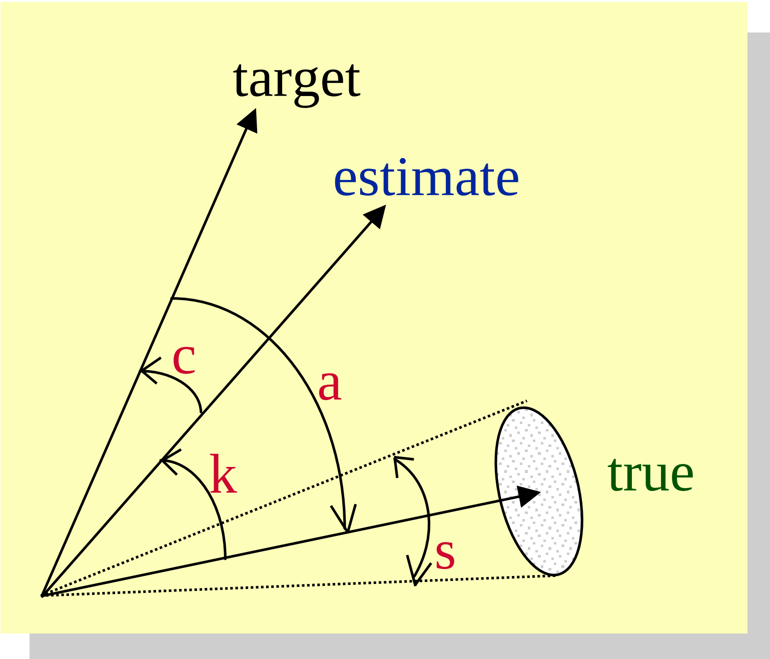
ATTITUDE DETERMINATION: Real-Time or Post-Facto knowledge, within a given tolerance, of the spacecraft attitude

ATTITUDE CONTROL: Maintenance of a desired, specified attitude within a given tolerance

ATTITUDE ERROR: "Low Frequency" spacecraft misalignment; usually the intended topic of attitude control

ATTITUDE JITTER: "High Frequency" spacecraft misalignment; usually ignored by ADCS; reduced by good design or fine pointing/optical control.

Pointing Control Definitions



target	desired pointing direction
true	actual pointing direction (mean)
estimate	estimate of true (instantaneous)
a	pointing accuracy (long-term)
s	stability (peak-peak motion)
k	knowledge error
c	control error

a = pointing accuracy = attitude error
s = stability = attitude jitter

Source:
G. Mosier
NASA GSFC

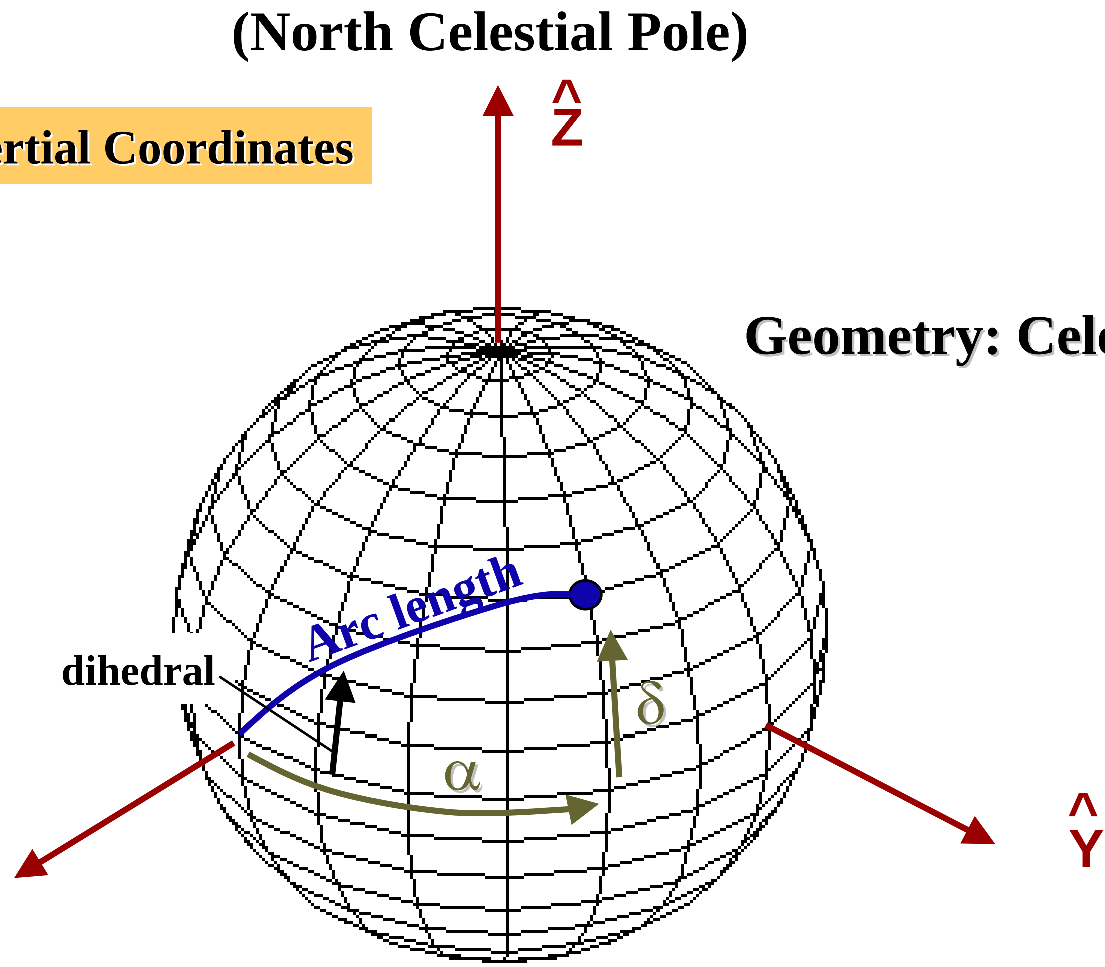
Attitude Coordinate Systems

GCI: Geocentric Inertial Coordinates

Cross product

$\hat{Y} = \hat{Z} \times \hat{X}$

Geometry: Celestial Sphere



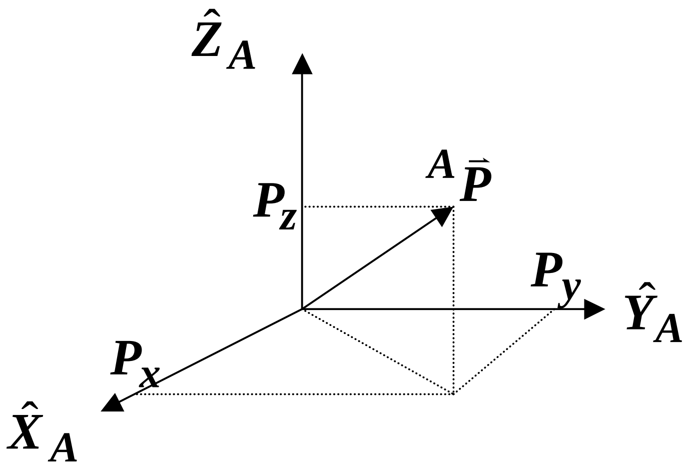
VERNAL EQUINOX

α : Right Ascension
δ : Declination

Inertial Coordinate System

X and Y are in the plane of the ecliptic

Attitude Description Notations



$\{\cdot\} = \text{Coordinate system}$

$\vec{P} = \text{Vector}$

${}^A\vec{P} = \text{Position vector w.r.t.} \{A\}$

$${}^A\vec{P} = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}$$

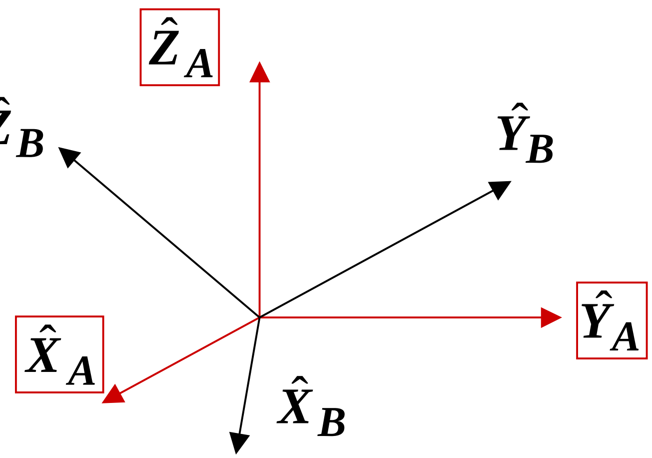
$$\text{Unit vectors of } \{A\} = \begin{bmatrix} \hat{X}_A & \hat{Y}_A & \hat{Z}_A \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Describe the orientation of a body:

(1) Attach a coordinate system to the body

(2) Describe a coordinate system relative to an inertial reference frame

Rotation Matrix

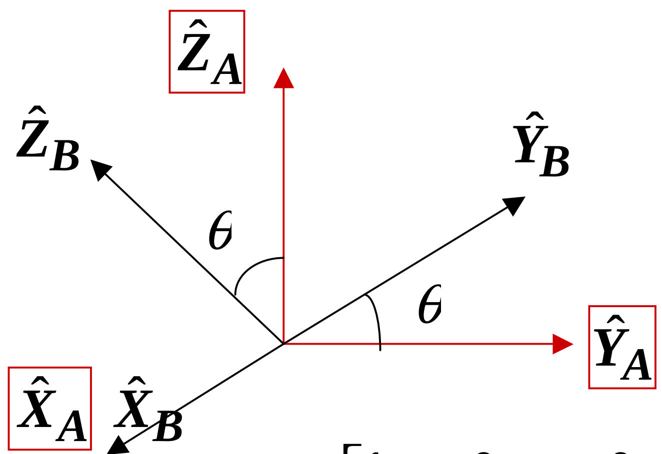


$\{A\} =$ **Reference coordinate system**

$\{B\} =$ **Body coordinate system**

Rotation matrix from {B} to {A}

$$ {}^{A}_{B}\boldsymbol{R} = \left[{}^{A}\hat{X}_B \quad {}^{A}\hat{Y}_B \quad {}^{A}\hat{Z}_B \right] $$



$$ {}^{A}_{B}\boldsymbol{R} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{cos}\theta & \mathbf{-sin}\theta \\ \mathbf{0} & \mathbf{sin}\theta & \mathbf{cos}\theta \end{bmatrix} $$

Special properties of rotation matrices:

(1) Orthogonal:

$$ \boldsymbol{R}^T \boldsymbol{R} = \boldsymbol{I}, \quad \boldsymbol{R}^T = \boldsymbol{R}^{-1} $$

(2) Orthonormal:

$$ \|\boldsymbol{R}\| = 1 $$

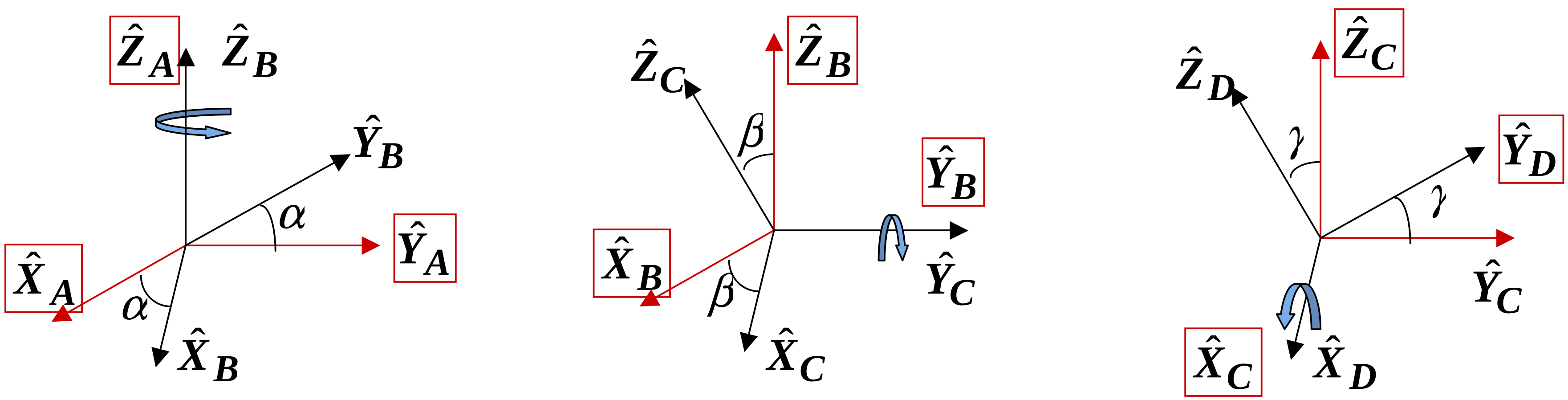
(3) Not commutative

$$ {}^{A}_{B}\boldsymbol{R}\, {}^{B}_{C}\boldsymbol{R} \neq {}^{B}_{C}\boldsymbol{R}\, {}^{A}_{B}\boldsymbol{R} $$

Euler Angles (1)

Euler angles describe a sequence of three rotations about different axes in order to align one coord. system with a second coord. system.

Rotate about $\hat{Z}_A$ by α | **Rotate about $\hat{Y}_B$ by β** | **Rotate about $\hat{X}_C$ by γ**



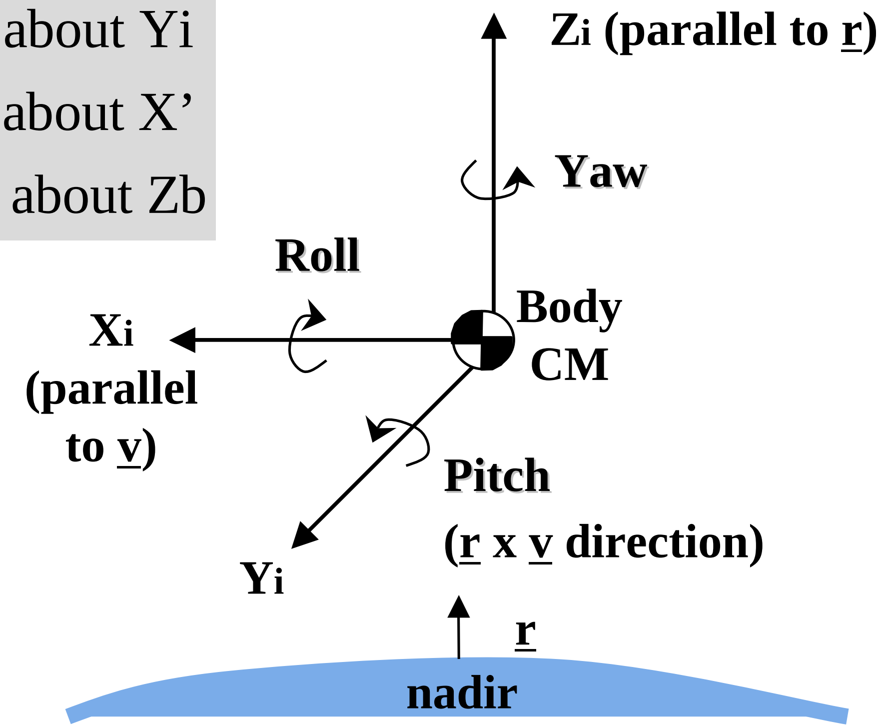
$$ {}^{A}_{B}R = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad {}^{B}_{C}R = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \quad {}^{C}_{D}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix} $$

$$ {}^{A}_{D}R = {}^{A}_{B}R \; {}^{B}_{C}R \; {}^{C}_{D}R $$

Euler Angles (2)

- Concept used in rotational kinematics to describe body orientation w.r.t. inertial frame
- Sequence of three angles and prescription for rotating one reference frame into another
- Can be defined as a transformation matrix body/inertial as shown: $T_{B/I}$
- Euler angles are non-unique and exact <u>sequence</u> is critical

θ about Yi
ϕ about X'
ψ about Zb



<u>Goal:</u> Describe kinematics of body-fixed frame with respect to rotating local vertical

(Pitch, Roll, Yaw) = (θ,φ,ψ) ⟶ Euler Angles

Note: $T_{B/I}^{-1} = T_{I/B} = T_{B/I}^{T}$

Transformation from Body to "Inertial" frame:

$$T_{B/I} = \underbrace{\begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{YAW}} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}}_{\text{ROLL}} \cdot \underbrace{\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}}_{\text{PITCH}}$$

Quaternions

- Main problem computationally is the existence of a singularity
- Problem can be avoided by an application of Euler's theorem:

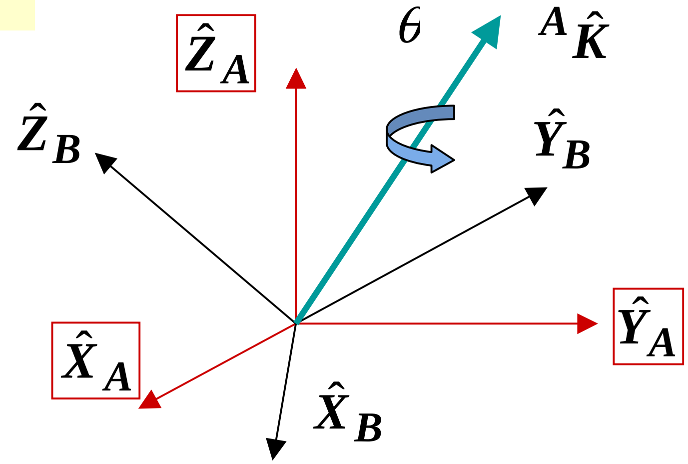
$$Q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} \vec{q} \\ q_4 \end{bmatrix}$$

$\vec{q}$ = A vector describes the axis of rotation.

q_4 = A scalar describes the amount of rotation.

EULER'S THEOREM

The Orientation of a body is uniquely specified by a vector giving the direction of a body axis and a scalar specifying a rotation angle about the axis.



A: Inertial
B: Body

$${}^A\hat{K} = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$$

$$q_1 = k_x \sin\left(\frac{\theta}{2}\right)$$

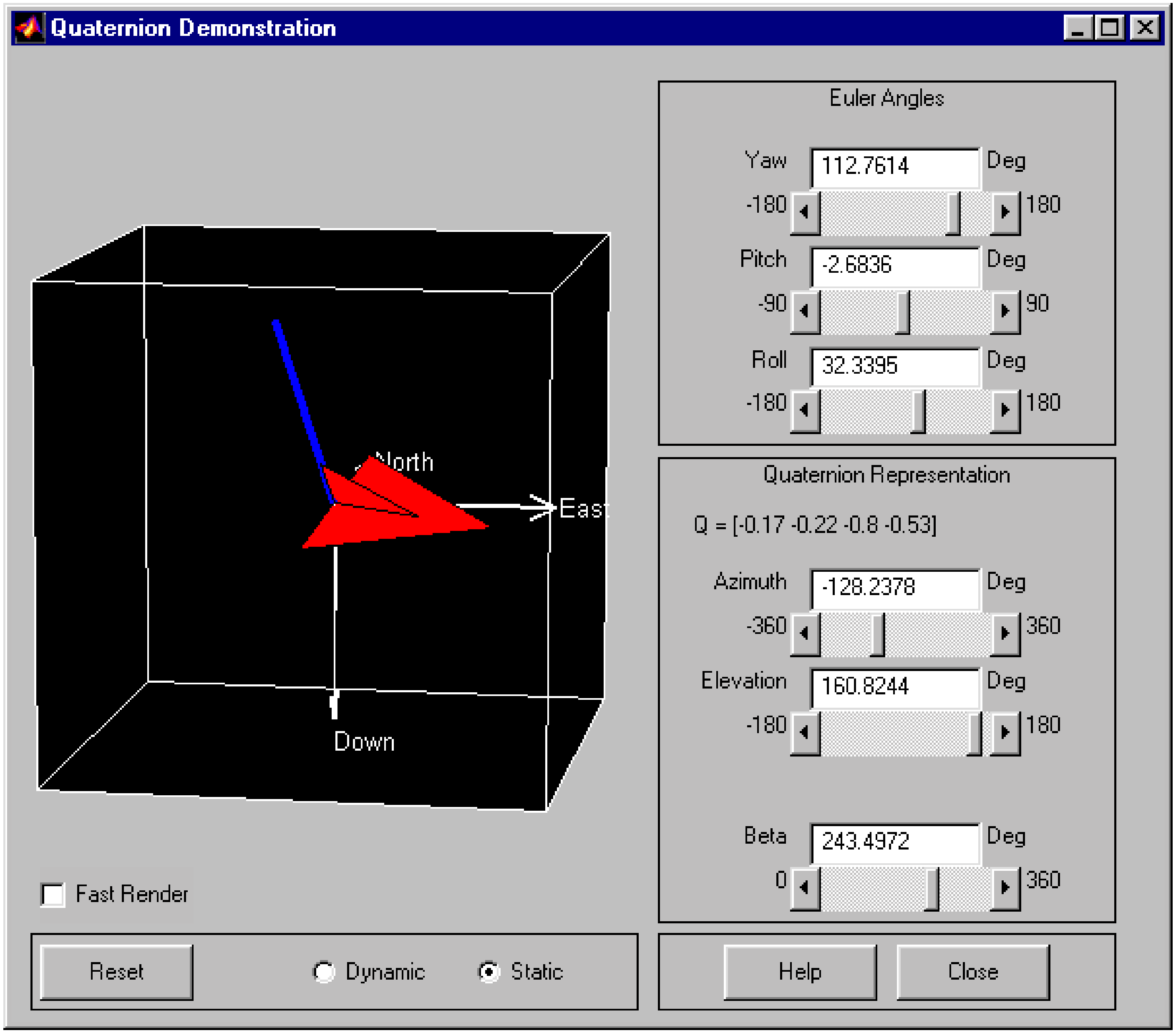
$$q_2 = k_y \sin\left(\frac{\theta}{2}\right)$$

$$q_3 = k_z \sin\left(\frac{\theta}{2}\right)$$

$$q_4 = \cos\left(\frac{\theta}{2}\right)$$

- Definition introduces a redundant fourth element, which eliminates the singularity.
- This is the **"quaternion"** concept
- Quaternions have no intuitively interpretable meaning to the human mind, but are computationally convenient

Quaternion Demo (MATLAB)



Comparison of Attitude Descriptions

Method	Euler Angles	Direction Cosines	Angular Velocity $\underline{\omega}$	Quaternions
Pluses	If given ϕ,ψ,θ then a unique orientation is defined	Orientation defines a unique dir-cos matrix **R**	Vector properties, commutes w.r.t addition	Computationally robust Ideal for digital control implement
Minuses	Given orient then Euler non-unique Singularity	6 constraints must be met, non-intuitive	Integration w.r.t time does not give orientation Needs transform	Not Intuitive Need transforms

Best for analytical and ACS design work

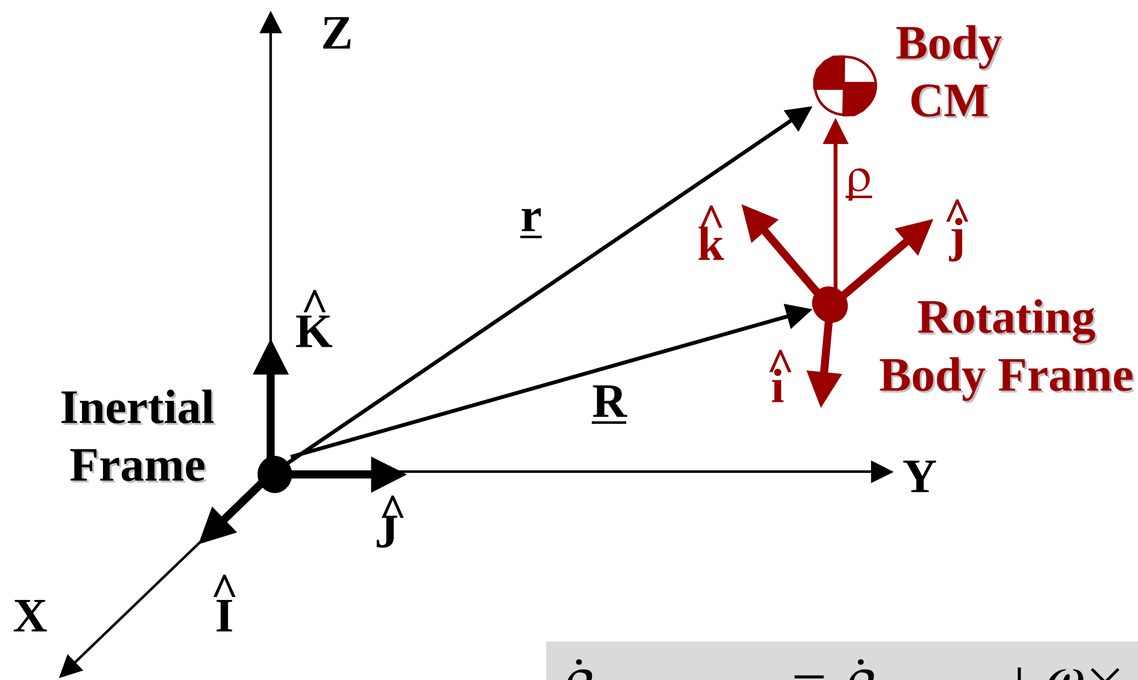
Must store initial condition

Best for digital control implementation

Rigid Body Kinematics

Time Derivatives:
(non-inertial)

$\underline{\omega}$ = Angular velocity of Body Frame



BASIC RULE: $\dot{\underline{\rho}}_{\text{INERTIAL}} = \dot{\underline{\rho}}_{\text{BODY}} + \underline{\omega} \times \underline{\rho}$

Applied to position vector r:

$$\underline{r} = \underline{R} + \underline{\rho}$$ **Position**

$$\dot{\underline{r}} = \dot{\underline{R}} + \dot{\underline{\rho}}_{\text{BODY}} + \underline{\omega} \times \underline{\rho}$$ **Rate**

Expressed in the Inertial Frame

$$\ddot{\underline{r}} = \ddot{\underline{R}} + \ddot{\underline{\rho}}_{\text{BODY}} + 2\underline{\omega} \times \dot{\underline{\rho}}_{\text{BODY}} + \dot{\underline{\omega}} \times \underline{\rho} + \underline{\omega} \times \left(\underline{\omega} \times \underline{\rho}\right)$$ **Acceleration**

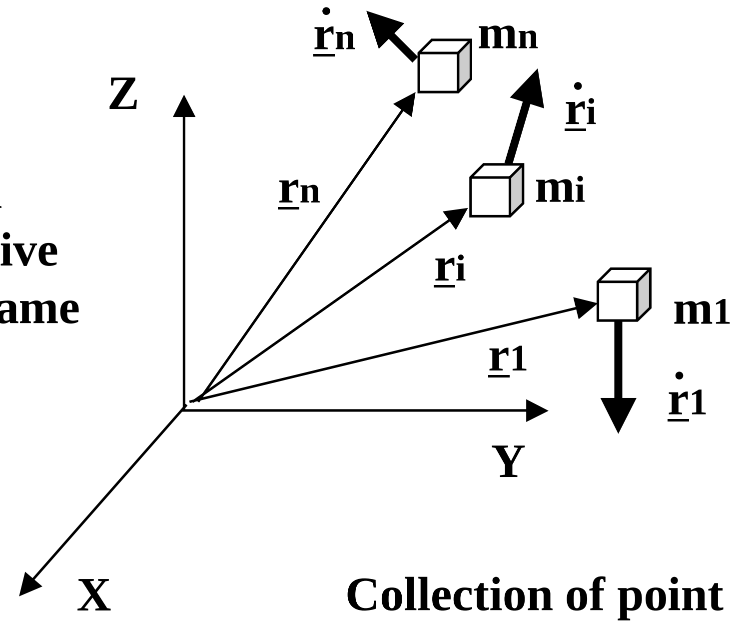
- $\ddot{\underline{R}}$: **Inertial accel of CM**
- $\ddot{\underline{\rho}}_{\text{BODY}}$: **relative accel w.r.t. CM**
- $2\underline{\omega} \times \dot{\underline{\rho}}_{\text{BODY}}$: **coriolis**
- $\dot{\underline{\omega}} \times \underline{\rho}$: **angular accel**
- $\underline{\omega} \times (\underline{\omega} \times \underline{\rho})$: **centripetal**

Angular Momentum (I)

Angular Momentum

$$\underline{H}_{\text{total}} = \sum_{i=1}^{n} \underline{r}_i \times m_i \underline{\dot{r}}_i$$

System in motion relative to Inertial Frame



Collection of point masses mi at ri

If we assume that

(a) Origin of Rotating Frame in Body CM
(b) Fixed Position Vectors ri in Body Frame (Rigid Body)

Angular Momentum Decomposition

Then :

$$\underline{H}_{\text{total}} = \underbrace{\left(\sum_{i=1}^{n} m_i\right) \underline{R} \times \underline{\dot{R}}}_{\substack{\text{ANGULAR MOMENTUM}\\ \text{OF TOTAL MASS W.R.T}\\ \text{INERTIAL ORIGIN}}} + \underbrace{\sum_{i=1}^{n} m_i \underline{\rho}_i \times \underline{\dot{\rho}}_i}_{\substack{\square\, \underline{H}_{\text{BODY}}\\ \text{BODY ANGULAR}\\ \text{MOMENTUM ABOUT}\\ \text{CENTER OFMASS}}}$$

Note that $\underline{\rho}_i$ is measured in the inertial frame

Angular Momentum (II)

For a RIGID BODY we can write:

$$\dot{\underline{\rho}}_i = \underbrace{\dot{\underline{\rho}}_{i,\text{BODY}}}_{\text{RELATIVE MOTION IN BODY}} + \underline{\omega} \times \underline{\rho}_i = \underline{\omega} \times \underline{\rho}_i$$

And we are able to write:

$$\underline{H} = I\underline{\omega}$$

RIIGID BODY, CM COORDINATES
$\underline{H}$ and $\underline{\omega}$ are resolved in BODY FRAME

"The vector of angular momentum in the body frame is the product of the 3x3 Inertia matrix and the 3x1 vector of angular velocities."

Inertia Matrix Properties:

Real Symmetric ; 3x3 Tensor ; coordinate dependent

$$I = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix}$$

$$I_{11} = \sum_{i=1}^{n} m_i \left(\rho_{i2}^2 + \rho_{i3}^2\right) \qquad I_{12} = I_{21} = -\sum_{i=1}^{n} m_i \rho_{i2} \rho_{i1}$$

$$I_{22} = \sum_{i=1}^{n} m_i \left(\rho_{i1}^2 + \rho_{i3}^2\right) \qquad I_{13} = I_{31} = -\sum_{i=1}^{n} m_i \rho_{i1} \rho_{i3}$$

$$I_{33} = \sum_{i=1}^{n} m_i \left(\rho_{i1}^2 + \rho_{i2}^2\right) \qquad I_{23} = I_{32} = -\sum_{i=1}^{n} m_i \rho_{i2} \rho_{i3}$$

Kinetic Energy and Euler Equations

Kinetic Energy

$$E_{\text{total}} = \underbrace{\frac{1}{2}\left(\sum_{i=1}^{n} m_i\right)\dot{R}^2}_{\text{E-TRANS}} + \underbrace{\frac{1}{2}\sum_{i=1}^{n} m_i \dot{\rho}_i^2}_{\text{E-ROT}}$$

For a RIGID BODY, CM Coordinates with $\underline{\omega}$ resolved in body axis frame

$$E_{\text{ROT}} = \frac{1}{2}\underline{\omega}\cdot\underline{H} = \frac{1}{2}\underline{\omega}^T I \underline{\omega}$$

$$\dot{\underline{H}} = \underline{T} - \underline{\omega}\times\left[\underline{\underline{I}}\,\underline{\omega}\right]$$

Sum of external and internal torques

In a BODY-FIXED, PRINCIPAL AXES CM FRAME: **Euler Equations**

$$\dot{H}_1 = I_1\dot{\omega}_1 = T_1 + (I_{22} - I_{33})\omega_2\omega_3$$

$$\dot{H}_2 = I_2\dot{\omega}_2 = T_2 + (I_{33} - I_{11})\omega_3\omega_1$$

$$\dot{H}_3 = I_3\dot{\omega}_3 = T_3 + (I_{11} - I_{22})\omega_1\omega_2$$

No general solution exists. Particular solutions exist for simple torques. Computer simulation usually required.

Torque Free Solutions of Euler's Eq.

TORQUE-FREE CASE:

An important special case is the torque-free motion of a (nearly) symmetric body spinning primarily about its symmetry axis

By these assumptions: $\omega_x, \omega_y << \omega_z = \Omega$ $I_{xx} \cong I_{yy}$

The components of angular velocity then become:

$$\omega_x(t) = \omega_{xo} \cos \omega_n t$$

$$\omega_y(t) = \omega_{yo} \cos \omega_n t$$

The ω_n is defined as the "natural" or "nutation" frequency of the body:

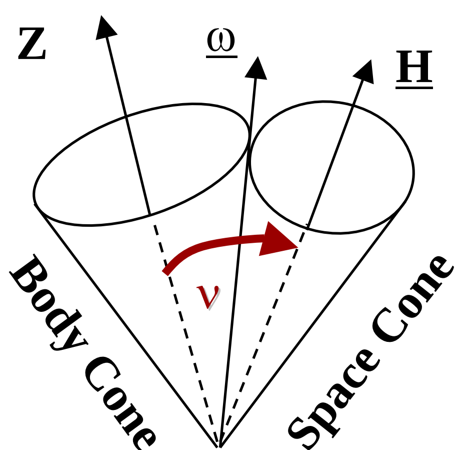
$$\omega_n^2 = K_x K_y \Omega^2$$

And the Euler equations become:

$$\dot{\omega}_x = -\underbrace{\frac{I_{zz} - I_{yy}}{I_{xx}}}_{K_x} \Omega \omega_y$$

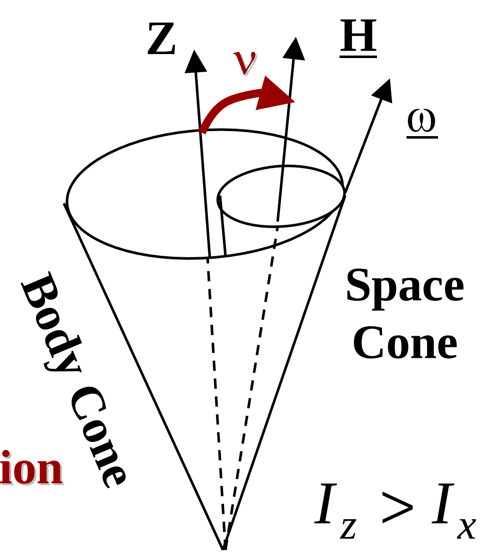
$$\dot{\omega}_y = \underbrace{\frac{I_{zz} - I_{xx}}{I_{yy}}}_{K_y} \Omega \omega_y$$

$$\dot{\omega}_z = 0$$



$I_z < I_x = I_y$

ν : nutation angle



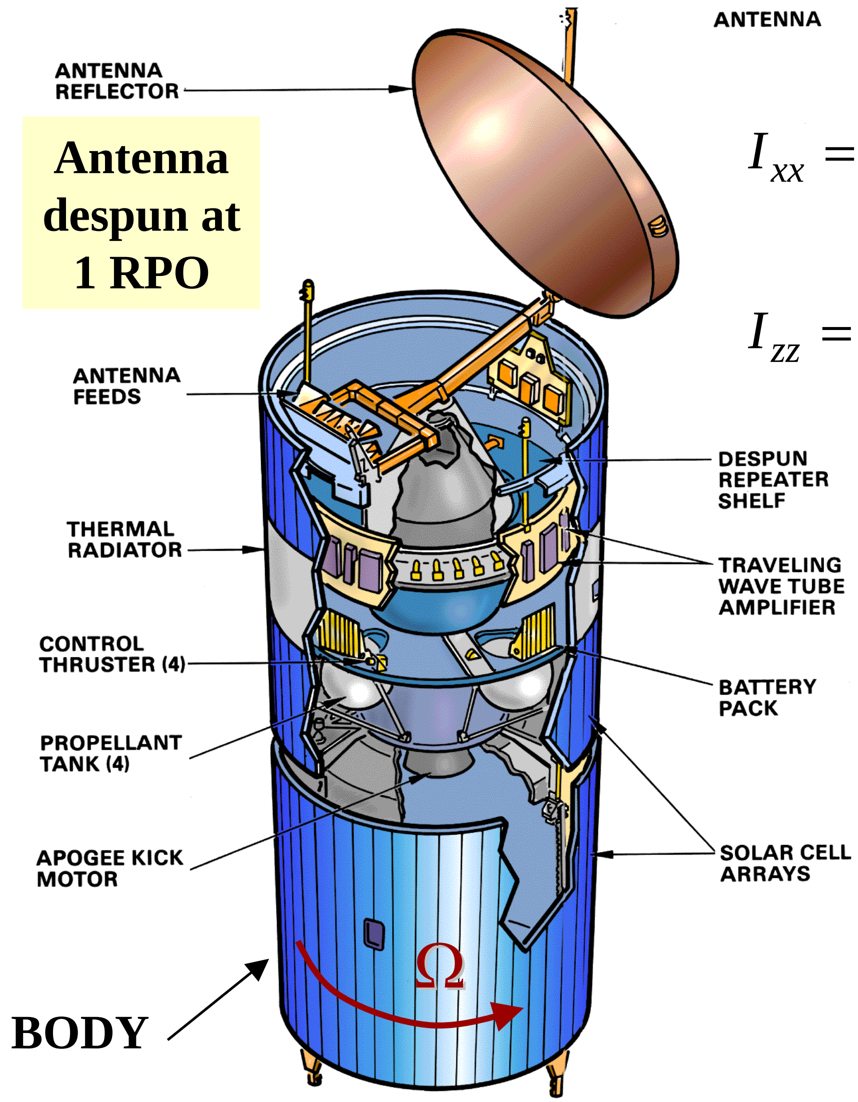
$I_z > I_x = I_y$

H and ω never align unless spun about a principal axis !

Spin Stabilized Spacecraft

UTILIZED TO STABILIZE SPINNERS

Antenna despun at 1 RPO

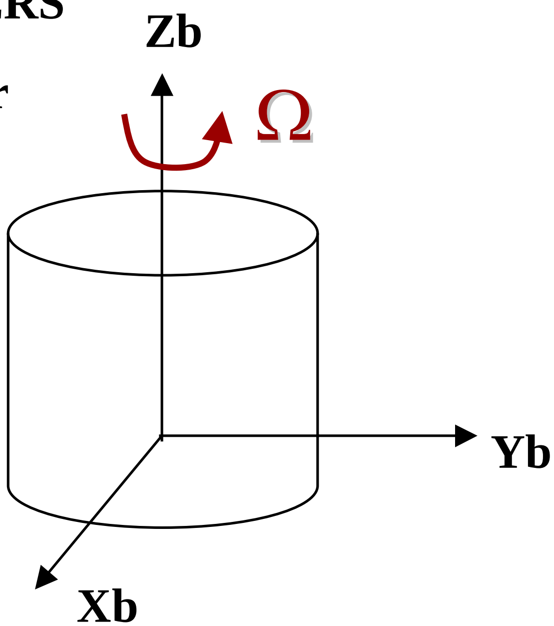


HS 376
SPACECRAFT CONFIGURATION

Perfect Cylinder

$$I_{xx} = I_{yy} = \frac{m}{4}\left(\frac{L^2}{3} + R^2\right)$$

$$I_{zz} = \frac{mR^2}{2}$$



DUAL SPIN

- Two bodies rotating at different rates about a common axis
- Behaves like simple spinner, but part is despun (antennas, sensors)
- requires torquers (jets, magnets) for momentum control and nutation dampers for stability
- allows relaxation of major axis rule

Disturbance Torques

Assessment of expected disturbance torques is an essential part of rigorous spacecraft attitude control design

Typical Disturbances

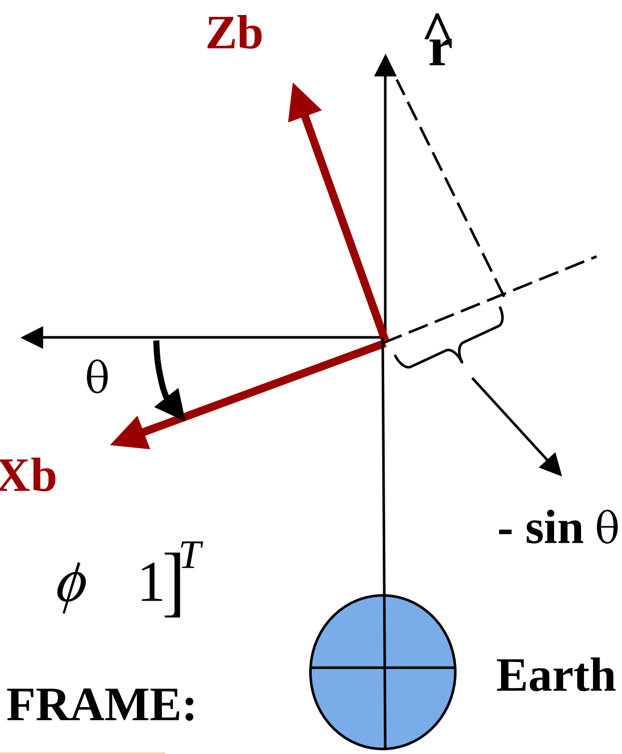
- Gravity Gradient: "Tidal" Force due to 1/r2 gravitational field variation for long, extended bodies (e.g. Space Shuttle, Tethered vehicles)
- Aerodynamic Drag: "Weathervane" Effect due to an offset between the CM and the drag center of Pressure (CP). Only a factor in LEO.
- Magnetic Torques: Induced by residual magnetic moment. Model the spacecraft as a magnetic dipole. Only within magnetosphere.
- Solar Radiation: Torques induced by CM and solar CP offset. Can compensate with differential reflectivity or reaction wheels.
- Mass Expulsion: Torques induced by leaks or jettisoned objects
- Internal: On-board Equipment (machinery, wheels, cryocoolers, pumps etc…). No net effect, but internal momentum exchange affects attitude.

Gravity Gradient

$$n = \sqrt{\mu / a^3} = \text{ORBITAL RATE}$$

<u>Gravity Gradient:</u>

1) $\perp$ Local vertical
2) 0 for symmetric spacecraft
3) proportional to $\propto 1/r^3$



Gravity Gradient Torques

$$\underline{T} = 3n^2 \cdot \hat{r} \times \left[\underline{\underline{I}} \cdot \hat{r} \right]$$

In Body Frame

Small angle approximation

$$\hat{r} = \left[-\sin\theta \quad \sin\phi \quad 1 - \sin^2\theta - \sin^2\phi \right]^T \cong \left[-\theta \quad \phi \quad 1 \right]^T$$

Resulting torque in BODY FRAME:

$$\therefore T \cong 3n^2 \begin{bmatrix} (I_{zz} - I_{yy})\phi \\ (I_{zz} - I_{xx})\theta \\ 0 \end{bmatrix}$$

<u>Typical Values:</u>
I=1000 kgm^2
n=0.001 s^{-1}
T= 6.7 x 10^{-5} Nm/deg

Pitch Libration freq.:

$$\omega_{lib} = n \sqrt{\frac{3(I_{xx} - I_{zz})}{I_{yy}}}$$

Aerodynamic Torque

$$\underline{T} = \underline{r} \times \underline{F}_a$$

$\underline{r}$ = Vector from body CM to Aerodynamic CP

$$F_a = \frac{1}{2}\rho V^2 S C_D$$

$\underline{F}_a$ = Aerodynamic Drag Vector in Body coordinates

Aerodynamic Drag Coefficient

$$1 \le C_D \le 2$$

Typically in this Range for Free Molecular Flow

S = Frontal projected Area

V = Orbital Velocity

ρ = Atmospheric Density

2×10^{-9} kg/m^3 (150 km)
3×10^{-10} kg/m^3 (200 km)
7×10^{-11} kg/m^3 (250 km)
4×10^{-12} kg/m^3 (400 km)

Exponential Density Model

Typical Values:

- $C_d = 2.0$
- $S = 5$ m^2
- $r = 0.1$ m
- $r = 4 \times 10^{-12}$ kg/m^3
- $T = 1.2 \times 10^{-4}$ Nm

Notes

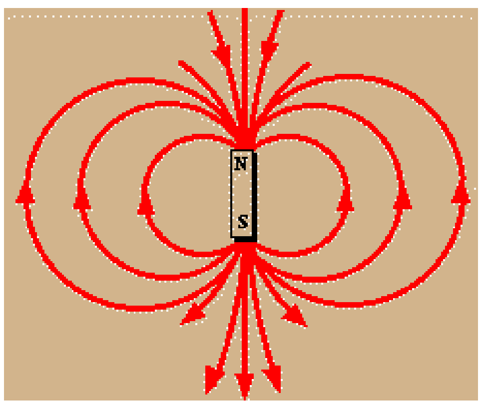
(1) $\underline{r}$ varies with Attitude
(2) ρ varies by factor of 5-10 at a given altitude
(3) C_D is uncertain by 50 %

Magnetic Torque

$$\underline{T} = \underline{M} \times \underline{B}$$

M = Spacecraft residual dipole in AMPERE-TURN-m2 (SI) or POLE-CM (CGS)

M = is due to current loops and residual magnetization, and will be on the order of 100 POLE-CM or more for small spacecraft.

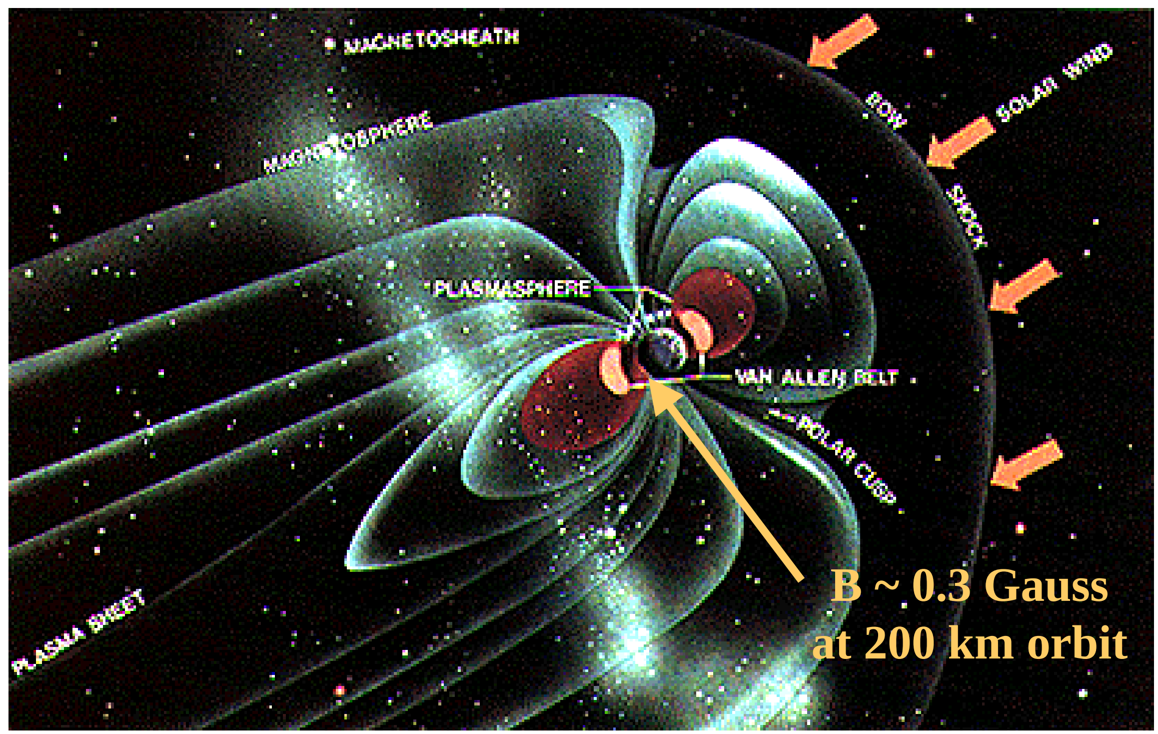


Typical Values:
$B = 3 \times 10^{-5}$ TESLA
$M = 0.1$ Atm^2
$T = 3 \times 10^{-6}$ Nm

B = Earth magnetic field vector in spacecraft coordinates (BODY FRAME) in TESLA (SI) or Gauss (CGS) units.

B varies as 1/r3, with its direction along local magnetic field lines.

Conversions:
1 Atm2 = 1000 POLE-CM , 1 TESLA = 104 Gauss



Solar Radiation Torque

$$\underline{T} = \underline{r} \times \underline{F}_s$$

$$F_s = (1 + K) P_s S$$

$$P_s = I_s / c$$

$$I_s = 1400 \text{ W/m}^2 \quad @ \ 1 \ \text{A.U.}$$

r = Vector from Body CM to optical Center-of-Pressure (CP)

$\underline{F}_s$ = Solar Radiation pressure in BODY FRAME coordinates

K = Reflectivity , 0 < K <1

S = Frontal Area

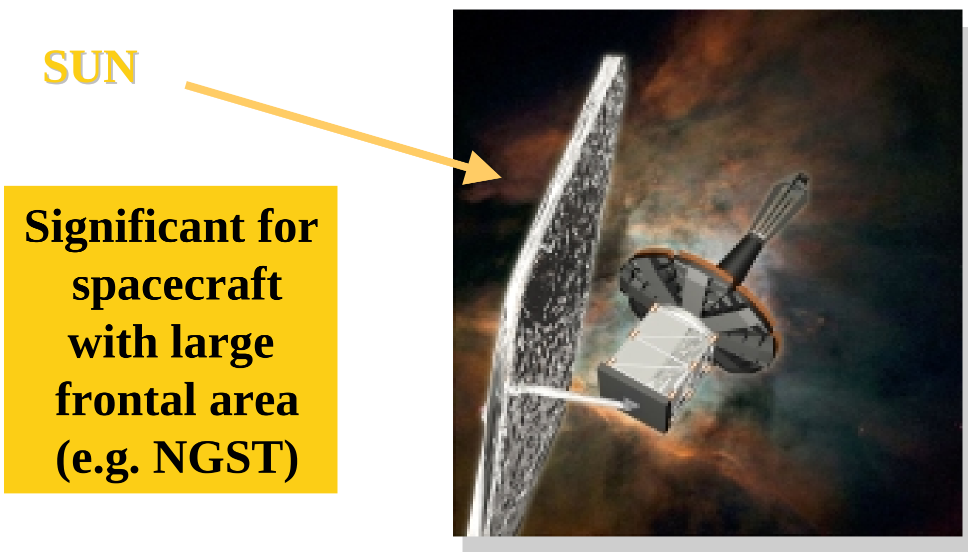
I_s = Solar constant, depends on heliocentric altitude

Notes:

(a) Torque is always $\perp$ to sun line
(b) Independent of position or velocity as long as in sunlight

Typical Values:
K = 0.5
S =5 m^2
r =0.1 m
T = 3.5 x 10^{-6} Nm

Significant for spacecraft with large frontal area (e.g. NGST)



Mass Expulsion and Internal Torques

Mass Expulsion Torque:

$$\underline{T} = \underline{r} \times \underline{F}$$

Notes:

(1) **May be deliberate (Jets, Gas venting) or accidental (Leaks)**

(2) **Wide Range of r, F possible; torques can dominate others**

(3) **Also due to jettisoning of parts (covers, cannisters)**

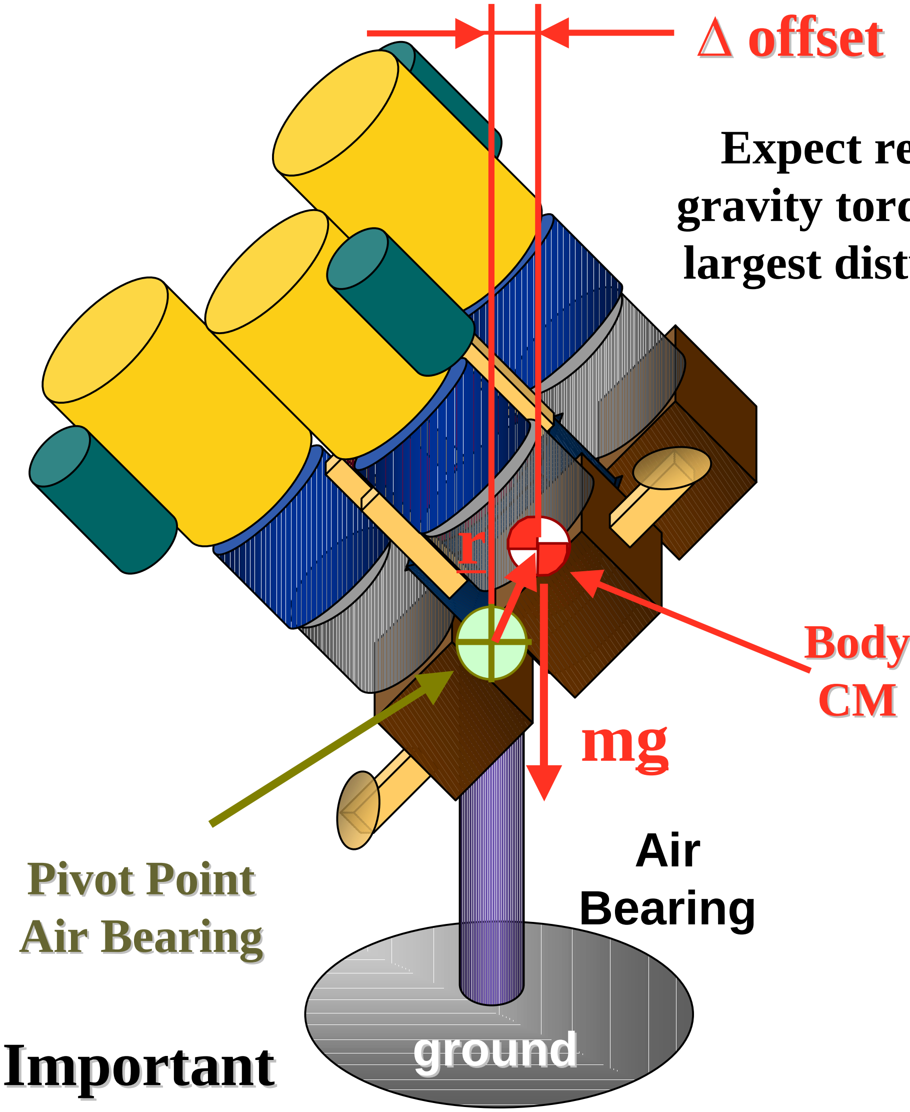
Internal Torque:

Notes:

(1) **Momentum exchange between moving parts has no effect on System H, but will affect attitude control loops**

(2) **Typically due to antenna, solar array, scanner motion or to deployable booms and appendages**

Disturbance Torque for CDIO



Expect residual gravity torque to be largest disturbance

Important to balance precisely !

Initial Assumption: $|T| = |\underline{r} \times m\underline{g}| \cong 0.001 \cdot 100 \cdot 9.81 \cong 1 \ \ [\text{Nm}]$

Passive Attitude Control (1)

Passive control techniques take advantage of basic physical principles and/or naturally occurring forces by designing the spacecraft so as to enhance the effect of one force, while reducing the effect of others.

SPIN STABILIZED

$$\dot{H} = |T| = rF$$

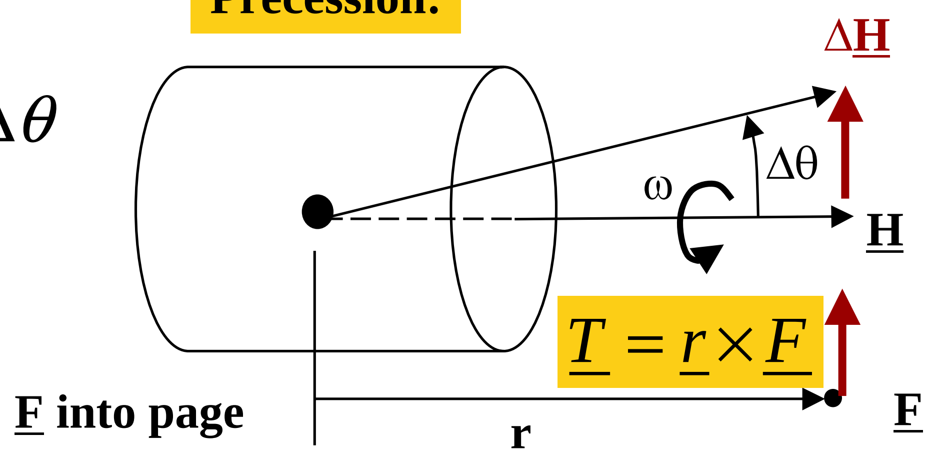
$$\dot{H} = \frac{dH}{dt} \cong \frac{\Delta H}{\Delta t}$$

$$\therefore \Delta H \cong rF\,\Delta t$$

- Requires Stable Inertia Ratio: Iz > Iy =Ix
- Requires Nutation damper: Eddy Current, Ball-in-Tube, Viscous Ring, Active Damping
- Requires Torquers to control precession (spin axis drift) magnetically or with jets
- Inertially oriented

Precession:

$$\Delta H = 2H \sin\frac{\Delta\theta}{2} \cong H\Delta\theta = I\omega\cdot\Delta\theta$$



Large ω = gyroscopic stability

$$\Delta\theta \cong \frac{rF\,\Delta t}{H} = \frac{rF}{I\omega}\Delta t$$

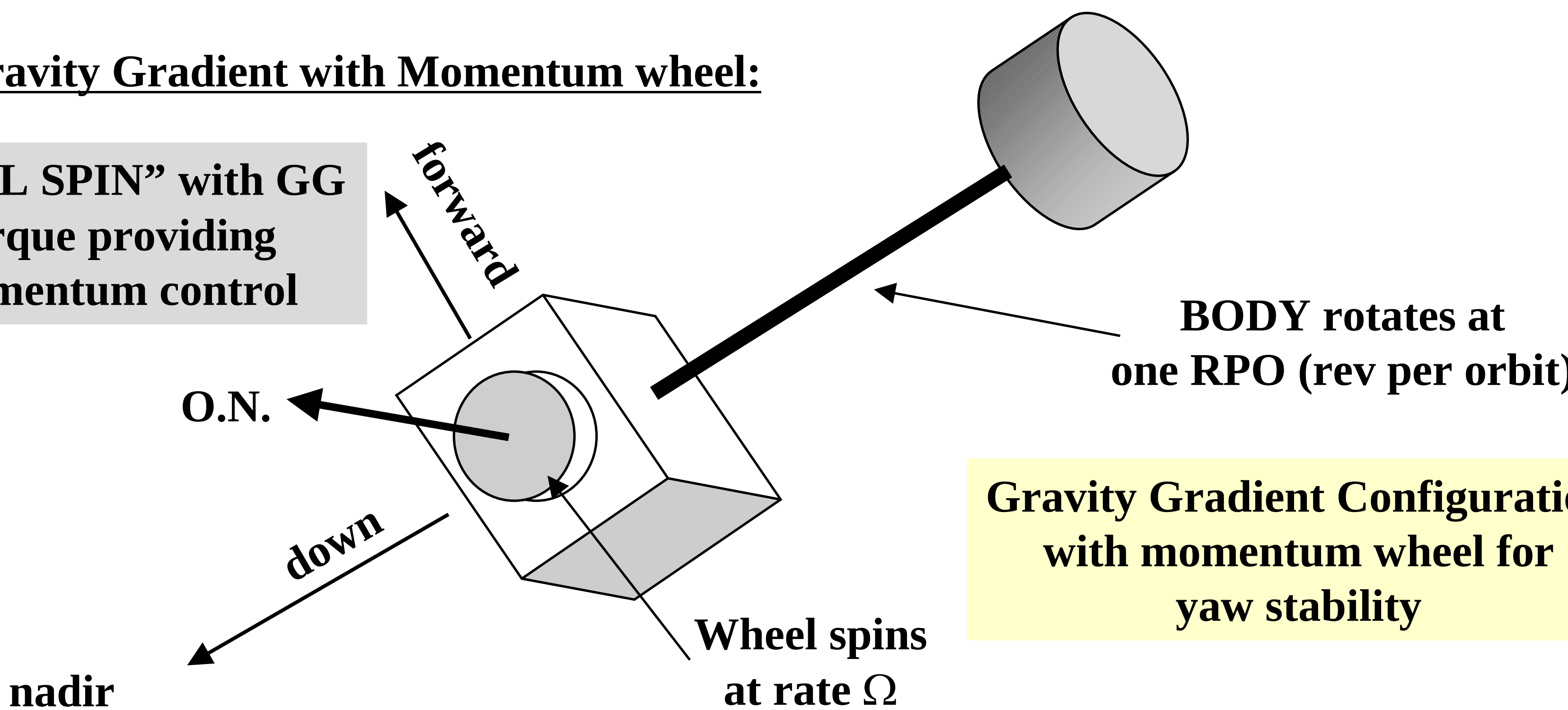
Passive Attitude Control (2)

GRAVITY GRADIENT

- Requires stable Inertias: $I_z << I_x, I_y$
- Requires Libration Damper: Eddy Current, Hysteresis Rods
- Requires no Torquers
- Earth oriented
- No Yaw Stability (can add momentum wheel)

Gravity Gradient with Momentum wheel:

"DUAL SPIN" with GG torque providing momentum control



Gravity Gradient Configuration with momentum wheel for yaw stability

Active Attitude Control

Active Control Systems directly sense spacecraft attitude and supply a torque command to alter it as required. This is the basic concept of feedback control.

- Reaction Wheels most common actuator
- Fast; continuous feedback control
- Moving Parts
- Internal Torque only; external still required for "momentum dumping"
- Relatively high power, weight, cost
- Control logic simple for independent axes (can get complicated with redundancy)

Typical Reaction (Momentum) Wheel Data:

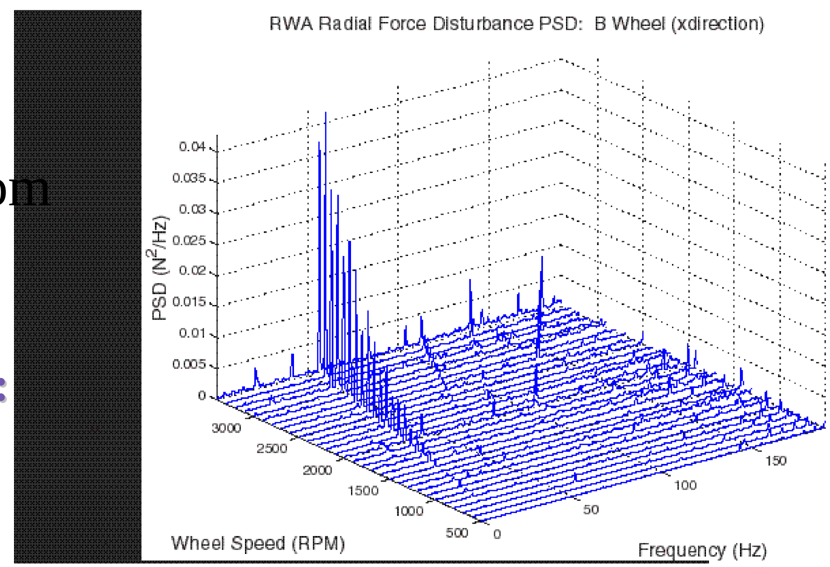
Operating Range: 0 +/- 6000 RPM
Angular Momentum @ 2000 RPM: 1.3 Nms
Angular Momentum @ 6000 RPM: 4.0 Nms
Reaction Torque: 0.020 - 0.3 Nm

Actuators: Reaction Wheels

- One creates torques on a spacecraft by creating equal but opposite torques on **Reaction Wheels** (flywheels on motors).
  - For three-axes of torque, three wheels are necessary. Usually use four wheels for redundancy (use wheel speed biasing equation)
  - If external torques exist, wheels will angularly accelerate to counteract these torques. They will eventually reach an RPM limit (~3000-6000 RPM) at which time they must be desaturated.
  - Static & dynamic imbalances can induce vibrations (mount on isolators)
  - Usually operate around some nominal spin rate to avoid stiction effects.

Ithaco RWA's (www.ithaco.com /products.html)

Waterfall plot:



Needs to be carefully balanced !

Actuators: Magnetic Torquers

Magnetic Torquers

- Often used for Low Earth Orbit (LEO) satellites
- Useful for initial acquisition maneuvers
- Commonly use for momentum desaturation (“dumping”) in reaction wheel systems
- May cause harmful influence on star trackers

- Can be used
  - for attitude control
  - to de-saturate reaction wheels
- Torque Rods and Coils
  - Torque rods are long helical coils
  - Use current to generate magnetic field
  - This field will try to align with the Earth’s magnetic field, thereby creating a torque on the spacecraft
  - Can also be used to sense attitude as well as orbital location

ACS Actuators: Jets / Thrusters

- Thrusters / Jets
  - Thrust can be used to control attitude but at the cost of consuming fuel
  - Calculate required fuel using "Rocket Equation"
  - Advances in micro-propulsion make this approach more feasible. Typically want I_{sp} > 1000 sec

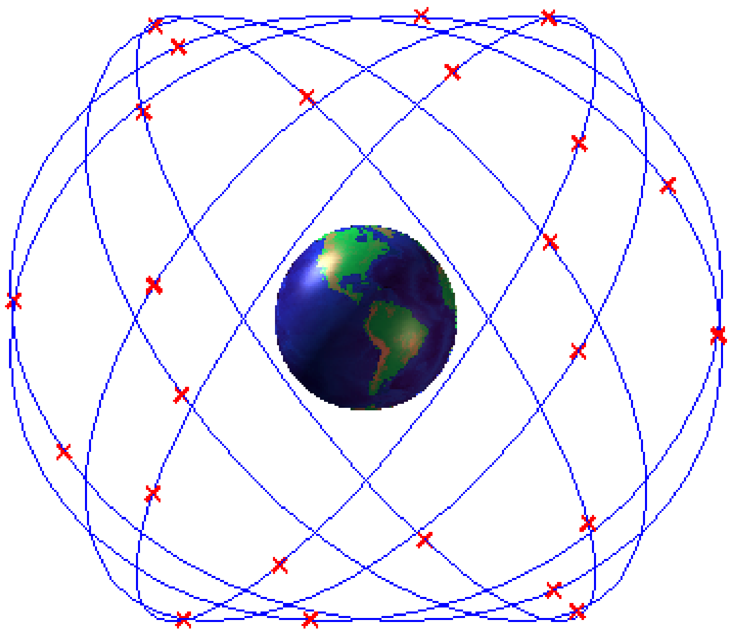
- Use consumables such as Cold Gas (Freon, N2) or Hydrazine (N2H4)
- Must be ON/OFF operated; proportional control usually not feasible: pulse width modulation (PWM)
- Redundancy usually required, makes the system more complex and expensive
- Fast, powerful
- Often introduces attitude/translation coupling
- Standard equipment on manned spacecraft
- May be used to "unload" accumulated angular momentum on reaction-wheel controlled spacecraft.

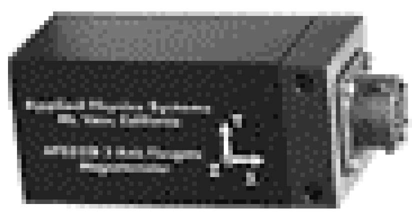
ACS Sensors: GPS and Magnetometers

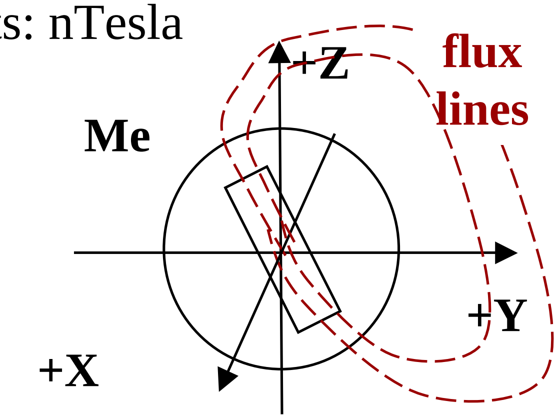
- Global Positioning System (GPS)
  - Currently 27 Satellites
  - 12hr Orbits
  - Accurate Ephemeris
  - Accurate Timing
    - Stand-Alone 100m
    - DGPS 5m
    - Carrier-smoothed DGPS 1-2m
- Magnetometers
  - Measure components Bx, By, Bz of ambient magnetic field B
  - Sensitive to field from spacecraft (electronics), mounted on boom
  - Get attitude information by comparing measured B to modeled B
  - Tilted dipole model of earth's field:

$$\begin{bmatrix} B_{north} \\ B_{east} \\ B_{down} \end{bmatrix} = \left(\frac{6378}{r_{km}}\right)^3 \begin{bmatrix} -C_\varphi & S_\varphi C_\lambda & S_\varphi S_\lambda \\ 0 & S_\lambda & -C_\lambda \\ -2S_\varphi & -2C_\varphi C_\lambda & -2C_\varphi S_\lambda \end{bmatrix} \begin{bmatrix} -29900 \\ -1900 \\ 5530 \end{bmatrix}$$

Where: C=cos , S=sin, ϕ=latitude, λ=longitude
Units: nTesla







ACS Sensors: Rate Gyros and IMUs

- Rate Gyros (Gyroscopes)
  - Measure the angular rate of a spacecraft relative to inertial space
  - Need at least three. Usually use more for redundancy.
  - Can integrate to get angle. However,
    - DC bias errors in electronics will cause the output of the integrator to ramp and eventually saturate (drift)
    - Thus, need inertial update

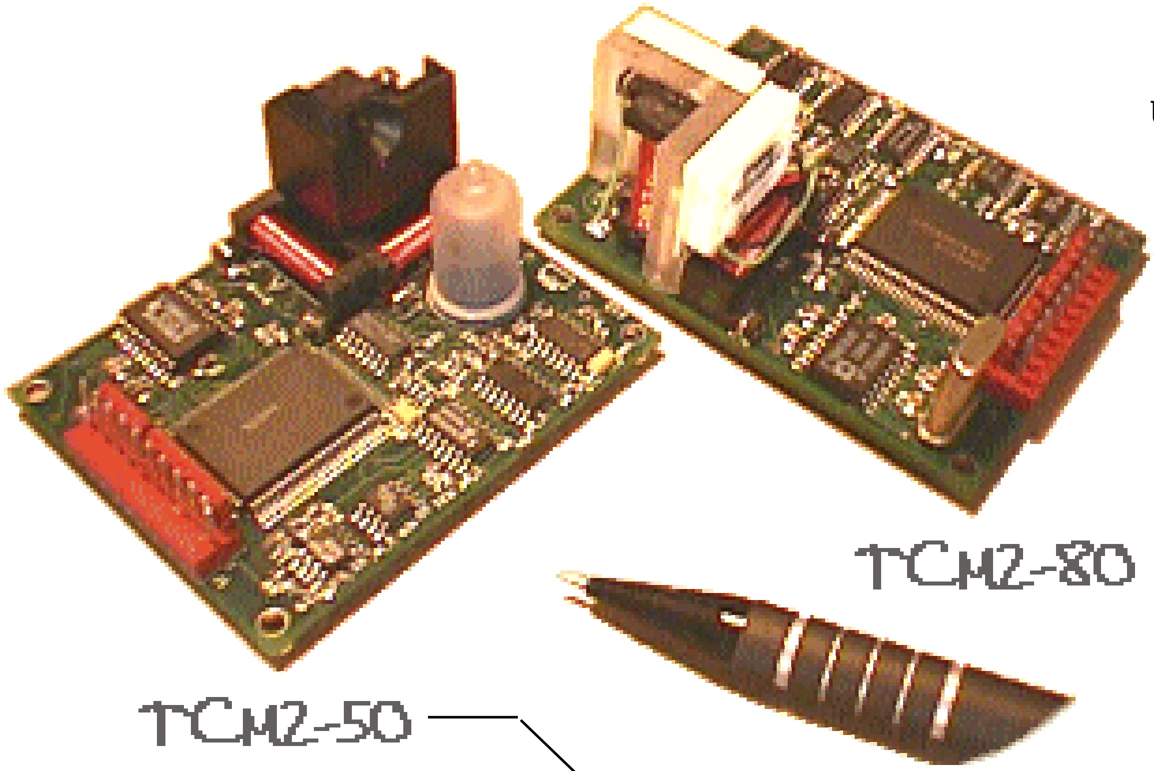
- Mechanical gyros (accurate, heavy)
- Ring Laser (RLG)
- MEMS-gyros

- Inertial Measurement Unit (IMU)
  - Integrated unit with sensors, mounting hardware,electronics and software
  - measure rotation of spacecraft with rate gyros
  - measure translation of spacecraft with accelerometers
  - often mounted on gimbaled platform (fixed in inertial space)
  - Performance 1: gyro drift rate (range: 0 .003 deg/hr to 1 deg/hr)
  - Performance 2: linearity (range: 1 to 5E-06 g/g^2 over range 20-60 g
  - Typically frequently updated with external measurement (Star Trackers, Sun sensors) via a Kalman Filter

ACS Sensor Performance Summary

Reference	Typical Accuracy	Remarks
Sun	1 min	Simple, reliable, low cost, not always visible
Earth	0.1 deg	Orbit dependent; usually requires scan; relatively expensive
Magnetic Field	1 deg	Economical; orbit dependent; low altitude only; low accuracy
Stars	0.001 deg	Heavy, complex, expensive, most accurate
Inertial Space	0.01 deg/hour	Rate only; good short term reference; can be heavy, power, cost

CDIO Attitude Sensing



Will not be able to use/afford STAR TRACKERS !

From where do we get an attitude estimate for inertial updates ?

Potential Solution: Electronic Compass, Magnetometer and Tilt Sensor Module

Specifications:

Heading accuracy: +/- 1.0 deg RMS @ +/- 20 deg tilt
Resolution 0.1 deg, repeatability: +/- 0.3 deg
Tilt accuracy: +/- 0.4 deg, Resolution 0.3 deg
Sampling rate: 1-30 Hz

Problem: Accuracy insufficient to meet requirements alone, will need FINE POINTING mode

Spacecraft Attitude Schemes

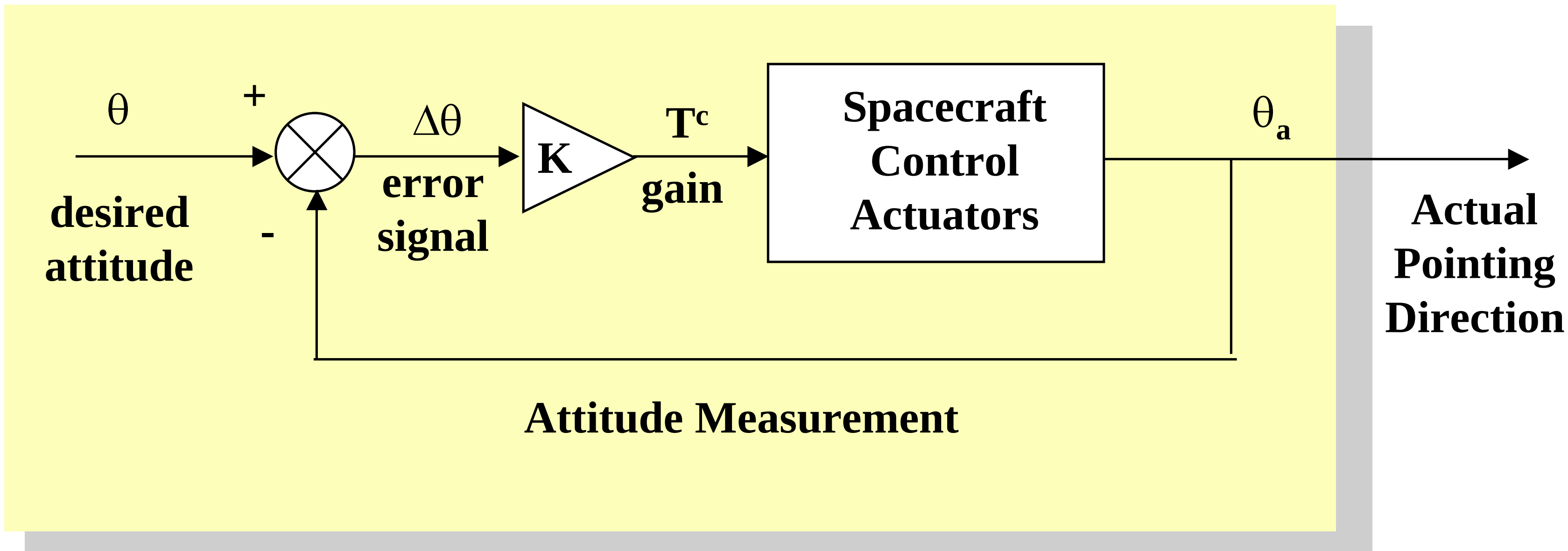
- Spin Stabilized Satellites
  - Spin the satellite to give it gyroscopic stability in inertial space
  - Body mount the solar arrays to guarantee partial illumination by sun at all times
  - EX: early communication satellites, stabilization for orbit changes
  - Torques are applied to precess the angular momentum vector
- De-Spun Stages
  - Some sensor and antenna systems require inertial or Earth referenced pointing
  - Place on de-spun stage
  - EX: Galileo instrument platform
- Gravity Gradient Stabilization
  - "Long" satellites will tend to point towards Earth since closer portion feels slightly more gravitational force.
  - Good for Earth-referenced pointing
  - EX: Shuttle gravity gradient mode minimizes ACS thruster firings
- Three-Axis Stabilization
  - For inertial or Earth-referenced pointing
  - Requires active control
  - EX: Modern communications satellites, International Space Station, MIR, Hubble Space Telescope

ADCS Performance Comparison

Method	Typical Accuracy	Remarks
Spin Stabilized	0.1 deg	Passive, simple; single axis inertial, low cost, need slip rings
Gravity Gradient	1-3 deg	Passive, simple; central body oriented; low cost
Jets	0.1 deg	Consumables required, fast; high cost
Magnetic	1 deg	Near Earth; slow ; low weight, low cost
Reaction Wheels	0.01 deg	Internal torque; requires other momentum control; high power, cost

3-axis stabilized, active control most common choice for precision spacecraft

ACS Block Diagram (1)



Feedback Control Concept: $T^c = K \cdot \Delta\theta$ Correction torque = gain x error

Force or torque is proportional to deflection. This is the equation, which governs a simple linear or rotational “spring” system. If the spacecraft responds “quickly we can estimate the required gain and system bandwidth.

Gain and Bandwidth

Assume control saturation half-width θ_{sat} at torque command T_{sat}, then

$$K \cong \frac{T_{sat}}{\theta_{sat}} \qquad \textbf{hence} \qquad \ddot{\theta} + \left(\frac{K}{I}\right)\theta_{sat} \cong 0$$

Recall the oscillator frequency of a simple linear, torsional spring:

$$\omega = \sqrt{\frac{K}{I}} \quad \text{[rad/sec]}$$

I = moment of inertia

This natural frequency is approximately equal to the system bandwidth. Also,

$$f = \frac{\omega}{2\pi} \quad \text{[Hz]} \quad \Rightarrow \quad \tau = \frac{1}{\text{f}} = \frac{2\pi}{\omega}$$

Is approximately the system time constant τ.

Note: we can choose any two of the set:

$$\ddot{\theta}, \theta_{sat}, \omega$$

EXAMPLE:

$$\theta_{sat} = 10^{-2} \quad \text{[rad]}$$

$$T_{sat} = 10 \quad \text{[Nm]}$$

$$I = 1000 \quad [\text{kgm}^2]$$

$$\therefore K = 1000 \quad \text{[Nm/rad]}$$

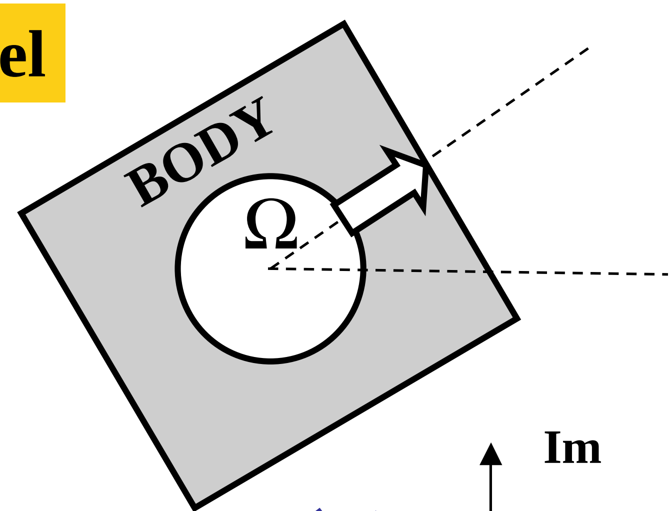
$$\omega = 1 \quad \text{[rad/sec]}$$

$$f = 0.16 \quad \text{[Hz]}$$

$$\tau = 6.3 \quad \text{[sec]}$$

Feedback Control Example

Pitch Control with a single reaction wheel



Rigid Body Dynamics

$$I\ddot{\theta} = T_w + T_{ext} = I\dot{\omega} = \dot{H}$$

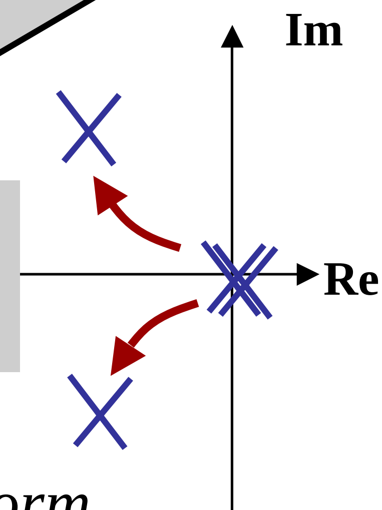
Wheel Dynamics

$$J(\dot{\Omega} + \ddot{\theta}) = -T_w = \dot{h}$$

Feedback Law, Choose

$$T_w = \underbrace{-K_p\theta}_{\text{Position feedback}} \underbrace{-K_r\dot{\theta}}_{\text{Rate feedback}}$$

Position feedback **Rate feedback**



Then:

$$\ddot{\theta} + (K_r / I)\dot{\theta} + \left(K_p / I\right) = 0 \quad \rightarrow \textit{Laplace Transform}$$

$$s^2 + (K_r / I)s + \left(K_p / I\right) = 0$$

$$s^2 + 2\zeta\omega s + \omega^2 = 0$$

Characteristic Equation

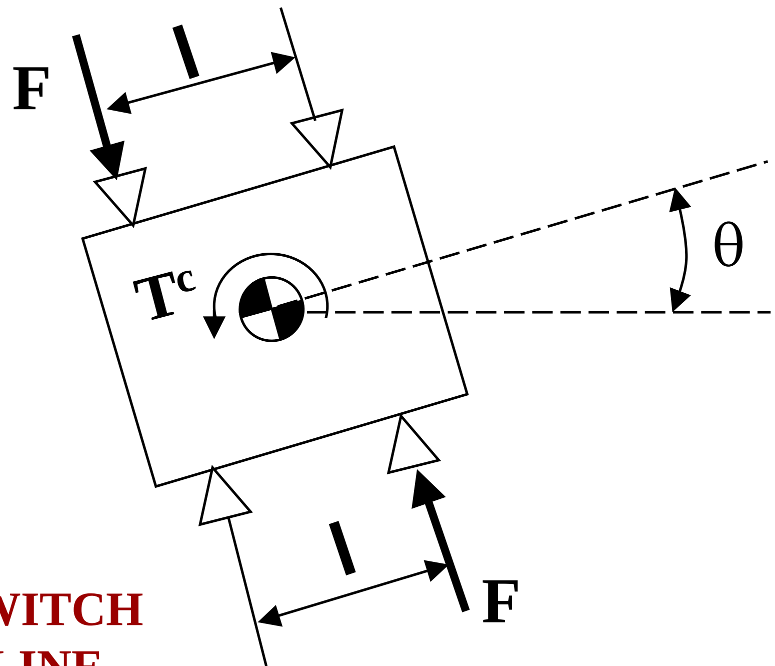
Nat. frequency

$$\omega = \sqrt{K_p / I}$$

damping

$$\zeta = \mathrm{K_r} / 2\sqrt{K_p I}$$

Jet Control Example (1)



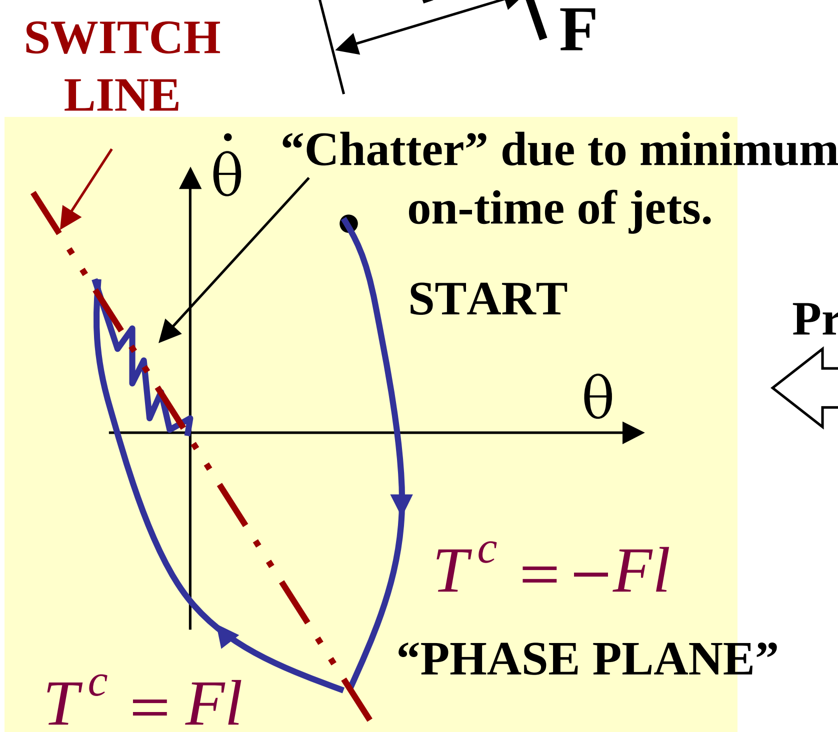
Introduce control torque $\mathbf{T^c}$ via force couple from jet thrust:

$$I\ddot{\theta} = T^c$$

Only three possible values for $\mathbf{T^c}$:

$$T^c = \begin{cases} Fl \\ 0 \\ -Fl \end{cases}$$

On/Off Control only



Problem

Can stabilize (drive θ to zero) by feedback law:

$$T^c = -Fl \cdot \operatorname{sgn}\left(\theta + \tau\dot{\theta}\right)$$

prediction term

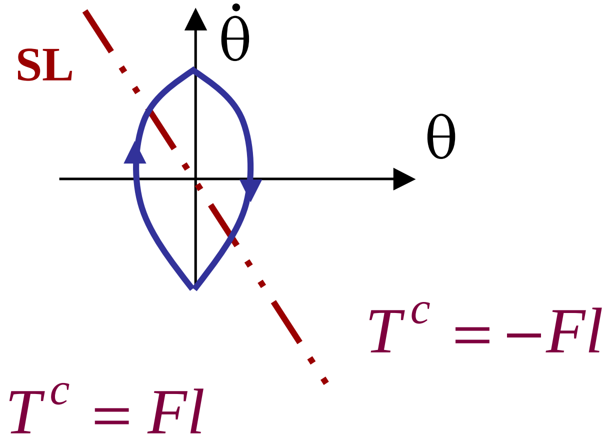
Where

$$\operatorname{sgn}(x) = \frac{x}{|x|}$$

τ = time constant

Jet Control Example (2)

"Chatter" leads to a "limit cycle", quickly wasting fuel



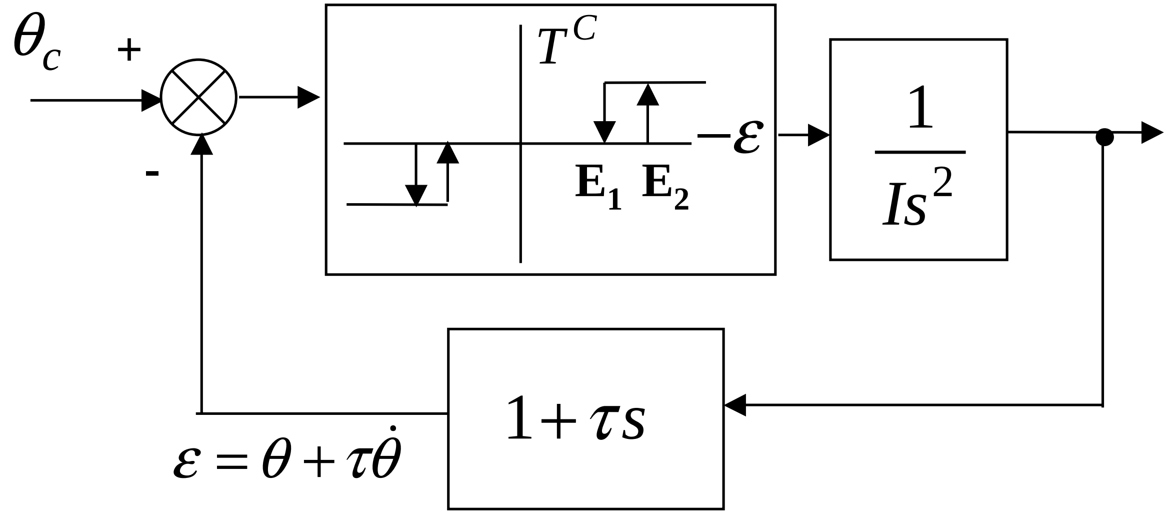
"PHASE PLANE"

At Switch Line: $\theta + \tau\dot{\theta} = 0$

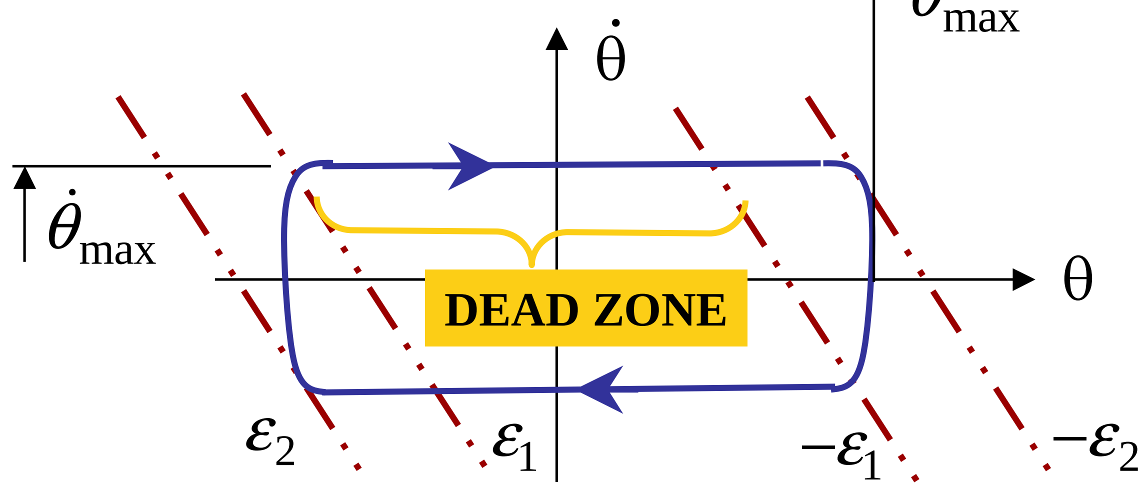
- **Low Frequency Limit Cycle**
- **Mostly Coasting**
- **Low Fuel Usage**
- **θ and $\dot{\theta}$ bounded**

Solution:

Eliminate "Chatter" by "Dead Zone" ; with Hysteresis:

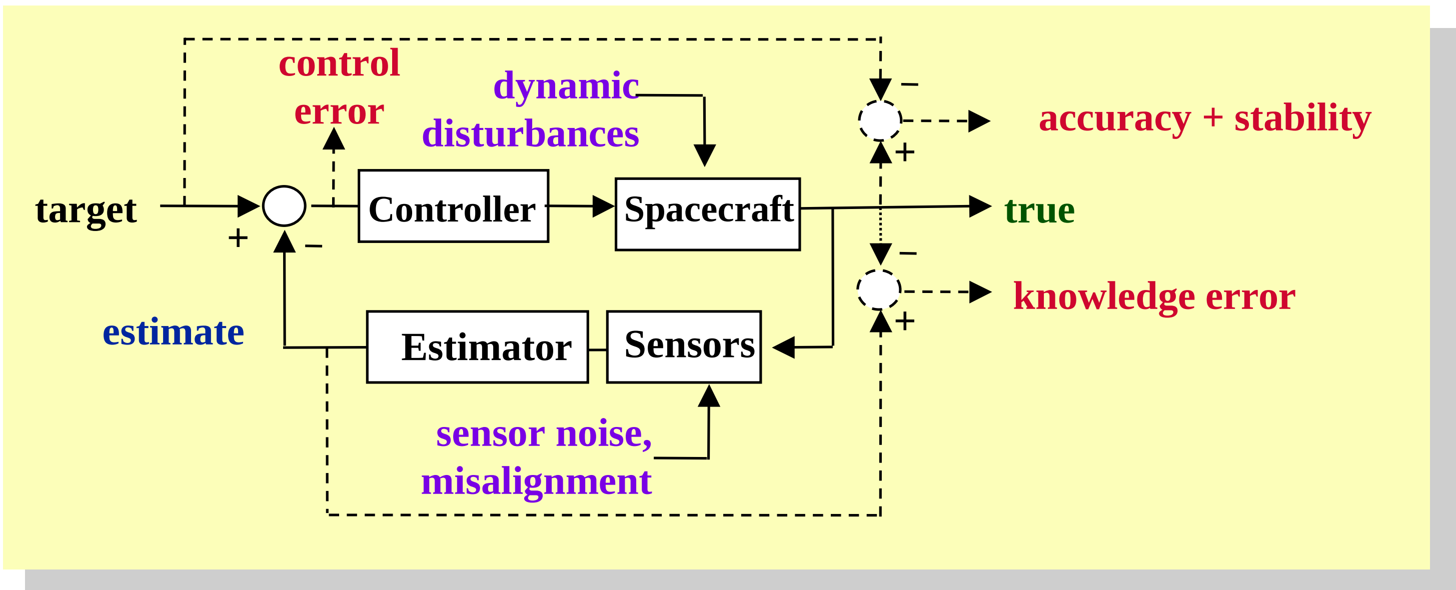


Results in the following motion:



ACS Block Diagram (2)

In the "REAL WORLD" things are somewhat more complicated:

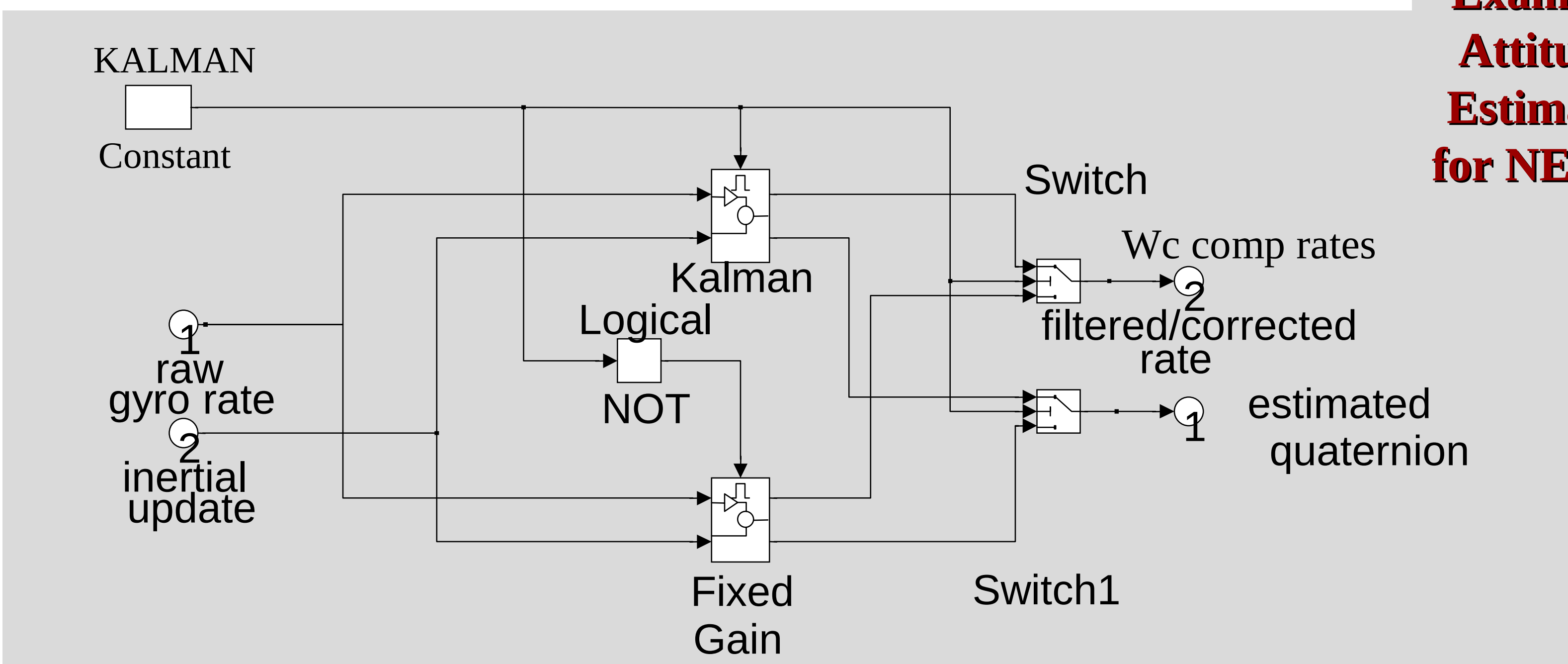


- Spacecraft not a RIGID body, sensor , actuator & avionics dynamics
- Digital implementation: work in the z-domain
- Time delay (lag) introduced by digital controller
- A/D and D/A conversions take time and introduce errors: 8-bit, 12-bit, 16-bit electronics, sensor noise present (e.g rate gyro @ DC)
- Filtering and estimation of attitude, never get q directly

Attitude Determination

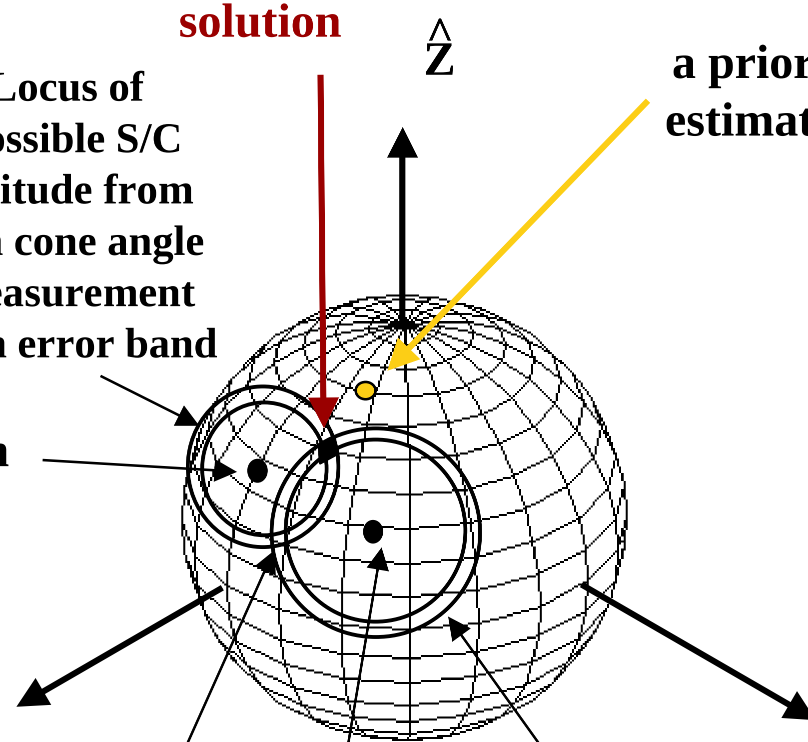
- Attitude Determination (AD) is the process of of deriving estimates of spacecraft attitude from (sensor) measurement data. Exact determination is NOT POSSIBLE, always have some error.
- Single Axis AD: Determine orientation of a single spacecraft axis in space (usually spin axis)
- Three Axis AD: Complete Orientation; single axis (Euler axis, when using Quaternions) plus rotation about that axis

Example: Attitude Estimator for NEXUS



Single-Axis Attitude Determination

- Utilizes sensors that yield an arc-length measurement between sensor boresight and known reference point (e.g. sun, nadir)
- Requires at least two independent measurements and a scheme to choose between the true and false solution
- Total lack of a priori estimate requires three measurements
- Cone angles only are measured, not full 3-component vectors. The reference (e.g. sun, earth) vectors are known in the reference frame, but only partially so in the body frame.



Three-Axis Attitude Determination

- Need two vectors (u,v) measured in the spacecraft frame and known in reference frame (e.g. star position on the celestial sphere)
- Generally there is redundant data available; can extend the calculations on this chart to include a least-squares estimate for the attitude
- Do generally not need to know absolute values

$$|u|,|v|$$

Define:

$$\hat{i} = u/|u|$$

$$j = (u \times v)/|u \times v|$$

$$\hat{k} = \hat{i} \times \hat{j}$$

Want Attitude Matrix T:

$$\underbrace{\begin{bmatrix} \hat{i}_B & \hat{j}_B & \hat{k}_B \end{bmatrix}}_{M} = T \cdot \underbrace{\begin{bmatrix} \hat{i}_R & \hat{j}_R & \hat{k}_R \end{bmatrix}}_{N}$$

So: $T = MN^{-1}$

<u>Note:</u> N must be non-singular (= full rank)

Effects of Flexibility (Spinners)

The previous solutions for Euler's equations were only valid for a RIGID BODY. When flexibility exists, energy dissipation will occur.

$$\underline{H} = \underline{\underline{I}}\,\underline{\omega}$$ ⇒ **CONSTANT**

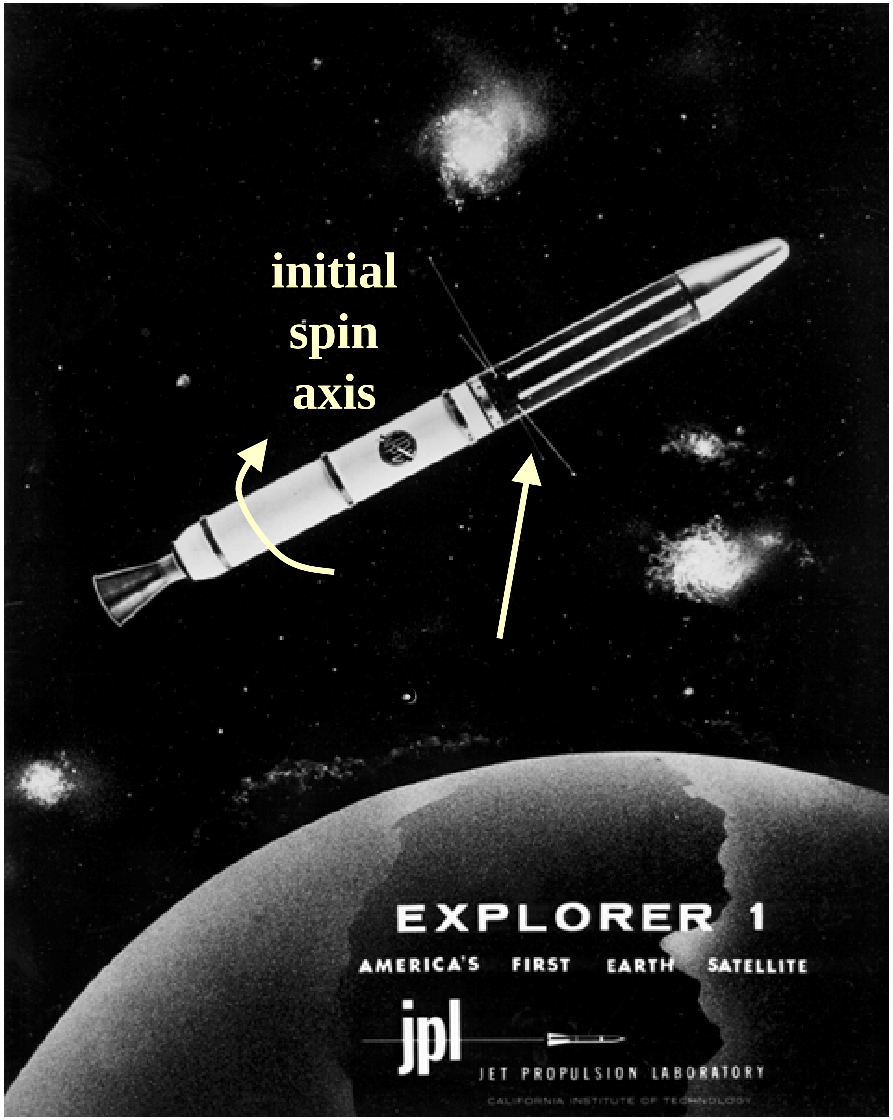
Conservation of Angular Momentum

$$E_{\mathrm{ROT}} = \frac{1}{2}\underline{\omega}^T \underline{\underline{I}}\,\underline{\omega}$$ ⇒ **DECREASING**

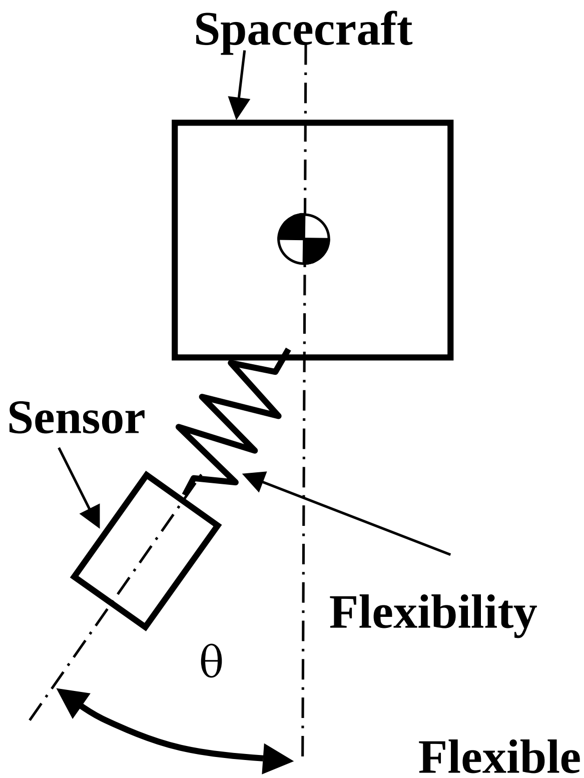
∴ **Spin goes to maximum I and minimum ω**

CONCLUSION: Stable Spin is only possible about the axis of maximum inertia.

Classical Example: EXPLORER 1

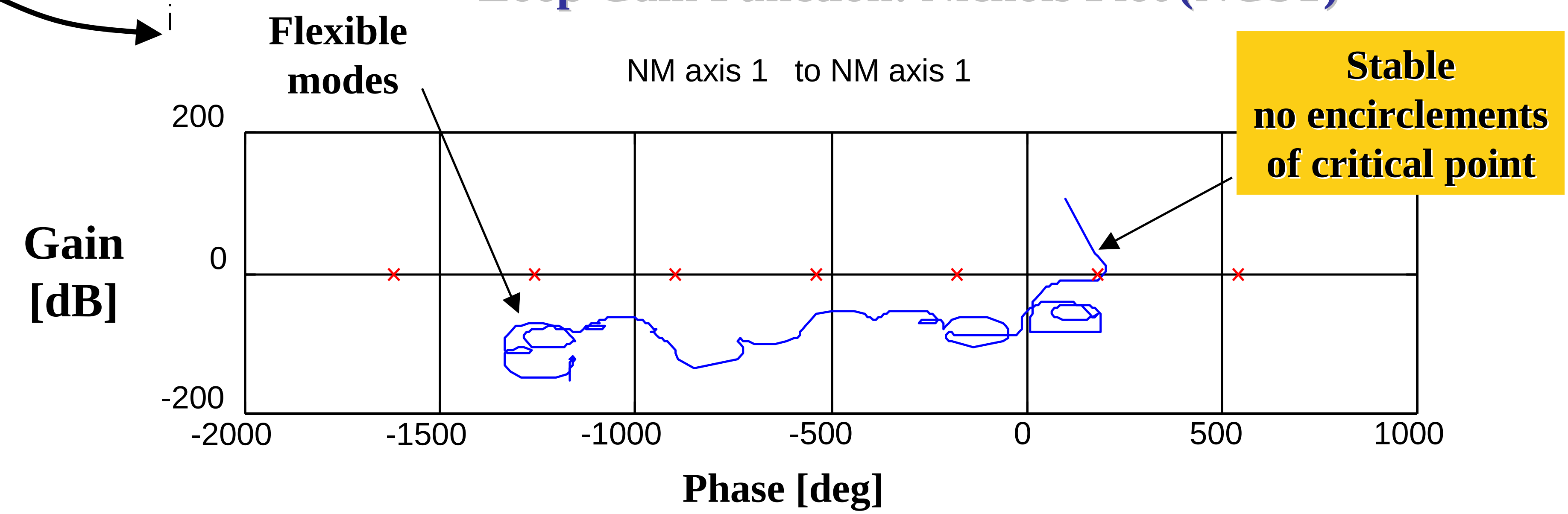


Controls/Structure Interaction



- Can't always neglect flexible modes (solar arrays, sunshield)
- Sensor on flexible structure, modes introduce phase loss
- Feedback signal "corrupted" by flexible deflections; can become unstable
- Increasingly more important as spacecraft become larger and pointing goals become tighter

Loop Gain Function: Nichols Plot (NGST)

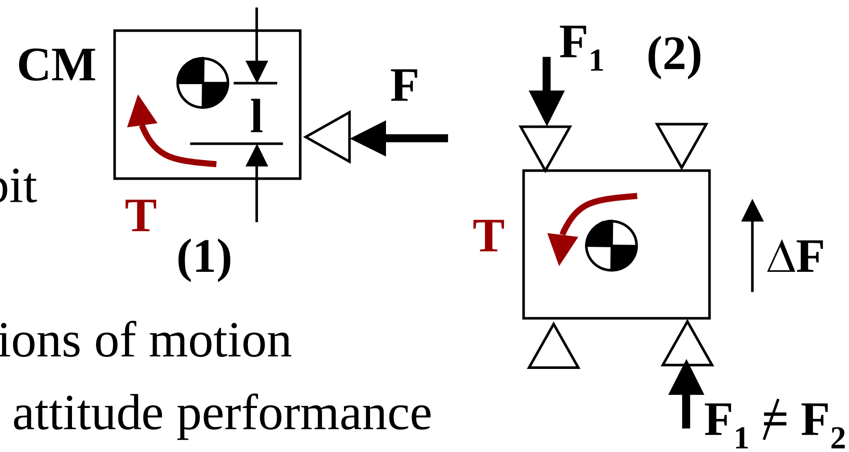


Other System Considerations (1)

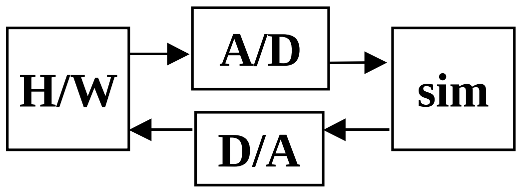
- Need on-board COMPUTER
  - Increasing need for on-board performance and autonomy
  - Typical performance (somewhat outdated: early 1990's)
  - 35 pounds, 15 Watts, 200K words, 100 Kflops/sec, CMOS
  - Rapidly expanding technology in real-time space-based computing
  - Nowadays get smaller computers, rad-hard, more MIPS
  - Software development and testing, e.g. SIMULINK Real Time Workshop, compilation from development environment MATLAB C, C++ to target processor is getting easier every year. Increased attention on software.
- Ground Processing
  - Typical ground tasks: Data Formatting, control functions, data analysis
  - Don't neglect; can be a large program element (operations)
- Testing
  - Design must be such that it can be tested
  - Several levels of tests: (1) benchtop/component level, (2) environmental testing (vibration,thermal, vacuum), (3) ACS tests: air bearing, hybrid simulation with part hardware, part simulated

Other System Considerations (2)

- Maneuvers
  - Typically: Attitude and Position Hold,Tracking/Slewing, SAFE mode
  - Initial Acquisition maneuvers frequently required
  - Impacts control logic, operations, software
  - Sometimes constrains system design
  - Maneuver design must consider other systems, I.e.: solar arrays pointed towards sun, radiators pointed toward space, antennas toward Earth
- Attitude/Translation Coupling
  - (1) Δ**v** from thrusters can affect attitude
  - **(2)** Attitude thrusters can perturb the orbit



- Simulation
  - Numerical integration of dynamic equations of motion
  - Very useful for predicting and verifying attitude performance
  - Can also be used as “surrogate” data generator
  - “Hybrid” simulation: use some or all of actual hardware, digitally simulate the spacecraft dynamics (plant)
  - can be expensive, but save money later in the program



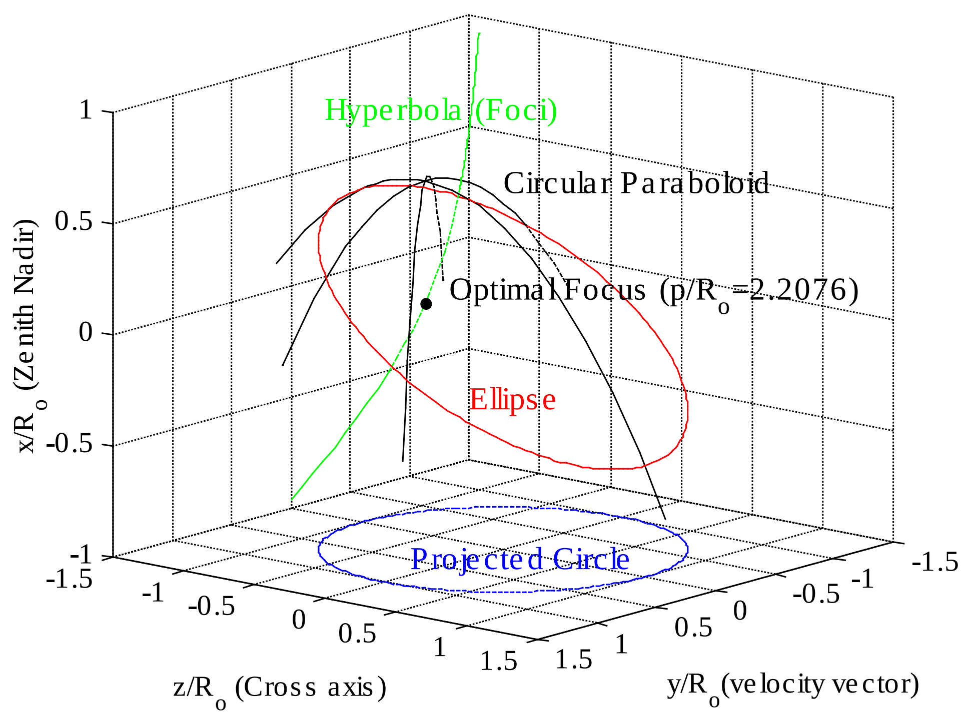
Future Trends in ACS Design

- Lower Cost
  - Standardized Spacecraft, Modularity
  - Smaller spacecraft, smaller Inertias
  - Technological progress: laser gyros, MEMS, magnetic wheel bearings
  - Greater on-board autonomy
  - Simpler spacecraft design
- Integration of GPS (LEO)
  - Allows spacecraft to perform on-board navigation; functions independently from ground station control
  - Potential use for attitude sensing (large spacecraft only)
- Very large, evolving systems
  - Space station ACS requirements change with each added module/phase
  - Large spacecraft up to 1km under study (e.g. TPF Able "kilotruss")
  - Attitude control increasingly dominated by controls/structure interaction
  - Spacecraft shape sensing/distributed sensors and actuators

Advanced ACS concepts

Visible Earth Imager using a Distributed Satellite System

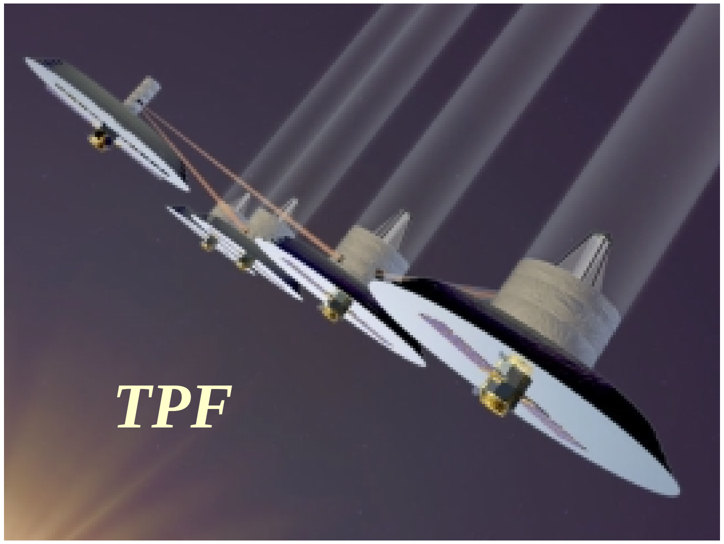
- No ΔV required for collector spacecraft
- Only need ΔV to hold combiner spacecraft at paraboloid's focus



Formation Flying in Space

- Exploit natural orbital dynamics to synthesize sparse aperture arrays using formation flying
- Hill's equations exhibit closed "free-orbit ellipse" solutions

$$\ddot{x} - 2\dot{y}n - 3n^2x = a_x$$
$$\ddot{y} + 2\dot{x}n = a_y$$
$$\ddot{z} + n^2z = a_z$$



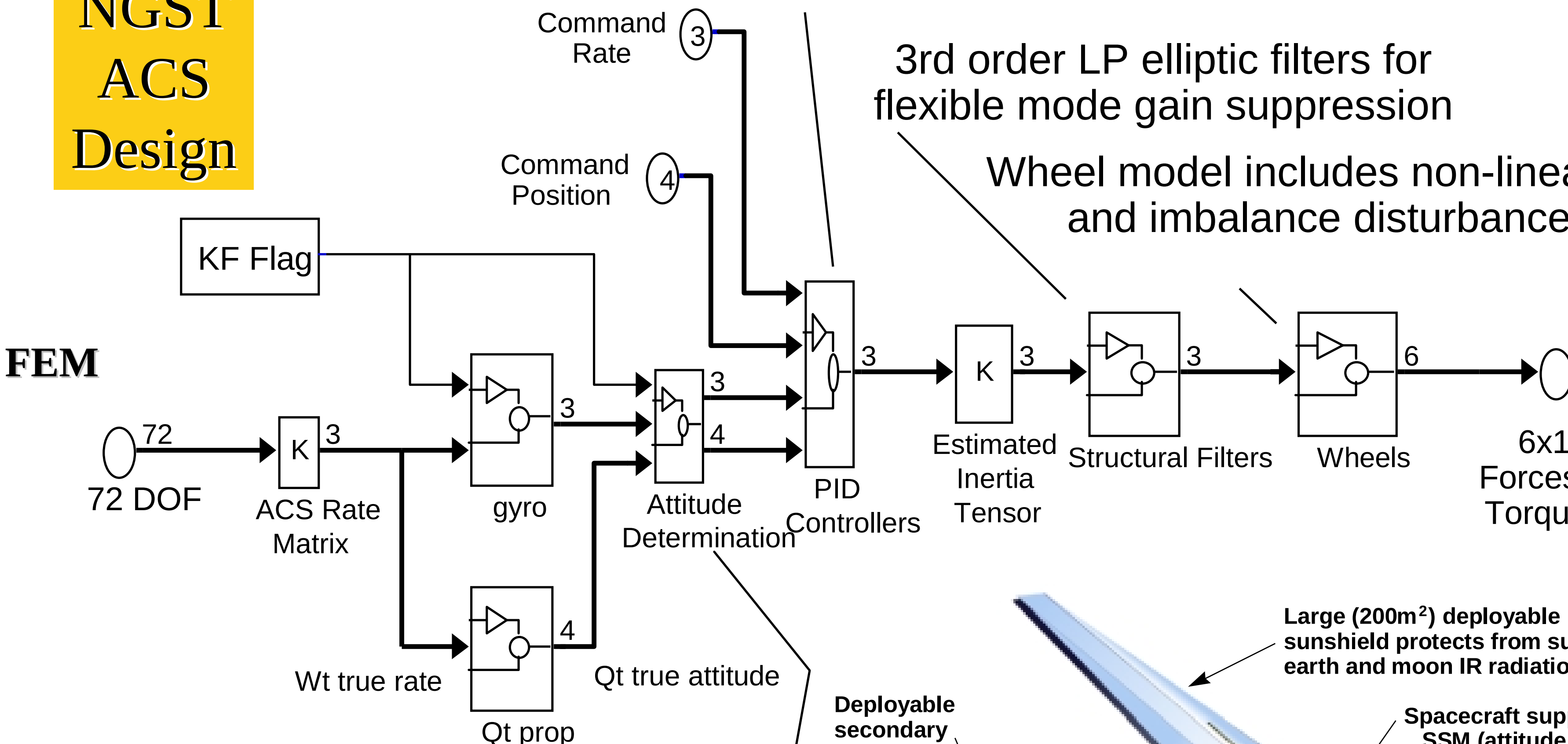
ACS Model of NGST (large, flexible S/C)

NGST ACS Design

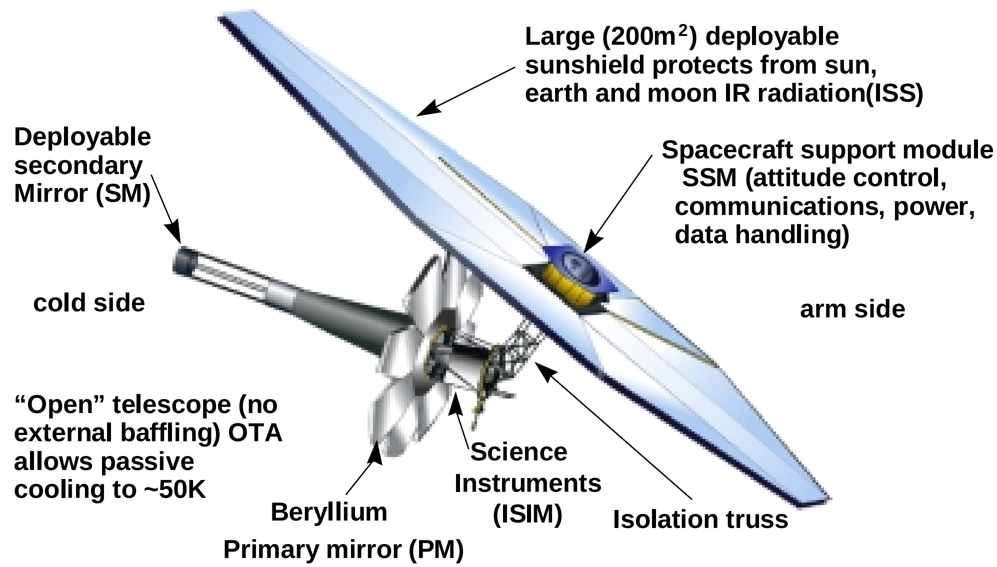
PID bandwidth is 0.025 Hz

3rd order LP elliptic filters for flexible mode gain suppression

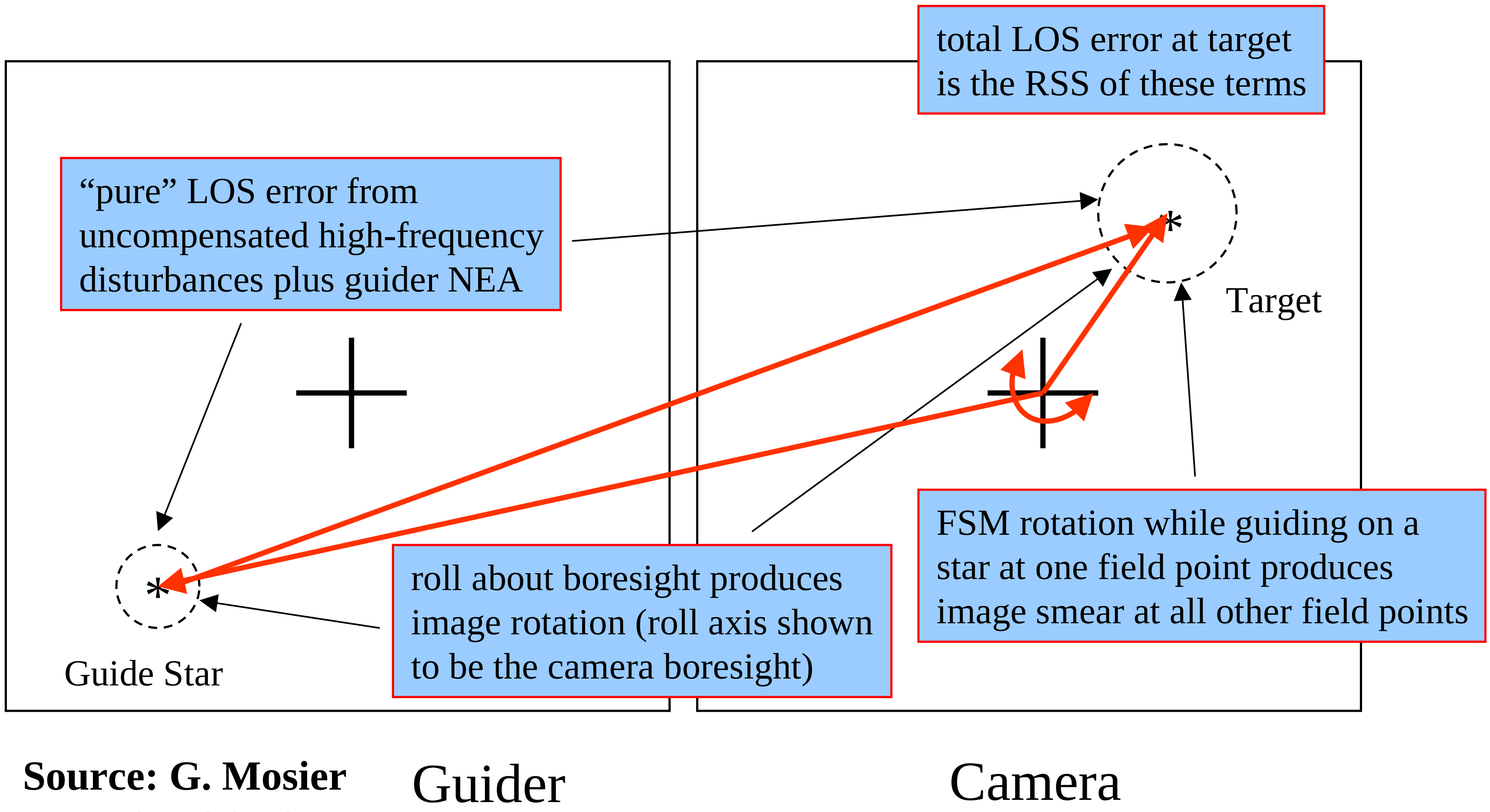
Wheel model includes non-linearities and imbalance disturbances



Kalman Filter blends 10 Hz IRU and 2 Hz ST data to provide optimal attitude estimate; option exists to disable the KF and inject white noise, with amplitude given by steady-state KF covariance into the controller position channel



Attitude Jitter and Image Stability



Source: G. Mosier NASA GSFC

Important to assess impact of attitude jitter ("stability") on image quality. Can compensate with fine pointing system. Use a guider camera as sensor and a 2-axis FSM as actuator.

Rule of thumb:
Pointing Jitter
RMS LOS < FWHM/10

E.g. HST: RMS LOS = 0.007 arc-seconds

References

- James French: AIAA Short Course: “Spacecraft Systems Design and Engineering”, Washington D.C.,1995
- Prof. Walter Hollister: 16.851 “Satellite Engineering” Course Notes, Fall 1997
- James R. Wertz and Wiley J. Larson: “Space Mission Analysis and Design”, Second Edition, Space Technology Series, Space Technology Library, Microcosm Inc, Kluwer Academic Publishers