Propulsion

Relationships:

$$T = \dot{m} u_e \qquad \Delta V = u_e \ln\left(\frac{M_o}{M_f}\right)$$

$$u_e = I_{sp} g_o \qquad g_o = 9.81 \ m/s^2$$

$$P_T = \frac{1}{2}\dot{m} u_e^2 = \frac{1}{2}\dot{m}(I_{sp} g_o)^2 = \frac{1}{2}\frac{T^2}{\dot{m}} = \frac{1}{2} T u_e = \frac{1}{2}\dot{m} I_{sp} g_o$$

(How does P scale w/T?)

Parts

- Thruster (nozzle)
- Tank
- propellant
- plumbing
  - Regulators
  - Tubing
  - Valves
- power

Energy Source

Pressure (mechanical)

chemical

Electrical

Nuclear

System Trades

$$M = M_{struct} + M_{Tank} + M_{prop} + M_{pow} + M_{Thruster} + M_{plumbing}$$

$$M_D = M$$

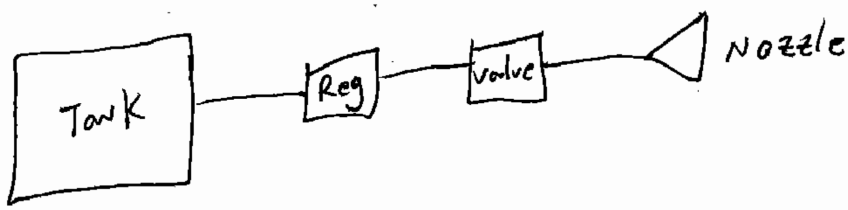
$$M_F = M - M_{prop}$$

$$\Delta V = I_{sp} g_o \ln\left(\frac{M_D}{M_F}\right)$$

- Maximize ΔV
- Reduce Mass
- Constrain Volume

] Goals

Cold Gas - N_2, CO_2, Xe, N_2O



$$M_o = M_{struct} + M_{prop} + M_{Tank} + M_{Bat}$$

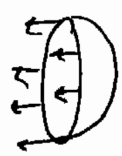
$$M_f = M_s + M_T + M_B$$

$$\frac{M_o}{M_f} = \frac{M_s/M_p + 1 + M_T/M_p + M_B/M_p}{M_s/M_p + M_T/M_p + M_B/M_p}$$

$$\alpha = \frac{M_B}{M_p}$$

$$\beta = \frac{M_s}{M_p} \qquad M_T = 4\pi R^2 t \rho_T \qquad t?$$

$$K = \frac{M_T}{M_p}$$



$$P \pi R^2 = 2\pi R t \sigma \Rightarrow t = \frac{PR}{2\sigma}$$

$$M_T = 4\pi R^3 \left(\frac{P}{2\sigma}\right) \rho_T$$

$$M_p = \frac{4}{3}\pi R^3 \rho_p$$

$$\frac{M_T}{M_p} = \boxed{\frac{3}{2}\,\frac{\rho_T R T}{\sigma} = K}$$

Not a function of pressure.
(physical explanation?)

$$\Delta V = I_{sp} g_0 \ln\left(\frac{1+\alpha+\beta+K}{\alpha+\beta+K}\right)$$

$$h_0 = C_p T_0 = C_p T_t + \frac{1}{2} V_t^2 = C_p T_E + \frac{1}{2} u_e^2$$

($\frac{1}{2}V_t^2 \to 0$; $C_p T_E \to 0$, full expansion)

$$u_e = \sqrt{2 C_p T}$$ (How can power augment?)

$$m_p h_0 + M_b E = \frac{1}{2} M_p u_e^2$$

$$u_e = \sqrt{2 C_p T + 2\alpha E} = I_{sp} g_0$$

$$\Delta V = \sqrt{2}\left(C_p T + \alpha E\right)^{1/2} \ln\left(\frac{1+\alpha+\beta+K}{\alpha+\beta+K}\right)$$

$$C_p = \frac{\gamma}{\gamma - 1}\frac{R}{M} \qquad \gamma \sim 1 + \frac{2}{f} \qquad f = \text{d.o.f.s}$$

(Low Temp)

N_2 •—• 3 Trans + 2 Rot = 5 $\quad \gamma = 1 + \frac{2}{5} = 1.4$

↓

monatomic?

Make K more generic

$$K = \frac{3}{2}\frac{\rho_F R T}{\sigma} = \frac{3}{2}\frac{\rho_T R T}{M\sigma} \qquad K_m = \frac{3}{2}\frac{\rho_T R T}{\sigma}$$

$$\boxed{K = K_m / M}$$

propellant →

$$\Delta V = \sqrt{2}\left(\frac{\gamma}{\gamma-1}\frac{R}{M}T + \alpha E\right)^{1/2} \ln\left(\frac{1+\alpha+\beta+K_m/M}{\alpha+\beta+K_m/M}\right)$$

$\alpha = 0$ Lower M => raise I_{sp}

lower M_o/M_f (for same β)

what does this mean?

Optimum M? (given β)

CO_2

- Stored as liquid at high enough pressures (800-900 psi @ room Temp)
- higher density (lower volume for given mass)
- Requires energy to vaporize ~ 100 J/g
- Insulated tank gets cold, Isp drops (Use batteries)

how to include vaporization energy?

$$M_p h_o - M_p \Delta H_v + M_b E = \frac{1}{2} M_p U_e^2$$

$$C_p T - \Delta H_v + \alpha E = \frac{1}{2} u_e^2$$

$$\boxed{\alpha_o E_o = \Delta H_v}$$

$$C_p T + (\alpha - \alpha_o) E = \frac{1}{2} u_e^2$$

$$\boxed{I_{sp} = \frac{\sqrt{2}}{g_o}\left[C_p T + (\alpha - \alpha_o) E\right]^{1/2}}$$

$\alpha < \alpha_o$

for [illegible]

Isp changes with Time.

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Revisit K, since $P = \rho RT$ doesn't apply.

$$K_{CO_2} = \frac{M_T}{M_P} \qquad M_T = 4\pi R^3 \left(\frac{P}{2\sigma}\right)\rho_T$$

$$M_P = \frac{4}{3}\pi R^3 \rho_P$$

$$K_{CO_2} = \frac{3}{2}\left(\frac{P}{\sigma}\right)\frac{\rho_T}{\rho_P} = \frac{3}{2}\left(\frac{P}{\rho}\right)_{CO_2}\frac{\rho_T}{\sigma} \qquad \left(\frac{P}{\rho}\right)_{CO_2}(300K) = 10{,}000\ J/kg$$

$= 10\ kJ/kg$

$$\left(\frac{\sigma}{\rho_T}\right)_{7075\ Al} = 61{,}000\ J/kg\ [FS=2]$$

$$K_{CO_2}(300K) \sim 0.25$$

$$K_{N_2} = \frac{3}{2}\frac{\rho_T RT}{\sigma M_{N_2}} = 2.1$$

$$C_p = \frac{8314(1.3)}{44\ \ 0.3} = 818$$

$$\frac{K_m}{K_{CO_2}} = 244 = \delta$$

$$K = \frac{K_{CO_2}}{M}\delta$$

$$\Delta V_{CO_2} = \sqrt{2}\left[C_pT + (\alpha-\alpha_0)E\right]^{1/2} \ln\left(\frac{1+\alpha+\beta+K_{CO_2}}{\alpha+\beta+K_{CO_2}}\right)$$

$$\alpha_0 = \frac{\Delta H_v}{E} = \frac{101\ kJ/kg\ (300K)}{245\ kJ/kg} = 0.413$$

$\alpha = \alpha_0$
$$\Delta V_{CO_2} = \sqrt{2}\left[C_pT\right]^{1/2} \ln\left(\frac{1.663+\beta}{0.663+\beta}\right)$$

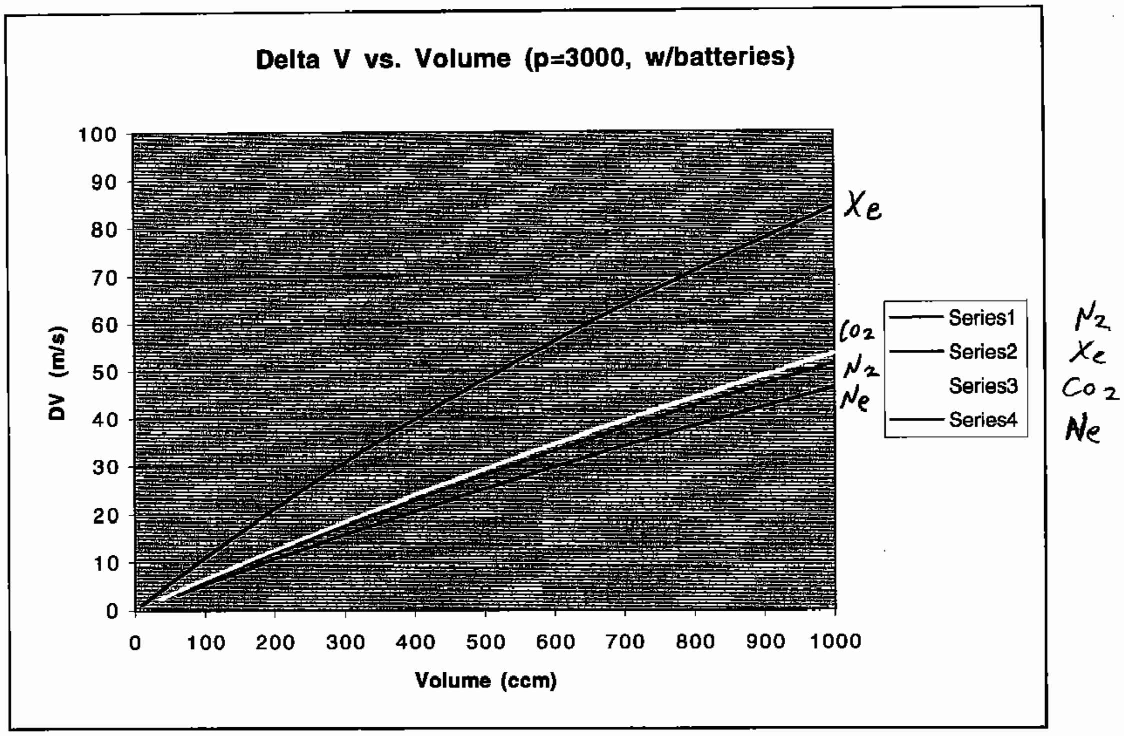
$$(\Delta V_{CO_2})_{max}\ (\beta=0) = 644\ m/s \qquad \text{(maximum value)}$$

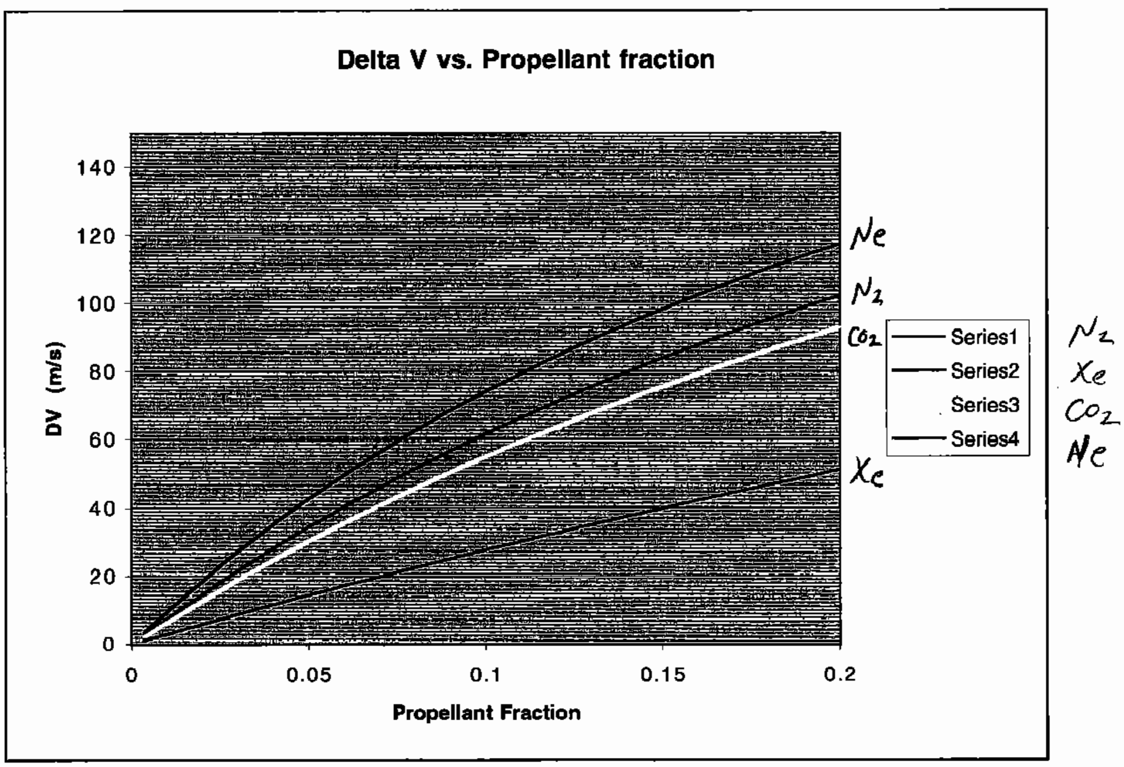
(6)

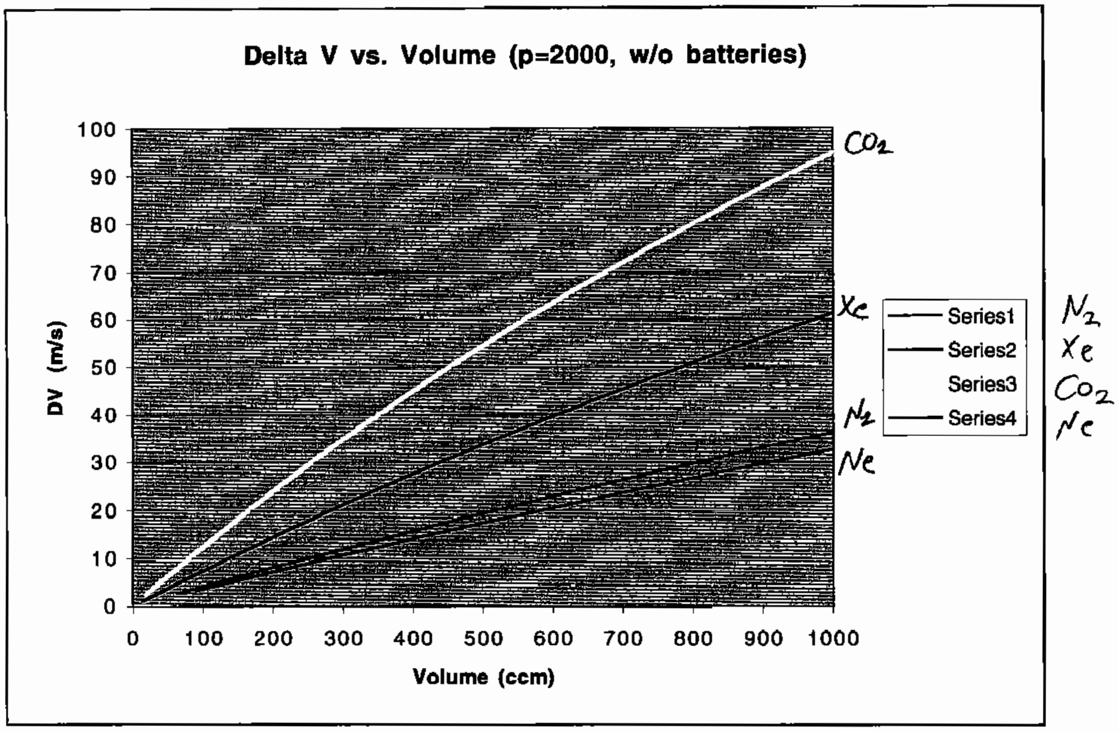
For N_2 $k = 2.1$ $C_p = \frac{8314}{29}\frac{(1.4)}{0.4} = 1000\ J/kg \cdot K$

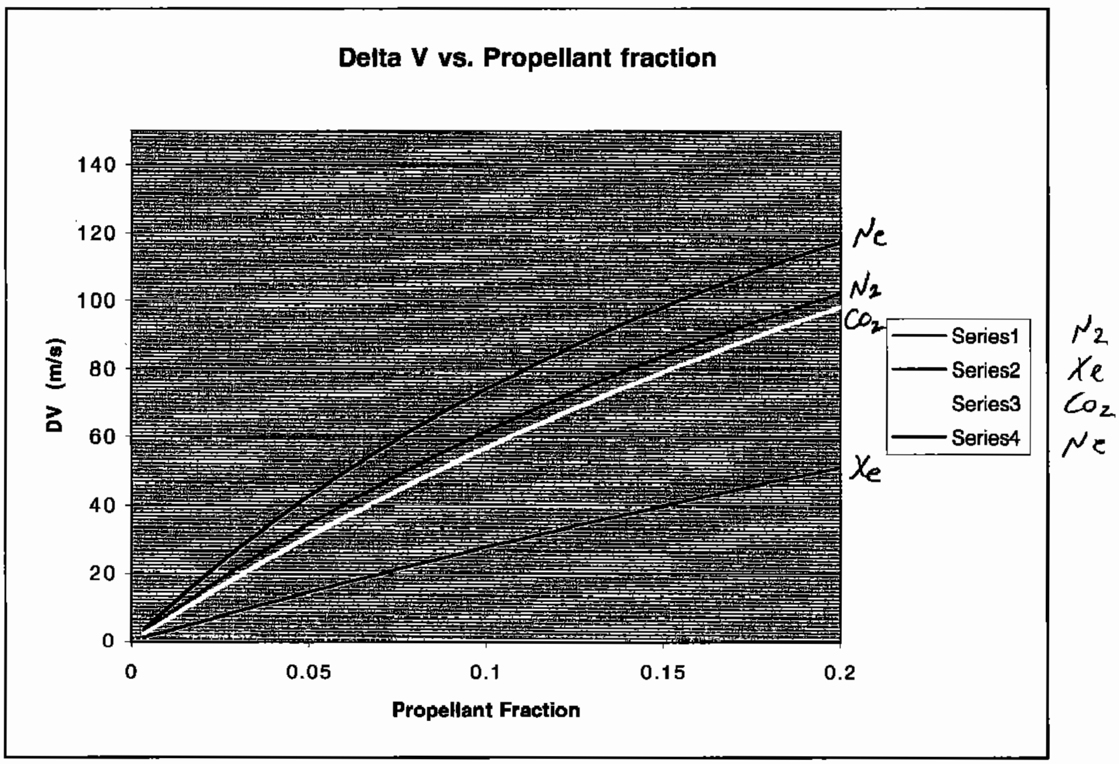
$$\Delta V_{max}\ (\alpha = 0, \beta = 0) = \sqrt{2}\left[(1000)(300)\right]^{1/2} \ln\left(\frac{1 + 2.1}{2.1}\right)$$

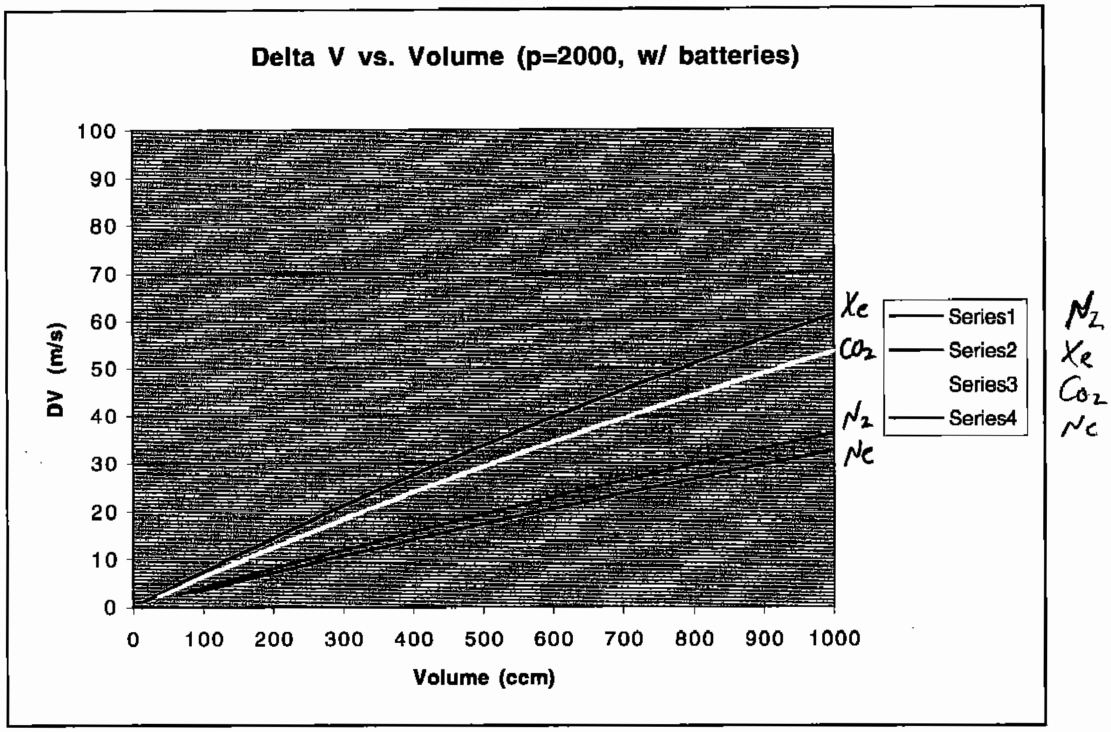
$$= 300\ m/s$$

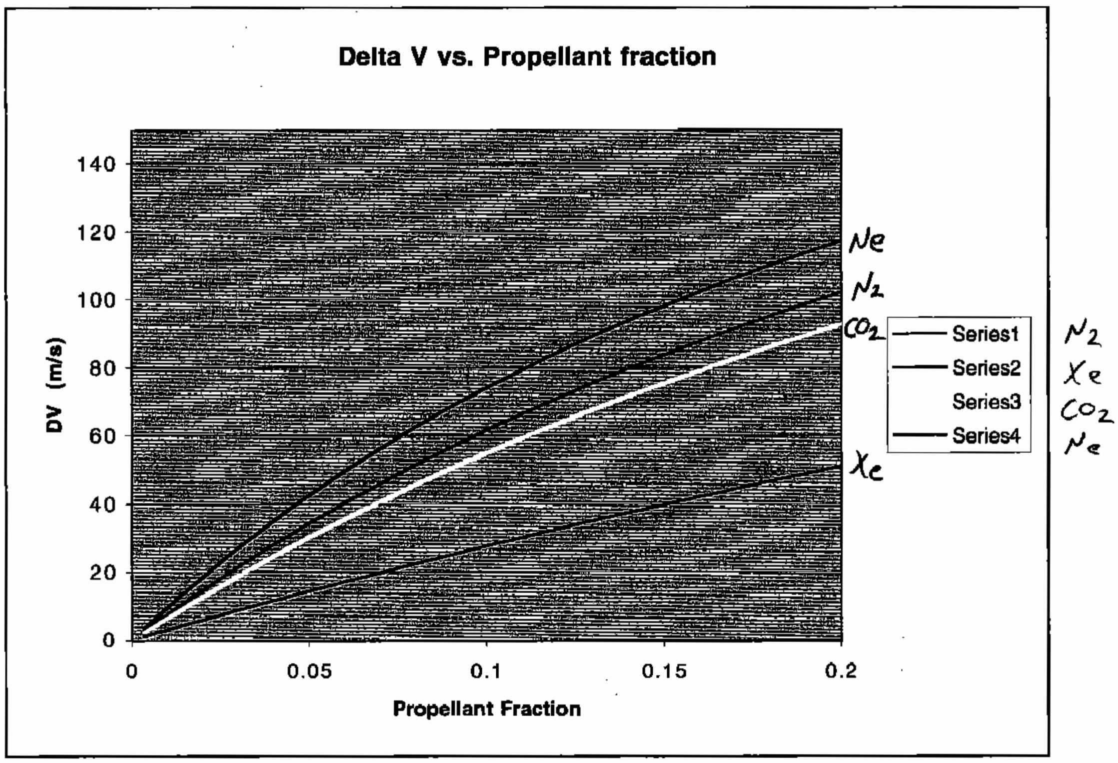

Delta V vs. Volume (p=3000, w/batteries)
DV (m/s)
Volume (ccm)
Series1
Series2
Series3
Series4
Xe
Co2
N2
Ne
N2
Xe
Co2
Ne

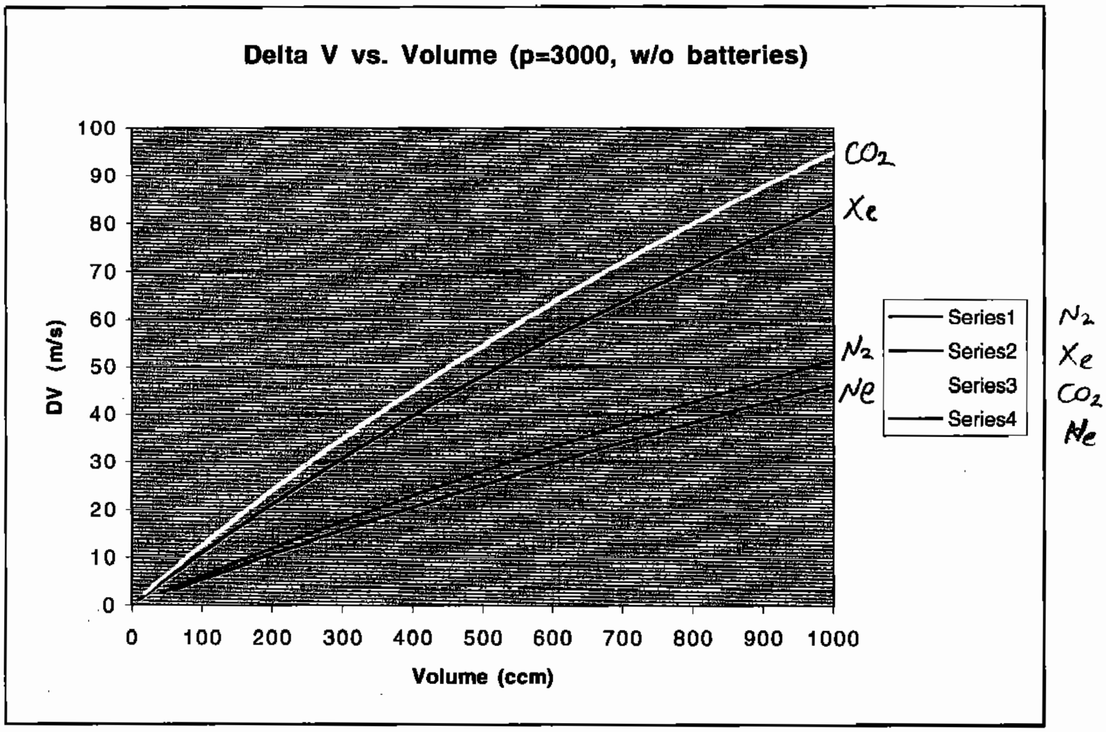

Delta V vs. Propellant fraction
DV (m/s)
Propellant Fraction
Series1
Series2
Series3
Series4
Ne
N2
Co2
Xe
N2
Xe
Co2
Ne

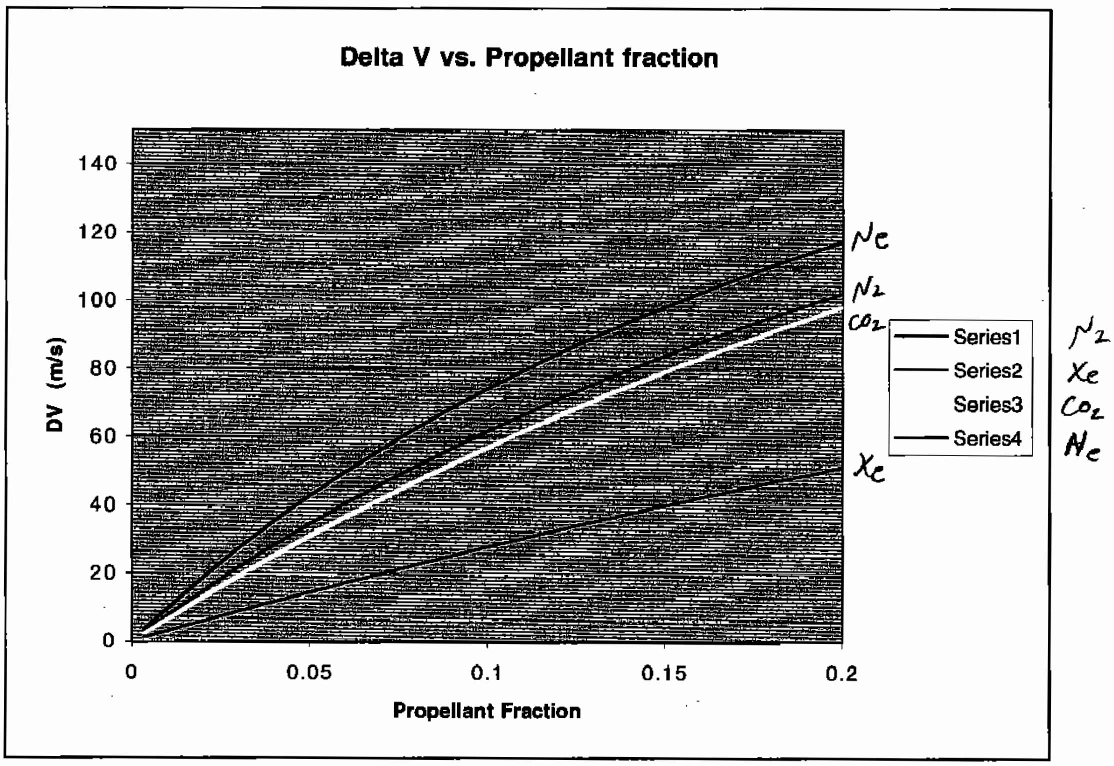

Delta V vs. Volume (p=2000, w/o batteries)
DV (m/s)
100
90
80
70
60
50
40
30
20
10
0
0
100
200
300
400
500
600
700
800
900
1000
Volume (ccm)
Series1
Series2
Series3
Series4
CO2
Xe
N2
Ne
N2
Xe
CO2
Ne


Delta V vs. Propellant fraction
DV (m/s)
140
120
100
80
60
40
20
0
0
0.05
0.1
0.15
0.2
Propellant Fraction
Series1
Series2
Series3
Series4
Ne
N2
CO2
Xe
N2
Xe
CO2
Ne

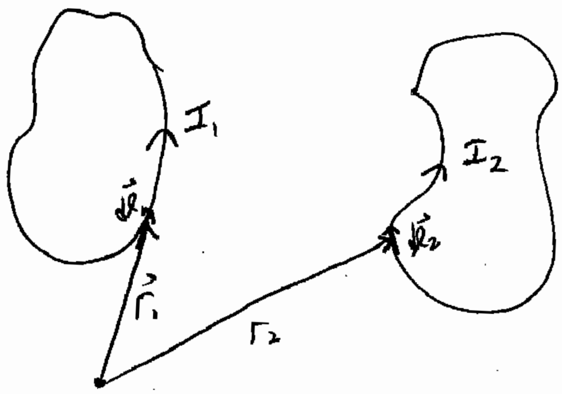

Delta V vs. Volume (p=2000, w/ batteries)
DV (m/s)
100
90
80
70
60
50
40
30
20
10
0
0
100
200
300
400
500
600
700
800
900
1000
Volume (ccm)
Xe
CO2
N2
Ne
Series1
Series2
Series3
Series4
N2
Xe
CO2
Ne


Delta V vs. Propellant fraction
DV (m/s)
140
120
100
80
60
40
20
0
0
0.05
0.1
0.15
0.2
Propellant Fraction
Ne
N2
CO2
Xe
Series1
Series2
Series3
Series4
N2
Xe
CO2
Ne


Delta V vs. Volume (p=3000, w/o batteries)
DV (m/s)
100
90
80
70
60
50
40
30
20
10
0
0
100
200
300
400
500
600
700
800
900
1000
Volume (ccm)
Series1
Series2
Series3
Series4
CO_2
Xe
N_2
Ne
N_2
Xe
CO_2
Ne


Delta V vs. Propellant fraction
DV (m/s)
140
120
100
80
60
40
20
0
0
0.05
0.1
0.15
0.2
Propellant Fraction
Series1
Series2
Series3
Series4
Ne
N_2
CO_2
Xe
N_2
Xe
CO_2
Ne

EMFF



$$F_2 = \frac{\mu_0}{4\pi} I_1 I_2 \oint_1 \oint_2 \frac{d\vec{l}_2 \times [d\vec{l}_1 \times (\vec{r}_2 - \vec{r}_1)]}{|\vec{r}_2 - \vec{r}_1|^3}$$

Generally hard to solve analytically

How, at "large" distances from the wire, the field can be expanded into powers of $1/r$, and retaining only the highest order term yields

$$B(r) = \frac{\mu_0}{4\pi}\left[-\frac{\vec{m}}{r^3} + \frac{3(\vec{m}\cdot\vec{r})\vec{r}}{r^5}\right] \quad \text{Dipole Field}$$

where $|\vec{m}| = IA$ wire around circuit time Area enclosed.

=> Dipole moment

Can be written as $B(r) = -\mu_0 \nabla \left(\frac{\vec{m}\cdot\vec{r}}{4\pi r^3}\right)$

so the $\phi = \frac{\vec{m}\cdot\vec{r}}{4\pi r^3}$ is a <u>scalar</u> potential

A dipole place in an external field has a potential energy of

$$U = -\vec{m}\cdot\vec{B}(r)$$

~~[illegible]~~

(2)

The Force on the dipole:

$$\vec{F} = -\nabla_r U = \vec{M}(\theta) \cdot \nabla \vec{B}(\vec{r})$$

↑ 2nd order Tensor

And the Torque:

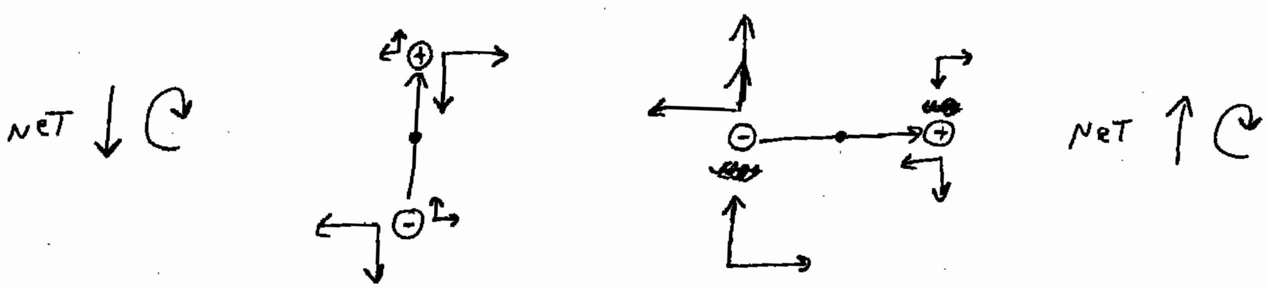
$$\vec{T} = -\nabla_\theta U = \nabla \vec{M}(\theta) \cdot \nabla \vec{B}(\vec{r})$$

↑ 2nd order Tensor

$$\vec{F}_A = \frac{3\mu_0}{4\pi}\left(\frac{m_A m_B}{r^4}\right)\left\{\left[5(\hat{m}_A\cdot\hat{r})(\hat{m}_B\cdot\hat{r}) - (\hat{m}_A\cdot\hat{m}_B)\right]\hat{r} - (\hat{m}_A\cdot\hat{r})\hat{m}_B - (\hat{m}_B\cdot\hat{r})\hat{m}_A\right\}$$

$$\vec{T}_A = \frac{\mu_0}{4\pi}\left(\frac{m_A m_B}{r^3}\right)\left\{3(\hat{m}_B\cdot\hat{r})(\hat{m}_A\times\hat{r}) - (\hat{m}_A\times\hat{m}_B)\right\}$$

Electrostatic Dipole Analogy (For field equivalent)



Add R/W to control Torque and all relative DoFs are controllable.