Minimum Energy Trajectories for Techsat 21 Earth Orbiting Clusters

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Objective and Outline

Objective : To determine the optimal trajectories to re-orient a cluster of spacecraft

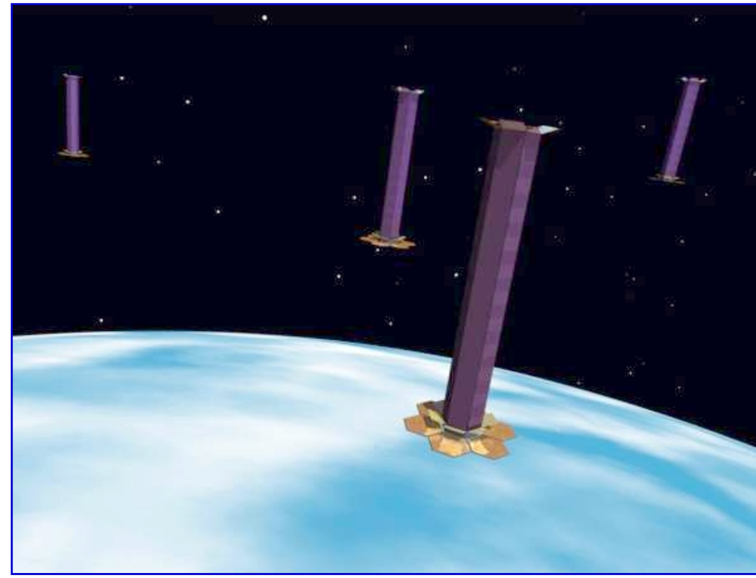
Motivation : To maximize the full potential of a cluster of spacecraft with minimal resources

Presentation Outline

- Techsat 21 Overview
- Optimal Control Formulation
  - Equations of Motions (Dynamics)
  - Propulsion System (Cost)
  - LQ Formulation
  - Terminal Constraints
- Results
  - Tolerance setting
  - Cluster Initialization
  - Cluster Re-sizing (Geolocation)
- Future Work
- Conclusions

Techsat 21

- **To explore the technologies required to enable a Distributed Satellite System**
- **Sparse Aperture Space Based Radar**
- **Full operational system of 35 clusters of 8 satellites to provide global coverage**
- **2003 Flight experiment with 3 spacecraft**
- **Spacecraft will be equipped with Hall Thrusters**
  - **2 large thrusters for orbit raising and de-orbit**
  - **10 micro-thrusters for full three-axis control**



* *Figure courtesy of AFOSR Techsat21 Research Review (29 Feb - 1 Mar 2000)*

Techsat 21 Flight Experiment

Number of Spacecraft	: 3
Spacecraft Mass	: 129.4 kg
Cluster Size	: 500 m
Orbital Altitude	: 600 km
Orbital Period	: 84 mins
Geo-location size	: 5000 m

Equations of Motions

- **First order perturbation about natural circular Keplerian orbit**
- **Modified Hill's Equations:**

$$a_x = \ddot{x} - \left(5c^2 - 2\right)n^2 x - 2\left(nc\right)\dot{y}$$

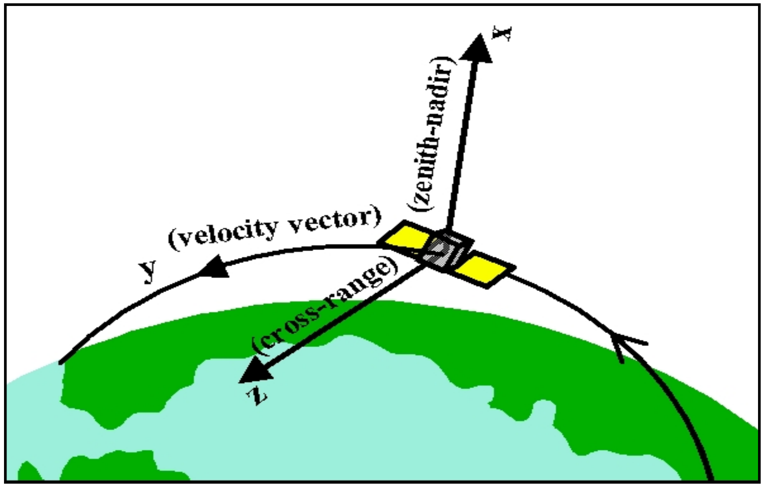
$$a_y = \ddot{y} + 2\left(nc\right)\dot{x}$$

$$a_z = \ddot{z} + k^2 z$$

where

$$s = \frac{3J_2 R_e^2}{8r_{ref}^2}\left[1 + 3\cos\left(2i_{ref}\right)\right] \qquad c = \sqrt{1+s}$$

$$k = n\sqrt{1+s} + \frac{3nJ_2 R_e^2}{2r_{ref}^2}\left[\cos\left(i_{ref}\right)\right]^2$$

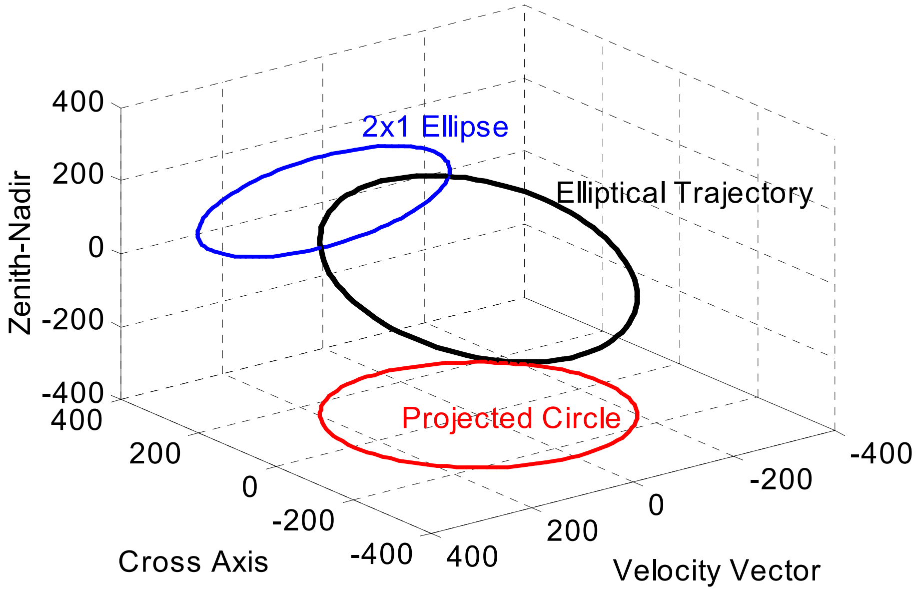


- **Possible trajectory for Techsat 21:**

$$x = A_o \cos(nt\sqrt{1-s})$$

$$y = -\frac{2\sqrt{1+s}}{\sqrt{1-s}} A_o \sin(nt\sqrt{1-s})$$

$$z = -\frac{2\sqrt{1+s}}{\sqrt{1-s}} A_o \cos(kt)$$



Propulsion Subsystem (Hall Thrusters)

- High specific impulse
  - low propellant expenditure
- Electrical power required:

$$P_e = \frac{m^2 u^2}{2 \dot{m} \eta}$$

where

m - mass of spacecraft (129.4 kg)

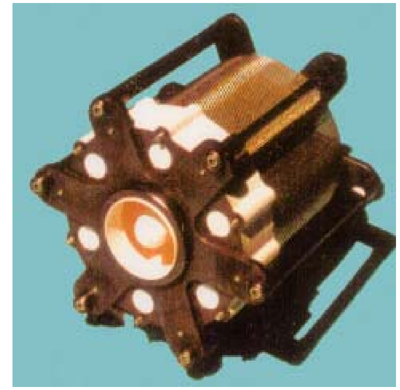
u - spacecraft acceleration (m/s)

$\dot{m}$ - mass flow rate of propellant (kg/s)

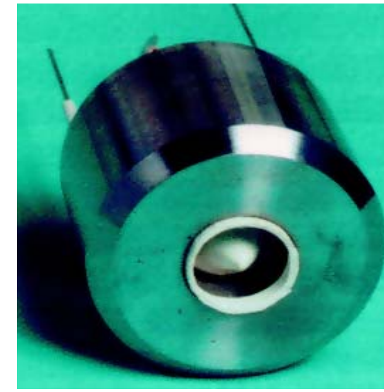
η - thruster efficiency (%)

- Objective is to minimize electrical energy required:

$$J = \int_{t_o}^{t_f} P_e dt$$



200 W Hall Thruster *



100 - 200 W Hall Thruster *

BHT-200-X2B Hall Thruster

Specific Impulse	: 1530 s
Thrust	: 10.5 mN
Mass flow rate	: 0.74 mg/s
Typical Efficiency	: 42%
Power Input	: 200 W

* *Figures courtesy of AFOSR Techsat21 Research Review (29 Feb - 1 Mar 2000)*

Optimal Control Theory

- **Linear Dynamics**

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

- **Quadratic Cost**

$$J = \int_{t_o}^{t_f} P_e dt \;\Rightarrow\; J(\mathbf{u}) = \frac{1}{2}\int_{t_o}^{t_f} \mathbf{u}^T\mathbf{R}\mathbf{u}\,dt$$

- **Augmented Cost (Method of Lagrange)**

$$J_a(\mathbf{u}) = \int_{t_o}^{t_f} \{\tfrac{1}{2}\mathbf{u}^T\mathbf{R}\mathbf{u} + \mathbf{p}^T[\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} - \dot{\mathbf{x}}]\}dt$$

- **First order variation**

$$\delta J_a(\mathbf{u}) = -\mathbf{p}^T(t_f)\delta\mathbf{x}_f + \int_{t_o}^{t_f} \{[\mathbf{p}^{*T}\mathbf{A} + \dot{\mathbf{p}}^{*T}]\delta\mathbf{x} + [\mathbf{u}^{*T}\mathbf{R} + \mathbf{p}^{*T}\mathbf{B}]\delta\mathbf{u} + [\mathbf{A}\mathbf{x}^* + \mathbf{B}\mathbf{u}^* - \dot{\mathbf{x}}^*]\delta\mathbf{p}\}dt = 0$$

Boundary Conditions

1. $\mathbf{x}(t_f) = \mathbf{x}_f$ specified terminal state	$\mathbf{x}^*(t_o) = \mathbf{x}_o$ $\mathbf{x}^*(t_f) = \mathbf{x}_f$
2. $\mathbf{x}(t_f)$ free	$\mathbf{x}^*(t_o) = \mathbf{x}_o$ $\mathbf{p}^*(t_f) = 0$
3. $\mathbf{x}(t_f)$ on the surface $\mathbf{m}(\mathbf{x}(t)) = 0$	$\mathbf{x}^*(t_o) = \mathbf{x}_o$ $-\mathbf{p}^*(t_f) = \sum_{i=1}^{k} \mathrm{d}_i[\frac{\partial m_i}{\partial \mathbf{x}}(\mathbf{x}^*(t_f))]$ $\mathbf{m}(\mathbf{x}^*(t_f)) = 0$

Linear Quadratic Controller

(t_o to t_f)

$$\dot{\mathbf{x}}^* = \mathbf{A}\mathbf{x}^* + \mathbf{B}\mathbf{u}^*$$

$$\dot{\mathbf{p}}^* = -\mathbf{A}^T\mathbf{p}^*$$

$$\mathbf{u}^* = -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{p}^*$$

Terminal Conditions (Multi-Spacecraft)

For each spacecraft (R_o projection on y-z plane):

- **Position Conditions**

$$m_1 = \left[\frac{y}{R_o}\right]^2 + \left[\frac{x\sin\gamma + z\cos\gamma}{(5/2)R_o\sin\gamma}\right]^2 - 1$$

$$m_2 = x\cos\gamma - z\sin\gamma$$

- **Velocity Conditions**

$$m_3 = \left[\frac{\dot{y}}{nR_o}\right]^2 + \left[\frac{\dot{x}\sin\gamma + \dot{z}\cos\gamma}{(5/2)nR_o\sin\gamma}\right]^2 - 1$$

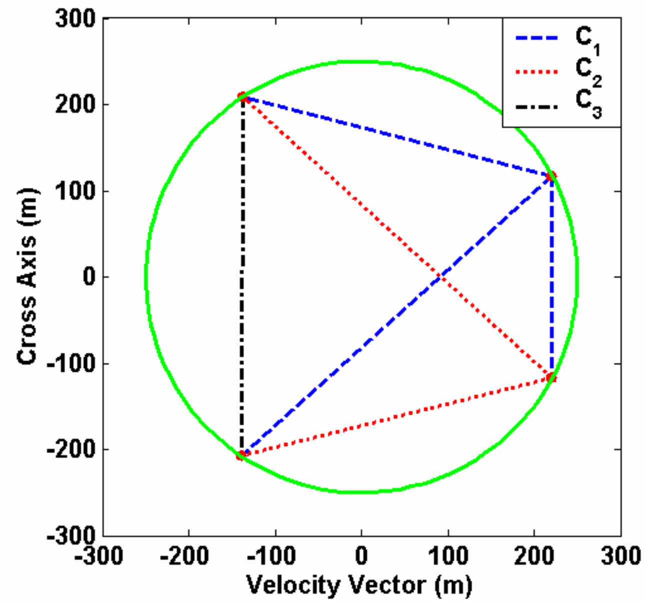
$$m_4 = \dot{x}\cos\gamma - \dot{z}\sin\gamma$$

- **Tying Condition**

$$m_5 = \dot{y}(x\sin\gamma + z\cos\gamma) - y(\dot{x}\sin\gamma + \dot{z}\cos\gamma) + \frac{5}{2}nR_o^2\sin\gamma$$

Phasing Condition (Cluster):

$$m_{5N+i} = \sum_{j=i}^{N}\left|\begin{bmatrix} y \\ z \end{bmatrix}_i - \begin{bmatrix} y \\ z \end{bmatrix}_j\right| - C_i$$



where

$$C_i = \sqrt{2}R_o\sum_{j=i}^{N}\sqrt{1-\cos\theta_{i,j}}$$ for $i = 1, 2, \ldots, N-1$

4 spacecraft example:

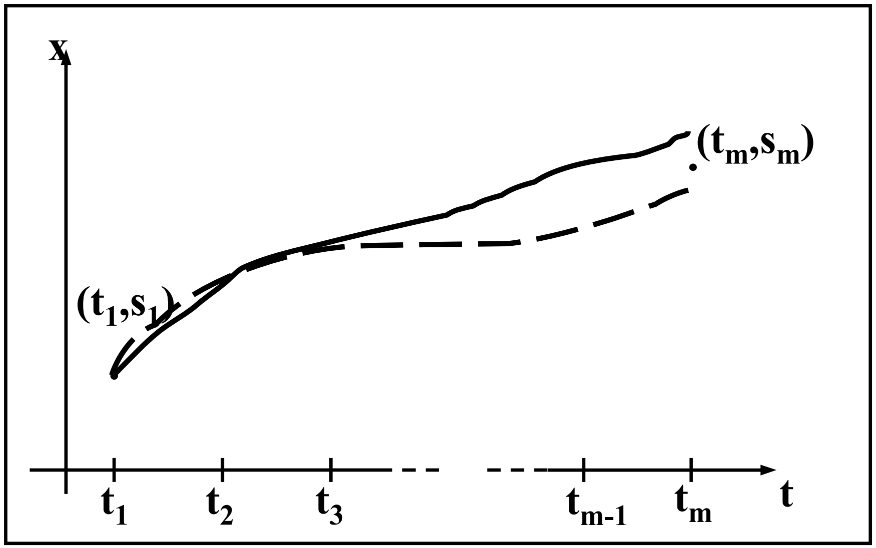
$C_1 = 4.35$ $C_2 = 3.42$ $C_3 = 1.67$

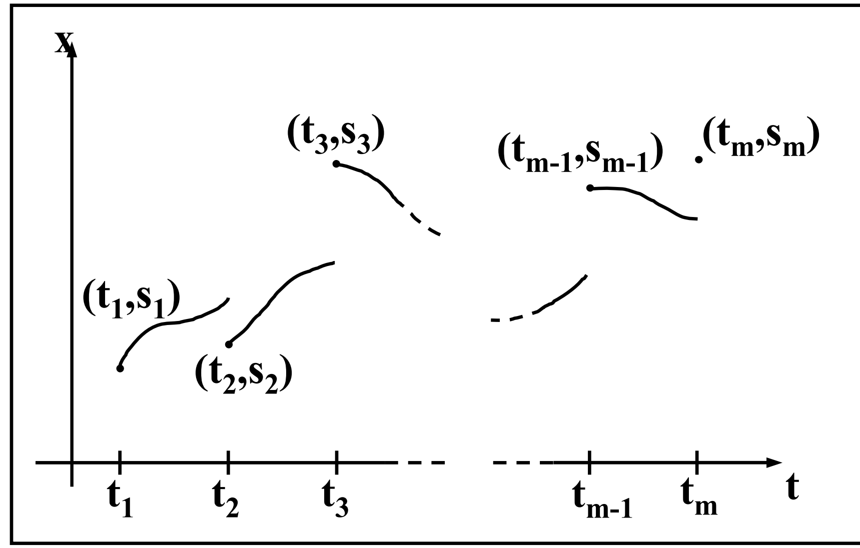
_N-th_ Condition (Total of 6_N_ conditions)

$$-p^*(t_f) = \sum_{i=1}^{6N-1} d_i\left[\frac{\partial m_i}{\partial x}(x^*(t_f))\right]$$

Multiple Shooting Method

Solving two point boundary value problems





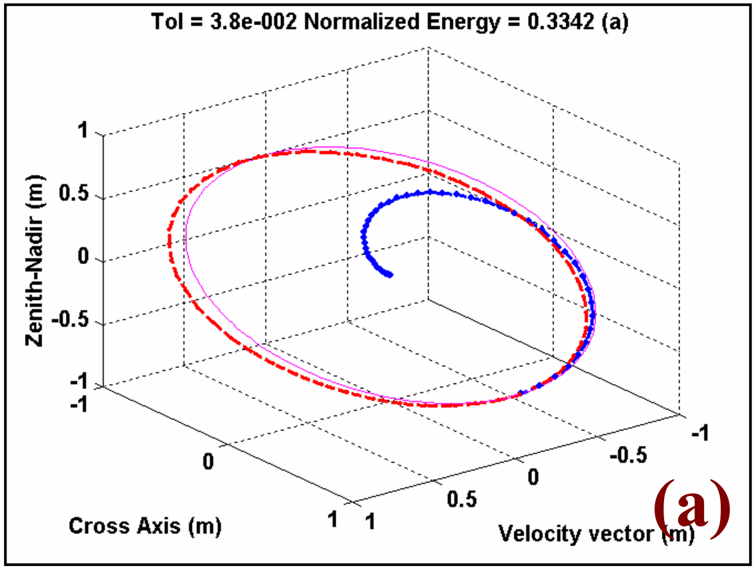
Simple shooting method

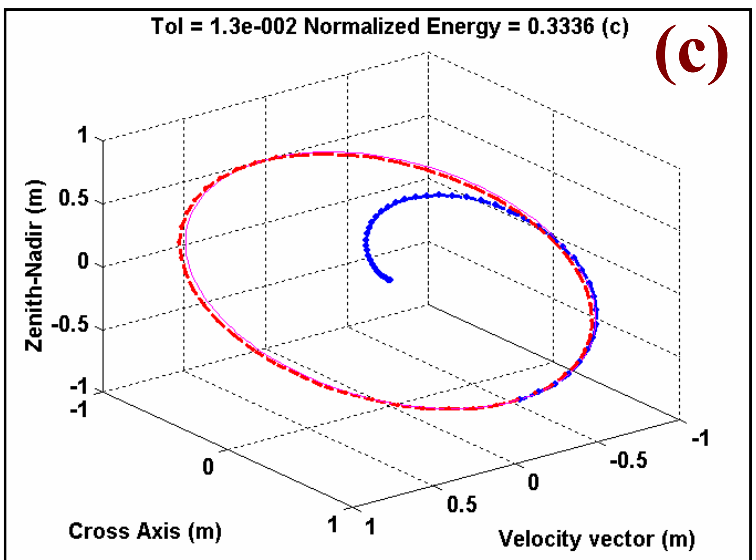
- Guess the missing states at t_o and compare the integrated states at t_f with terminal constraints
- Numerically unstable - errors are amplified due to integration

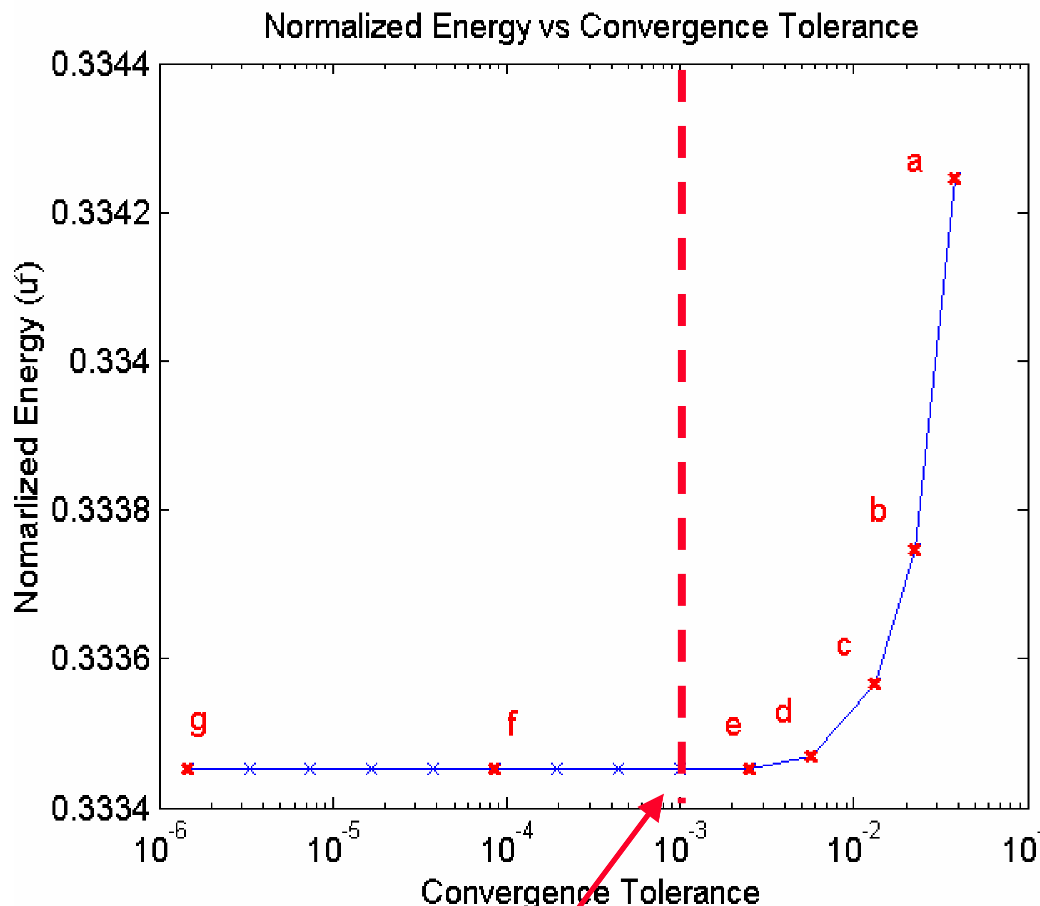
Multiple shooting method

- Guess states at t_k and compare the integrated states at t_{k+1} with states at t_{k+1}
- Numerically more stable
- Computationally expensive

Tolerance Setting

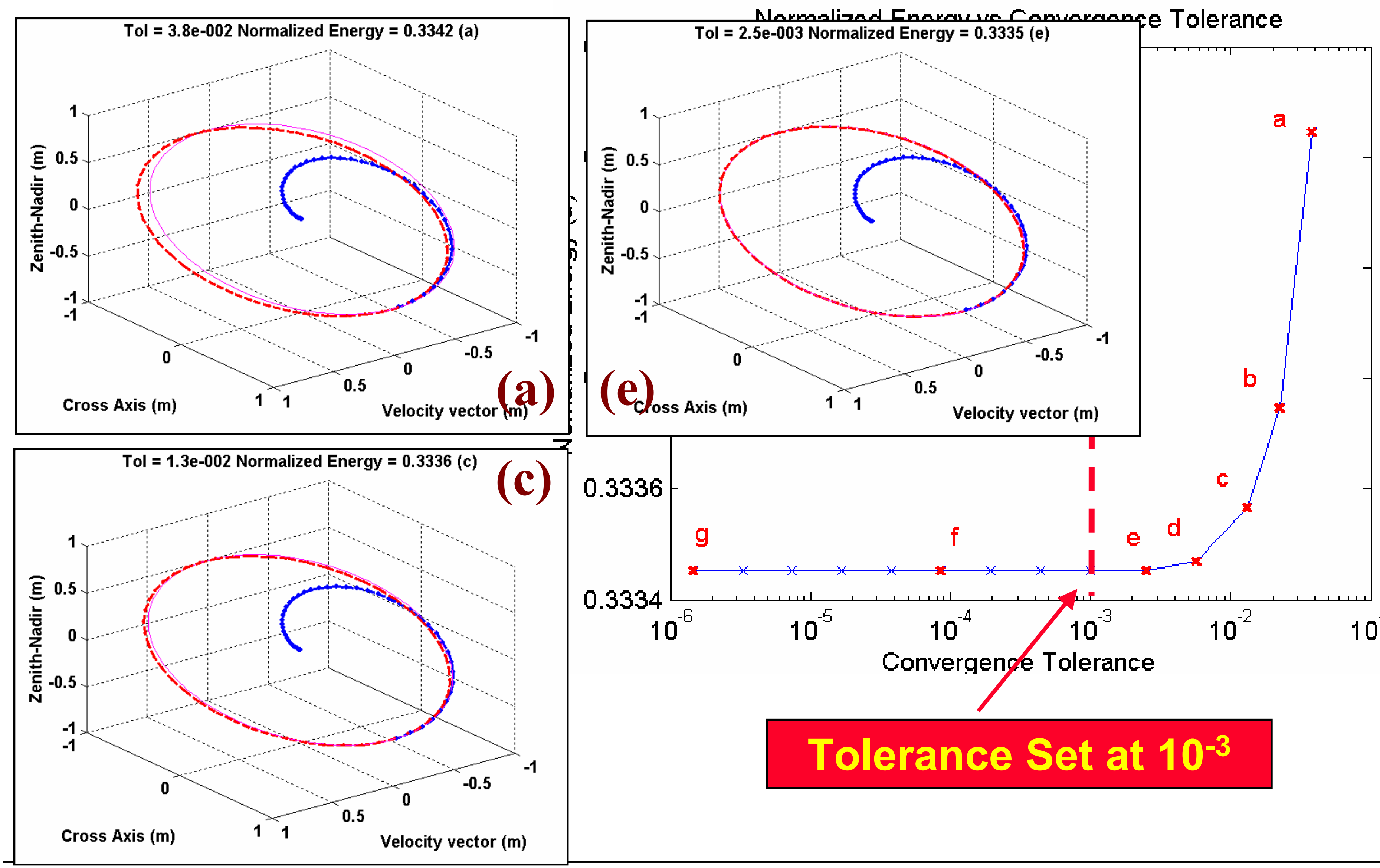






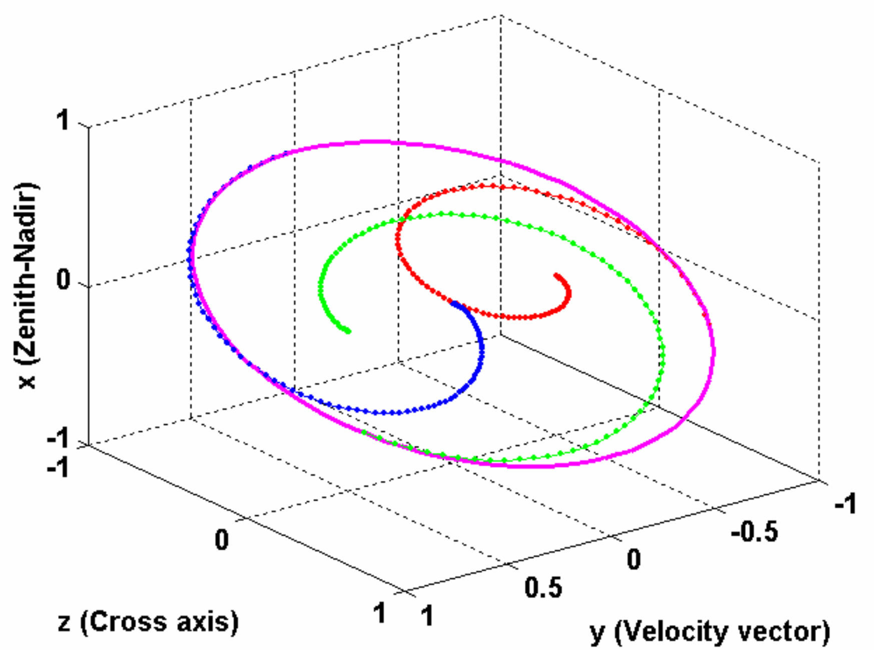
Tolerance Set at 10^{-3}

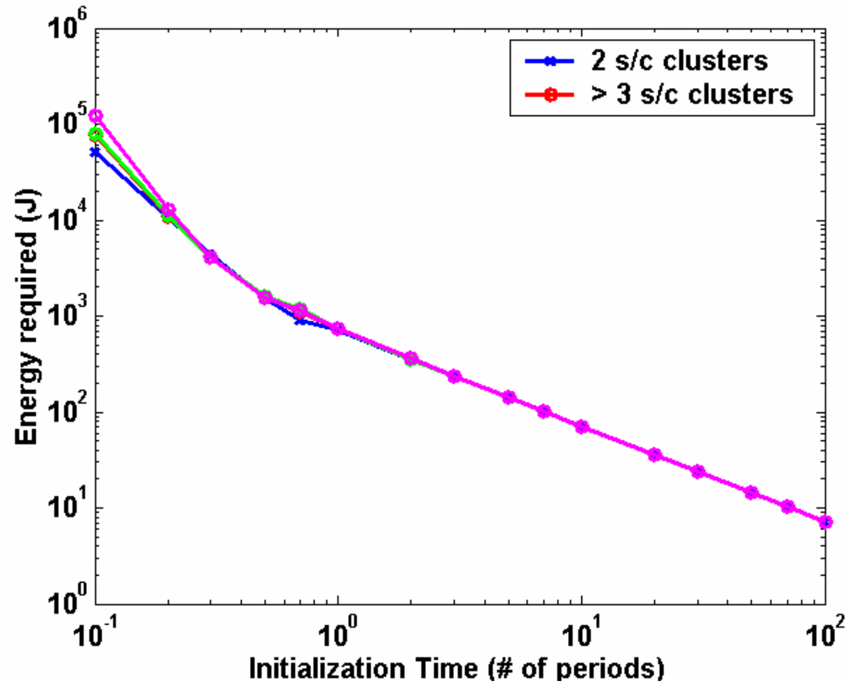
Tolerance Setting



Cluster Initialization (1)

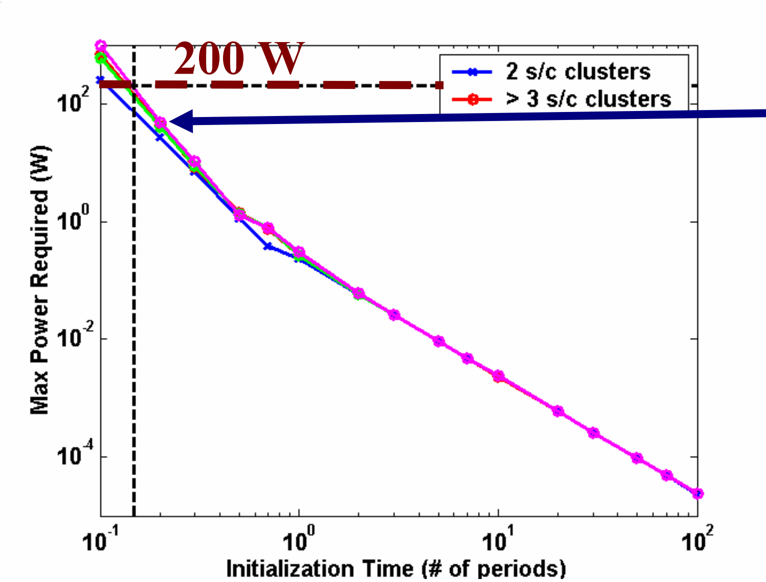
- **Cluster initialization from Hill's origin to R_o = 250 m**

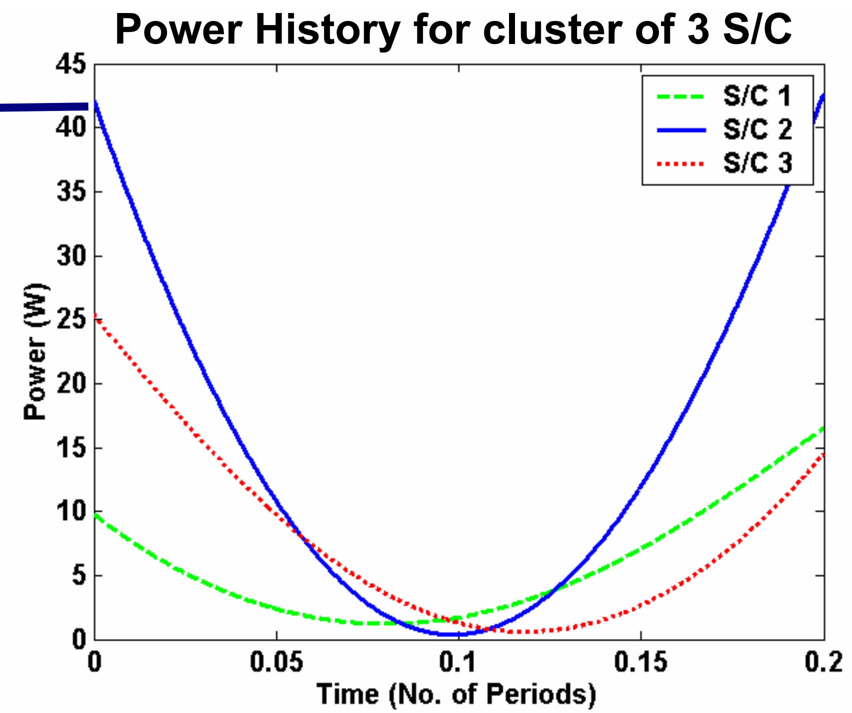


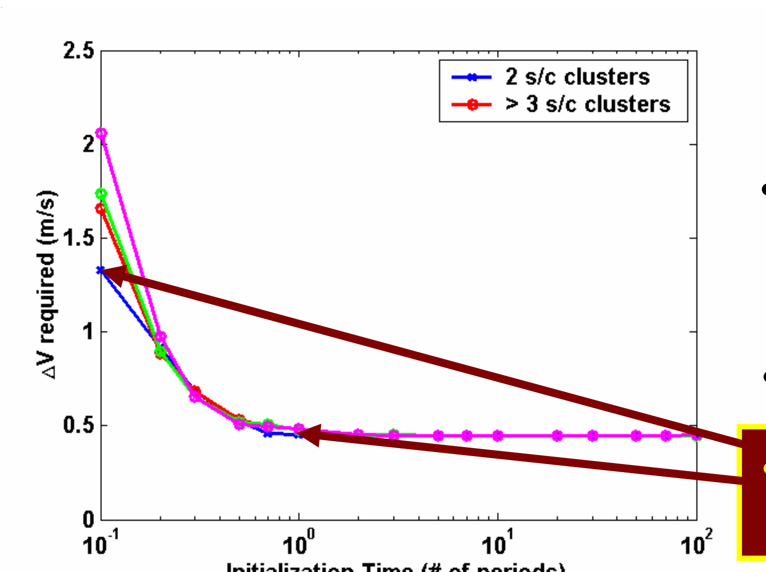


- **In general, average energy required are similar for different *N* spacecraft clusters**
- **Slight differences in energy requirements are due to the more stringent constrains placed on phasing the array (eg. $E_{2sc} < E_{3sc}$)**
- **Average energy required decay rapidly as a function of initialization time**

Cluster Initialization (2)



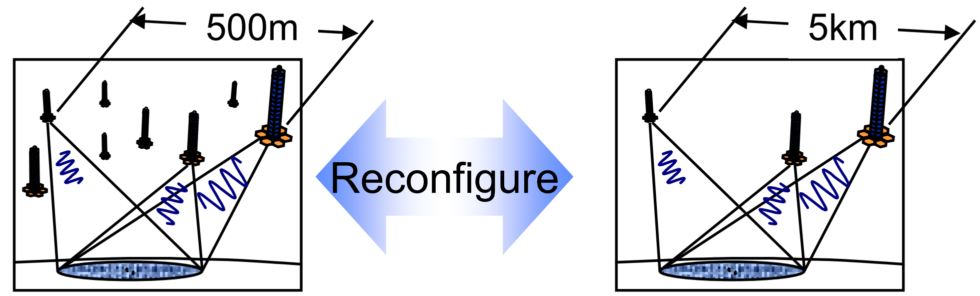




- Peak power required is below Techsat 21 maximum (200 W) for initialization periods greater than 0.2 period
- ΔV required asymptotes to ~ 0.45 m/s
- **Recommend initialization time of 1 period due to significant ΔV savings (67%)**

Cluster Re-sizing (1)

- **Objective of Techsat 21 Geo-location mission is to provide 10-50 m geo-location accuracy**
- **Geo-location accuracy is inversely proportional to size of cluster**
- **Re-size cluster to an elliptical trajectory of 2.5 km to achieve approximately 10 m ground resolution**
- **Example application is to quickly locate a lost pilot (Time critical mission)**



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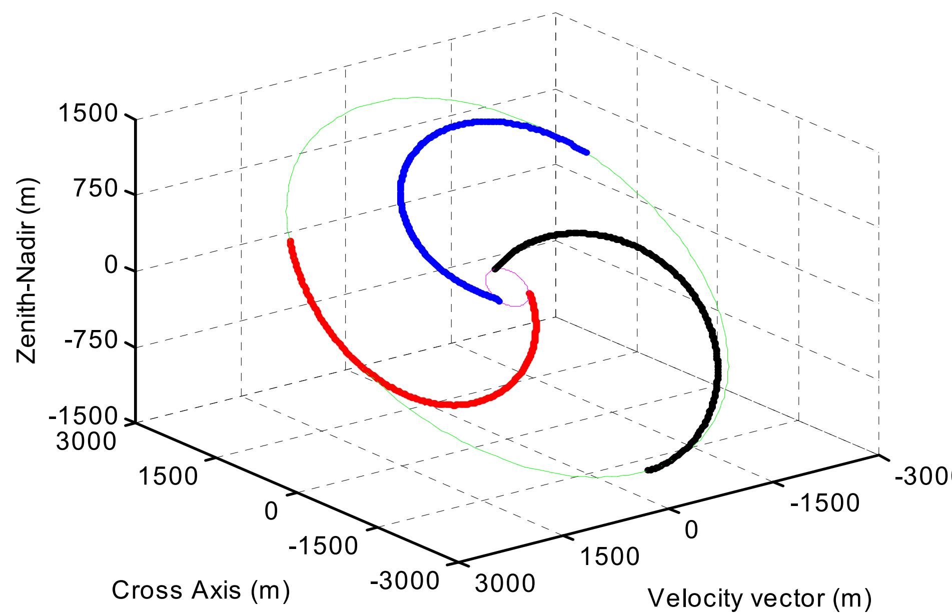
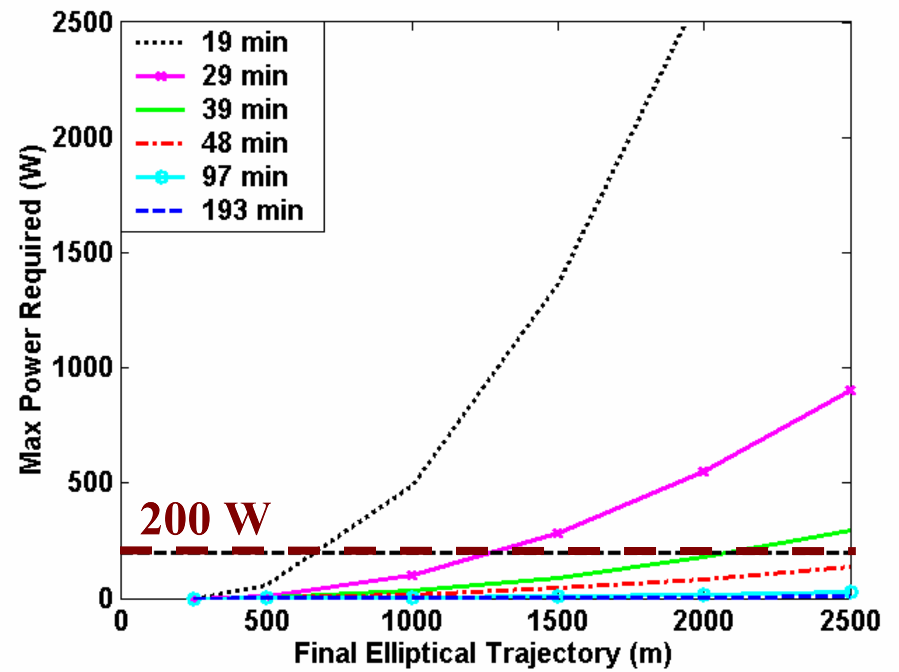


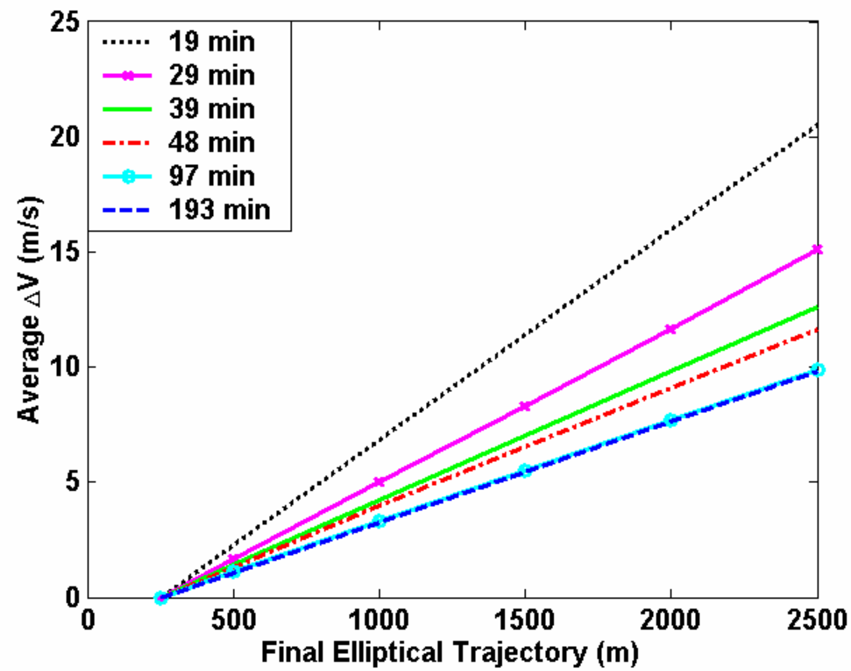
Optimal Cluster Re-sizing

Cluster Re-sizing (2)

Maximum Power Required For Geo-location



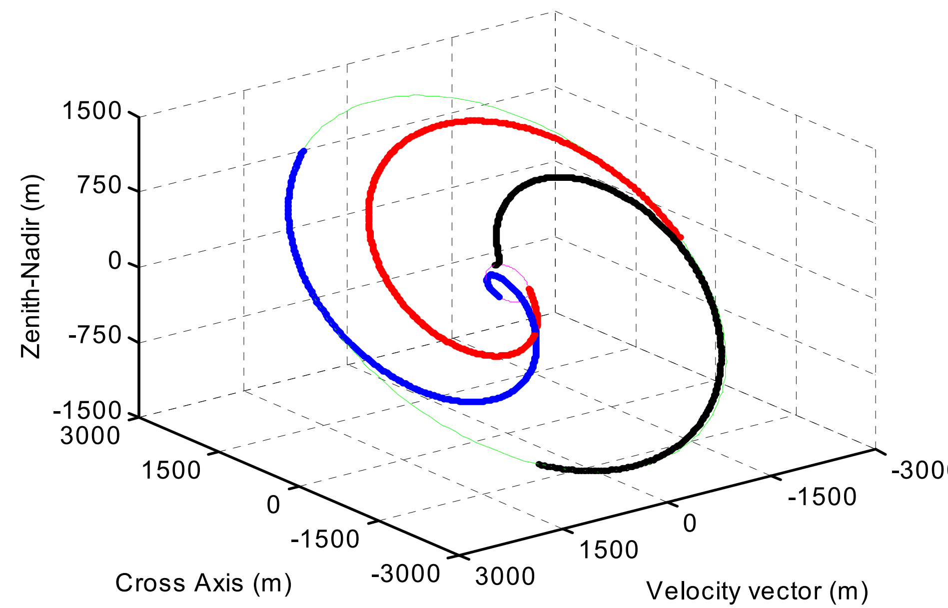
Corresponding ΔV Required

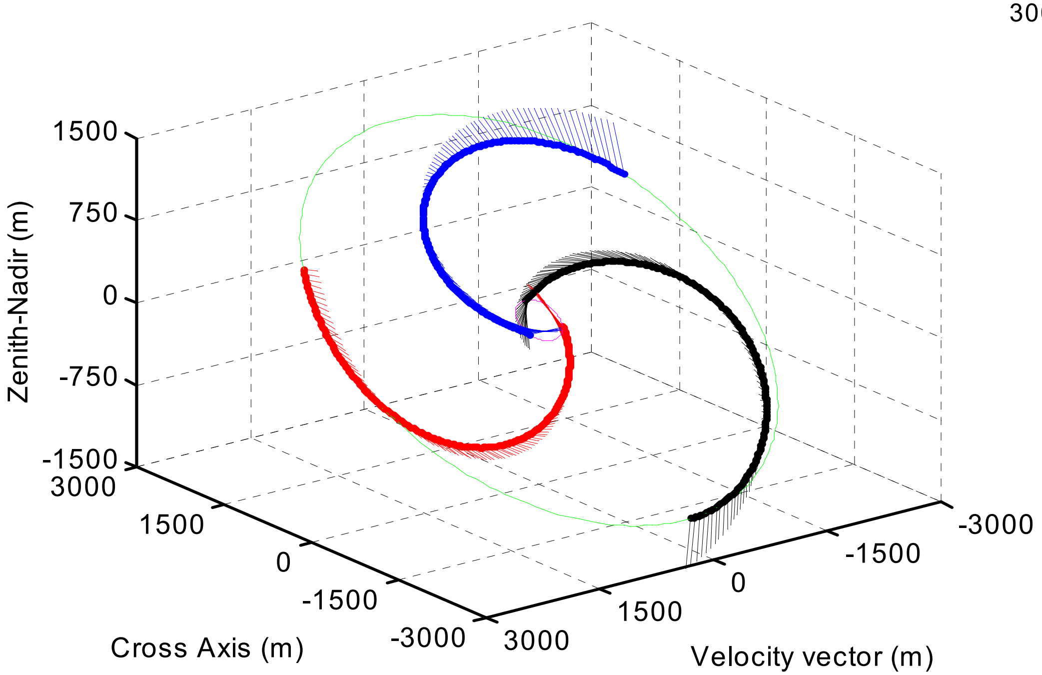


- **Minimum re-sizing time of 0.5 periods (48 mins) is required for Techsat 21 geo-location**
- **Maximum size of 1250 m can be attained if re-sizing time of 30 minutes is allowed**
- **Minimum ΔV of 10 m/s is required to perform Techsat 21 geo-location operation (25% of total ΔV budgeted)**
- **Significant ΔV savings can be achieved by increasing re-sizing time to at least 1 period (97 mins)**

Future Considerations

- Solutions obtained are only guaranteed to be local minimum - not global optimum
- Must check for minimum energy trajectories

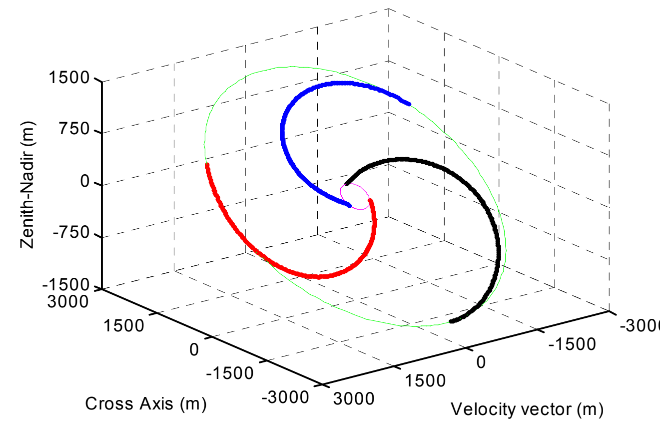


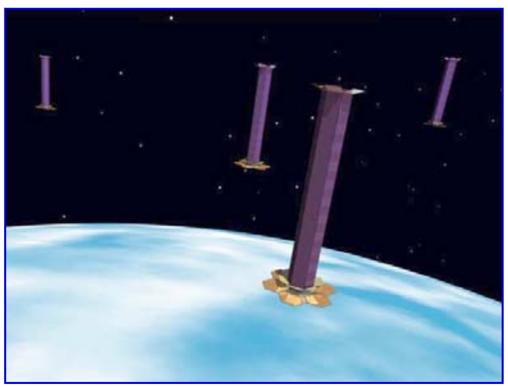


- Plume contamination due to thruster firings
- Penalize thruster firings at other spacecraft
- Penalize firings in the plane of elliptical trajectory

Conclusions

- Developed a tool to
  - determine minimum energy trajectories
  - evaluate minimum resources required for cluster re-configuration
  - size power subsystem for propulsion





- Techsat 21 Cluster initialization
  - achievable even with a short initialization time
  - recommend an initialization time of at least 1 period due to significant ΔV savings

- Techsat 21 Geo-location problem
  - a minimum re-orientation time of at least 1 period
  - extremely high ΔV expenditure operation