16.61 Homework Assignment #2

Clearly define the frames you are using, the angles between the frames, and the rates of these angles (as necessary). Be clear and consistent in your vector and matrix notation. You will not get full marks on these questions if your approach is not systematic and clear.

1. Given that the components of $\vec{A}$ are $A_1 = [\ 2\ \ -1\ \ 8\ ]^T$ in Frame 1 and the components of $\vec{B}$ are $B_2 = [\ -6\ \ 3\ \ 0\ ]^T$ in Frame 2, find $\vec{A} \cdot \vec{B}$ and $\vec{A} \times \vec{B}$ if Frame 2 is obtained by rotating Frame 1 through an angle of 60° about the $\vec{1}_x$-axis (which is one notation for the x-axis of Frame 1).

2. Do problem 2.6 on page 63 of Greenwood's *Principles of Dynamics*

3. A helicopter is moving vertically with speed 2 ft/s and acceleration 5 ft/s². At the same time, the body of the helicopter is rotating about a vertical axis with a constant angular velocity of 1.4 rad/s. The tail rotor (radius 2.5 ft) is rotating at a constant rate of 180 rad/s relative to the body. Find the absolute acceleration of a point at the tip of the tail rotor at the instant the blade is in the vertical position. The hub of the tail rotor is 25 ft behind the axis of rotation.

4. You all have probably seen figure skaters do those spins where they pull in their arms and ramp up to amazing angular rates. It is easy to explain this phenomenon using *conservation of angular momentum*, but an inquiring mind might ask *what forces are being applied to their body to cause them to spin up?*.

   To analyze this question, consider the body (no arms) to have constant moment of inertia I_o – this would be like the turntables in the problems done in class. Assume that their body has angular velocity ω and angular acceleration $\dot{\omega}$. Now consider their arms to be point masses (m) that move inwards along tracks (on opposite sides of the center) at constant relative velocities v_r. Given this information, show that the tangential force acting on the mass m is

$$F = m\dot{\omega}r - 2m\omega v_r$$

   What direction does this force act? Note that this force is exerted on the mass by the track, so $-F$ must be acting on the track by the mass. The result then is that the two arms would act as a *force couple* about the center creating a moment

$$M = 2Fr \equiv I_o\dot{\omega}$$

The final step is to recall that the inertia for the overall system is $I = I_o + 2mr^2$. Combining the above and noting that $v_r = -dr/dt$, show that

$$-\omega dI = I d\omega \Rightarrow \frac{d\omega}{\omega} + \frac{dI}{I} = 0$$

which implies that $\log \omega + \log I = \text{constant} \Rightarrow I\omega = \text{constant}$ which, is the conservation of angular momentum.

Use your results from this analysis to support the claim that: *It is the torque of the coriolis forces due to the inward moving arms that accelerates the spin of the skater.*