16.512, Rocket Propulsion
Prof. Manuel Martinez-Sanchez
**Lecture 9: Liquid Cooling**

## Cooling of Liquid Propellant Rockets

We consider only bi-propellant liquid rockets, since monopropellants tend to be small and operate at lower temperatures. In a bi-propellant rocket, both the oxidizer and the fuel streams are in principle available for cooling the most exposed parts of the chamber and nozzle prior to being injected. This is called "regenerative cooling", because the heat loss from the gas is recovered ("regenerated") into the liquid, so no heat escapes. This is not to say no thermodynamic loss is incurred, though (heat is transferred from very hot gas to cool liquid, which implies irreversibility and loss of work potential).

Of the two streams, the fuel is normally used for cooling. This is for two reasons:

(a) Fuels tend to have higher specific heats, so more heat is removed for a given $\Delta T$ of the coolant, and

(b) Leakage from an oxidizer stream into the normally fuel-rich combustion gas can produce a local flame that can be catastrophic, whereas leakage from a fuel line into the same fuel-rich gas is inert. In addition, exposing hot metal to oxygen or strong oxidants always carries some risk of accelerated chemical attack, or even ignition. Some exceptions do exist where oxidizers are used for cooling, though.

A typical arrangement is as shown below (Figure 1).

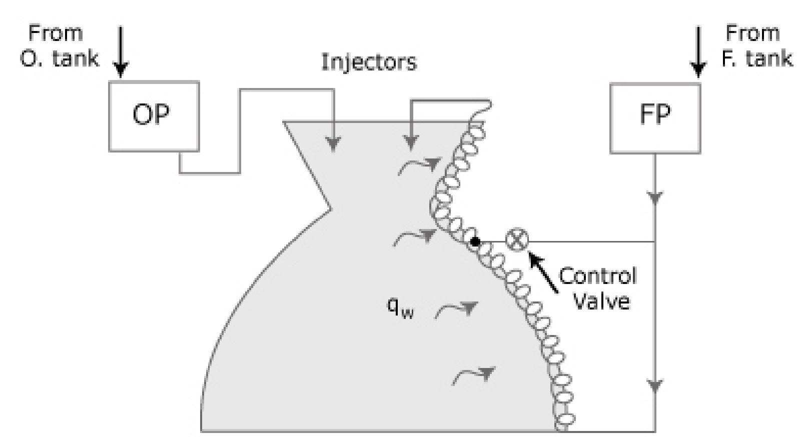

Figure 1

The fuel at high pressure from the fuel pump (FP) is sent through a series of narrow passages carved into the nozzle and chamber walls, picks up the wall heat flux from the gas, and is delivered eventually to the injector manifold. Since the nozzle region is the most thermally loaded one, often the coolant flow is split, with one part entering at the nozzle exit and another providing extra cool fluid by entering just downstream of the throat.

A typical construction for the cooling channels is shown in Fig. 2.

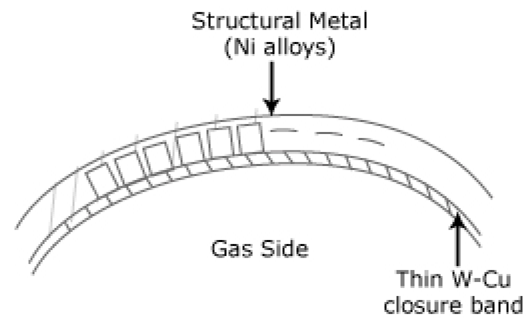

Figure 2

The load-bearing part of the structure is milled longitudinally with channels of varying depth and width (to obtain varying liquid velocity), and a high thermal conductivity thin layer of a Copper alloy is then brazed on the inside.

Design Considerations

Two aspects need to be verified in the design of the cooling system:

(a) The coolant should have sufficient thermal capacity to absorb the heat load without exceeding some critical temperature, which may be a chemical decomposition limit (thermal cracking for hydrocarbons) or the boiling point (although, with care, boiling can be sometimes tolerated or exploited for its strong heat absorption properties).

Suppose $Q_{LOSS}$ is the calculated total heat loss from the gas. As seen in a previous lecture, this amounts to 1-3% of $\dot{m} c_p T_c$, more for the smaller engines. Suppose also the fuel only is used as coolant, with a flow rate $\dot{m}_F$. The $O/F$ ratio is defined as $O/F = \dot{m}_{ox} / \dot{m}_F = \frac{\dot{m} - \dot{m}_F}{\dot{m}_F}$, and so $\dot{m}_F = \frac{\dot{m}}{1 + O/F}$. If the liquid fuel has a specific heat $c_{cool}$, its temperature rise $\Delta T$ from inlet to exit of the cooling circuit will be given by

$$Q_{LOSS} = \dot{m}_F c_{cool} \Delta T \qquad (1)$$

$$\dot{m} c_p T_c \left( \frac{Q_{LOSS}}{Q_{TOT}} \right) = \frac{\dot{m}_F}{1 + O/F} c_{cool} \Delta T$$

$$\Delta T = \left(\frac{Q_{LOSS}}{Q_{TOT}}\right)\left(\frac{c_p}{C_{cool}}\right)\left(1+\frac{O}{F}\right)T_c \qquad (2)$$

which must be kept within limits. For example, say $\frac{Q_{LOSS}}{Q_{TOT}} = 0.02$, $\frac{c_p}{c_{cool}} = 1$, $O/F = 4$, $T_c = 3000K$; we obtain $\Delta T = 0.02 \times 1 \times 5 \times 3000 = 300\ K$. This may or may not be acceptable; for a cryogenic coolant it would most likely be, but a hydrocarbon fuel, exiting the pump at 300K will then leave the cooling circuit at 600K, probably too high for chemical stability.

(b) The local cooling rate at the most exposed location (the throat) must be sufficient to avoid decomposition or boiling even at the contact point of the liquid with the wall. The thermal situation in a cut through the front wall of the cooling passages is as schematizes in Fig. 3.

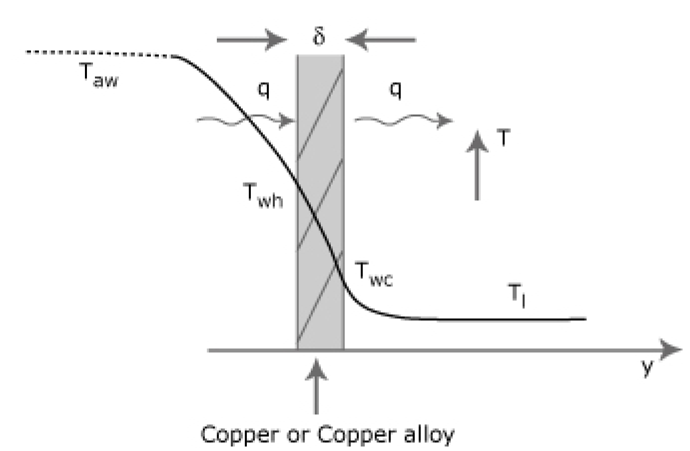

Figure 3

The $T_{aw}$ temperature is shown dashed because, as we know it is not the actual gas temperature outside the gas boundary layer, but is the one driving heat. The liquid bulk is at a temperature $T_l$, which is below that of the wetted wall ($T_{wc}$), because heat has to be driven through according to

$$q = h_l \left(T_{wc} - T_l\right) \qquad (3)$$

where $h_l$ is the liquid-side film coefficient, that can be calculated, for instance, from Bartz formula using liquid properties. The same heat flux is supplied from the gas through the gas-side film coefficient:

$$q = h_g \left( T_{aw} - T_{wh} \right) \tag{4}$$

and yet the same flux must cross the wall by conduction:

$$q = k \frac{T_{wh} - T_{wc}}{\delta} \tag{5}$$

where k is the wall thermal conductivity, and $\delta$ its thickness.

Re-writing (3)-(5) as $\begin{cases} T_{aw} - T_{wh} = \dfrac{q}{h_g} \\ T_{wh} - T_{wc} = \dfrac{\delta}{k} q \\ T_{wc} - T_l = \dfrac{q}{h_l} \end{cases}$

and adding, we obtain

$$T_{aw} - T_l = \left( \frac{1}{h_g} + \frac{\delta}{k} + \frac{1}{h_l} \right) q \ ;$$

$$\boxed{q = \frac{T_{aw} - T_l}{\dfrac{1}{h_g} + \dfrac{1}{h_l} + \dfrac{\delta}{k}}} \tag{6}$$

which we can then use to calculate intermediate temperatures from (3), (4), (5). Clearly, what we have done is adding the series “thermal impedances” $\dfrac{1}{h_g}$, $\dfrac{\delta}{k}$ and $\dfrac{1}{h_l}$ of the gas boundary layer, the metal, and the liquid.

## Stresses in Cooled Nozzle Walls

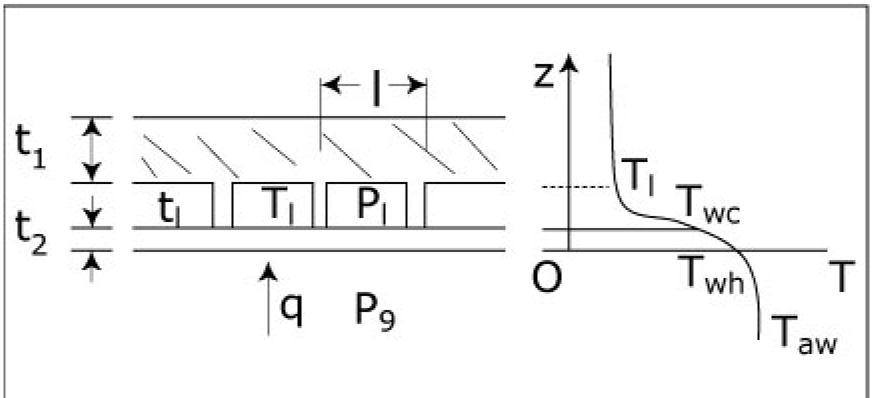

Fig. 1

Ribs carry weak in-plane stress.

Wall must carry the large hoop stress due to $P_g$, $P_l$. In addition, hot side will expand more, forcing back (cold) side to higher tension. So, use high $\sigma_{ul}$ steel for back (1) layer. Front (2) layer needs to be good thermal conductor; use Cu or W-Cu alloy (higher strength). Cu has higher expansion coefficient $\alpha_{Cu} > \alpha_{steel}$, which adds to the effect of higher T, and ends up putting this layer in compression. This can be relieved by hot assembly, so that the Cu is pre-stretched when cold.

Plane strain At any z, within one of the materials.

$$\varepsilon = (1-\nu)\frac{\sigma(z)}{E} + \alpha\left[T(z) - T_0\right] \qquad (1)$$

where $T_0$ can be interpreted as the temperature at which the strain $\varepsilon$ is defined to be zero, with zero stress. Since the shape remains planar, $\varepsilon = \text{constant}$ (at least within the layer).

Write (1) for both layers. We now "assemble" them with a tight fit, but zero stresses, at $T_0$, which from now on means the assembly temperature. Upon heating or cooling, thermal stresses will arrive, even with no loading or T gradients.

Both layers now have the same (constant) $\varepsilon$

$\varepsilon$ =0 by definition at assembly

$$(1-\nu_1)\frac{\sigma_1(z)}{E_1} + \alpha_1\left[T_1(z) - T_0\right] = \varepsilon \qquad (2)$$

$$(1-\nu_2)\frac{\sigma_2(z)}{E_2} + \alpha_2\left[T_2(z) - T_0\right] = \varepsilon \qquad (3)$$

For metals, $\nu$ varies little, so take $\nu_1 = \nu_2 = \nu$.

The temperature $T_1(z)$ will be nearly constant (adiabatic outer condition). $T_2(z)$ will vary linearly with z, according to

$$-k_2 \frac{dT_2}{dz} = q \tag{4}$$

We write (3) at $z = 0$, $z = t_2$ and subtract:

$$(1-\nu)\frac{\sigma_{2c} - \sigma_{2h}}{E_2} = \alpha_2 \left[T_{wh} - T_{wc}\right] \tag{5}$$

and integrate (4) to

$$q = k_2 \frac{T_{wh} - T_{wc}}{t_2} \tag{6}$$

so that

$$\boxed{\sigma_{2c} - \sigma_{2h} = \frac{\alpha_2 E_2}{1-\nu} \frac{t_2}{k_2} q} \tag{7}$$

Also (2) reads

$\varepsilon = (1-\nu)\dfrac{\sigma_1}{E_1} + \alpha_1\left[T_l - T_0\right]$, which can be combined with

$$\varepsilon = (1-\nu)\frac{\sigma_{2c}}{E_2} + \alpha_2\left[T_{wc} - T_0\right]$$

to give $(1-\nu)\left(\dfrac{\sigma_1}{E_1} - \dfrac{\sigma_{2c}}{E_2}\right) = \alpha_2 T_{wc} - \alpha_1 T_l - (\alpha_2 - \alpha_1) T_0$ (8)

Heat transfer from wall 2 to liquid gives

$$q = h_l \left(T_{wc} - T_l\right) \tag{9a}$$

or $$T_{wc} = T_l + \frac{q}{h_l} \tag{9b}$$

Substitute into (8)

$$(1-\nu)\left(\frac{\sigma_1}{E_1} - \frac{\sigma_{2c}}{E_2}\right) = (\alpha_2 - \alpha_1)(T_l - T_0) + \alpha_2 \frac{q}{h_l}$$

or

$$\boxed{\frac{\sigma_1}{E_1} - \frac{\sigma_{2c}}{E_2} = \frac{\alpha_2}{1-\nu}\ \frac{q}{h_l} + \frac{(\alpha_2 - \alpha_1)(T_l - T_0)}{1-\nu}} \qquad (10)$$

In all of this, q is taken as a given. It can be calculated from the given $T_{aw}$, $T_l$, plus $h_g$, $h_l$ and $t_2$, $k_2$:

$$q = \frac{T_{aw} - T_l}{\frac{1}{h_g} + \frac{1}{h_l} + \frac{t_2}{k_2}} \qquad (11)$$

Equations (7) and (10) relate $\sigma_{2h}$, $\sigma_{2c}$, $\sigma_1$. We need one more equation

Force balance

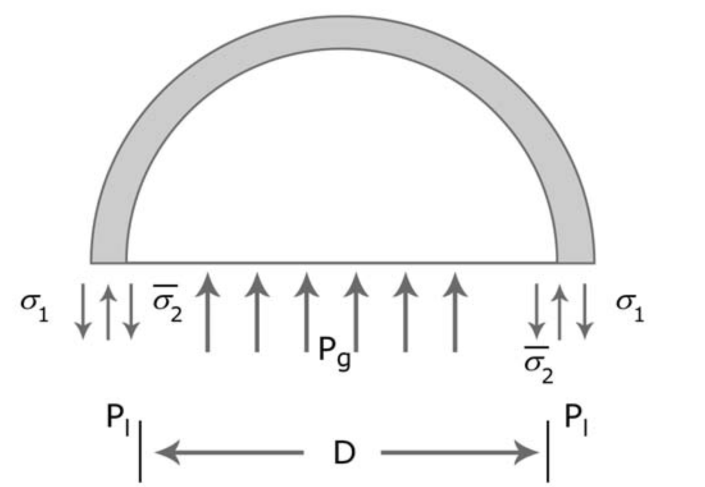

Figure 2

Since $T_2(z)$ is linear in z, so will $\sigma_2(z)$. For force calculations, then, we can use the mean value

$$\bar{\sigma}_2 = \frac{\sigma_{2c} + \sigma_{2h}}{2} \qquad (12)$$

The net balance then is

$$2\sigma_1 t_1 + 2\bar{\sigma}_2 t_2 = P_g D + 2P_l t_l \qquad (13)$$

which is our 3rd equation, together with (7), (10). Notice that, since $t_l \ll D$, $P_l$ has only a minor effect on stresses (except in the ribs).

From here, we either solve for $\sigma_{2h}$, $\sigma_{2c}$, $\sigma_1$ (given geometry, $P_g$, $P_l$, q) or, for design, go the reverse route and decide on the geometry for assigned stresses. We will pursue here the second approach.

Design

As noted, $\sigma_1$ will be positive and high, whereas $\sigma_{2h}$ (and less so $\sigma_{2c}$) will be negative, and probably high too. We then take the view that

$$\sigma_1 = \sigma_{1_{ult,tens.}} / S \qquad (14)$$

(S=safety factor ~ 1.5) and

$$\left(-\sigma_{2h}\right) = \text{least of} \begin{cases} \sigma_{2_{ult,comp.}} / S & (15a) \\ \sigma_{2_{buckling}} / S & (15b) \end{cases}$$

and use these conditions to determine $t_1$, $t_2$. The selection of $t_2$ is complicated by the fact that $\left(-\sigma_{2h}\right)$ will decrease with $t_2$, but so will (quadratically) $\sigma_{2_{buckling}}$:

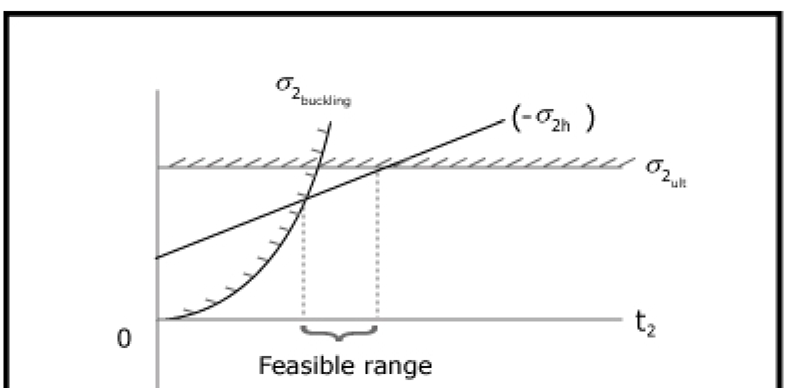

Figure 3

For buckling, use a simple clamped-beam formulation:

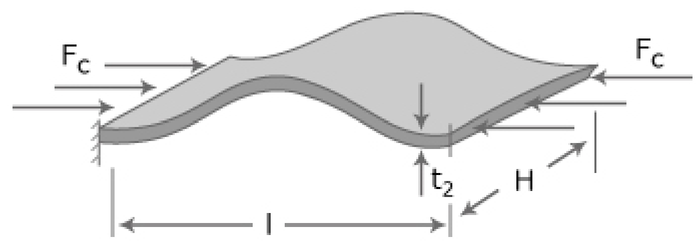

Figure 4

$$F_c = \frac{E_2 I \pi^2}{(l/2)^2} = \frac{4\pi^2 E_2 I}{l^2}$$

where $I = \frac{1}{12} H t_2^3$.

Dividing by $A = Ht_2$,

$$\sigma_{2_{buckling}} = \frac{4\pi^2 E_2 \left(\frac{1}{12} H t_2^3\right)}{l^2 \left(Ht_2\right)} \quad \sigma_{2_{buckling}} = \frac{\pi^2}{3}\left(\frac{t_2}{l}\right)^2 E_2 \tag{16}$$

To proceed, start by eliminating $\sigma_{2c}$ between (7) and (10):

$$\frac{\sigma_1}{E_1} - \frac{\sigma_{2h}}{E_2} - \frac{\alpha_2}{1-\nu}\frac{t_2}{k_2} q = \frac{\alpha_2}{1-\nu}\frac{q}{h_l} + \frac{\left(\alpha_2 - \alpha_1\right)\left(T_l - T_0\right)}{1-\nu}$$

$$\left(-\sigma_{2h}\right) = -\frac{E_2}{E_1}\sigma_1 + \frac{\alpha_2 E_2}{1-\nu}\left(\frac{1}{h_l} + \frac{t_2}{k_2}\right) q + \frac{E_2\left(\alpha_2 - \alpha_1\right)\left(T_l - T_0\right)}{1-\nu} \tag{17}$$

or, recalling (11),

$$\left(-\sigma_{2h}\right) = -\frac{E_2}{E_1}\sigma_1 + \frac{\alpha_2 E_2}{1-\nu}\frac{\frac{1}{h_l} + \frac{t_2}{k_2}}{\frac{1}{h_g} + \frac{1}{h_l} + \frac{t_2}{k_2}}\left(T_{aw} - T_l\right) + \frac{\left(\alpha_2 - \alpha_1\right)E_2\left(T_l - T_0\right)}{1-\nu} \tag{18}$$

This displays several effects:

(a) Once $\sigma_1 = \frac{\sigma_{1_{ult.}}}{S}$ is fixed, $-\sigma_{2h}$ will increase with $t_2$, although weakly, since $\frac{t_2}{k_2} \ll \frac{1}{h_g}, \frac{1}{h_l}$ (the Cu wall offers little thermal impedance, compared to the two boundary layers).

(b) This $(\sigma_{2h})$ (middle term in (18)) arises from the heating of layer 2 relative to 1, due to heat flowing in.

(c) The positive stress $\sigma_1$ (to counter $P_g$ mostly) relieves this tendency to compress layer 2, and might even reverse it.

(d) The last term in (18) arising from differential expansion coefficients $\alpha_2 - \alpha_1$, could be used as a design aid. If 2=Cu, 1=steel, $\alpha_2 = 2.3 \times 10^{-5}\,K^{-1}$, $\alpha_1 = 1.4 \times 10^{-5}\,K^{-1}$, so $\alpha_2 - \alpha_1 > 0$.

Then, if $-\sigma_{2h}$ is still too high despite $\sigma_1$, we could <u>increase</u> $\underline{T_0}$, if possible above $T_l$, to reduce $-\sigma_{2h}$. This implies <u>assembly at high temperature</u>.

We can use $\left\{\begin{matrix}(18)\\(16)\end{matrix}\right\}$ to construct a plot like Figure 3, and select a viable $t_2$. Once this is done, equation (7) gives $\sigma_{2c}$, equation (12) gives $\bar{\sigma}_2$, and equation (13) gives $t_1$.

<u>Some data</u>:

| Material | E $(P_a)$ (at 500K) | $d\left(k^{-1}\right)$ | $\sigma_{ult}$ $(P_a)$ (at 500K) | $\nu$ | $Z = \frac{(1-\nu)\sigma_{ult.}}{E\alpha}\left({}^{o}K\right)$ |
|---|---|---|---|---|---|
| Cu | $0.95 \times 10^{11}$ | $2.3 \times 10^{-5}$ | $1.1 \times 10^{8}$ | 0.3 | 35 |
| St. Steel 302 | $1.61 \times 10^{11}$ | $1.8 \times 10^{-5}$ | $4.6 \times 10^{8}$ | 0.3 | 111 |
| Ti | $1.63 \times 10^{11}$ | $1.7 \times 10^{-5}$ | $5.4 \times 10^{8}$ | 0.3 | 136 |
| Alloy Steel (SAE x4130) | $1.09 \times 10^{11}$ | $1.4 \times 10^{-5}$ | $5.1 \times 10^{8}$ | 0.3 | 234 |

The last column is a "figure of merit" extracted from equation (18) to give a preliminary rough idea of materials expansion stress. The higher Z, the higher the $\Delta T$ to reach $\sigma_{ult.}$ in a double strip of this material subject to differential heating $\Delta T$.

Example $P_g = 100\,atm \simeq 10^7\ P_a$ (neglect $P_e$ effect)

$D = 0.3m$; $l = 4mm$

$h_g = 24000\,W/m^2/K$;

$h_l = 2.76 \times 10^5\,W/m^2/K$;

$k_2 = 360\,W/m/K$

$T_{aw} = 3200K$, $T_l = 400K$

$$\sigma_1 = \frac{5.1 \times 10^8}{1.5}\ P_a\left(\text{alloy steel}\right)\ ;\ E_1 = 1.09 \times 10^{11}\ P_a\ ;\ \alpha_1 = 1.4 \times 10^{-5}\ K^{-1}$$

$$-\sigma_{2_{ult,comp.}} = \frac{1.1 \times 10^8}{1.5}\ P_a\left(Cu\right)\ ;\quad E_2 = 0.95 \times 10^{11}\ P_a\ ;\ \alpha_2 = 2.3 \times 10^{-5}\ K^{-1}$$

Substituting into (18),

$$-\sigma_{2h} = -2.96 \times 10^8 + 8.740 \times 10^9\,\frac{1.303 \times 10^{-3} + t_2}{16.30 \times 10^{-3} + t_2} + 1.221 \times 10^6\,(\underline{400} - T_0) \qquad (19)$$

and, from (16)

$$\sigma_{2_{buckling}} = 1.95 \times 10^{16}\ t_2^2 \qquad (t_2 \text{ in m.})$$

Following are some calculated results:

**$T_0 = 297K$**

| $t_2\,(mm)$ | 0 | 0.2 |
|---|---|---|
| $-\sigma_{2h}\,(P_a)$ | $5.33 \times 10^8$ | $6.26 \times 10^8$ |
| $-\sigma_{2_{buckling}}\,(P_a)$ | 0 | $7.80 \times 10^8$ |

COMMENTS

Since $-\sigma_{ult,com} = \frac{1.1 \times 10^8}{1.5}$, assembling at room $T_0$ is not acceptable <u>for any</u> $\underline{t_2}$.

**$T_0 = 500K$**

| $t_2\,(mm)$ | 0 | 0.2 |
|---|---|---|
| $-\sigma_{2h}\,(P_a)$ | $2.81 \times 10^8$ | $3.78 \times 10^8$ |
| $-\sigma_{2,buckl.}\,(P_a)$ | 0 | $7.80 \times 10^8$ |

Closer, but still no solution

| | | | | | |
|---|---|---|---|---|---|
| $T_0 = 700K$ | $t_2\,(mm)$ | 0 | 0.2 | 0.1 | With assembly at $T_0$ =700K, a <u>very thin</u> $t_2 \simeq 0.1\,mm$ is acceptable, but may be questionable on robustness. Also, <u>$\sigma_{2c}$ is too high</u> |
| | $-\sigma_{2h}\,(P_a)$ | $0.36\times10^8$ | $1.34\times10^8$ | $0.85\times10^8$ | |
| $q\left(w/m^2\right) = 6.15\times10^7$ | $-\sigma_{2,buckl.}\,(P_a)$ | 0 | $7.80\times10^8$ | $1.95\times10^8$ | |
| | $+\sigma_{2c}\,(P_a)$ | | | $+4.48\times10^8$ | |
| | $t_1\,(mm)$ | | | 4.40 | |

| | | | | | |
|---|---|---|---|---|---|
| $T_0 = 800K$ | $t_2\,(mm)$ | 0 | 0.2 | 0.3 | If $T_0$ =800K, is feasible, then $t_2 = 0.3\,mm$ is acceptable. $\sigma_{2c}$ also OK (in tension) |
| | $-\sigma_{2h}\,(P_a)$ | $-0.86\times10^8$ | $0.12\times10^8$ | $0.60\times10^8$ | |
| $q = 6.07\times10^7$ | $-\sigma_{2,buckl.}\,(P_a)$ | 0 | $7.8\times10^8$ | $1.76\times10^8$ | |
| | $+\sigma_{2c}\,(P_a)$ | | | $9.79\times10^7$ | |
| | $t_1\,(mm)$ | | | 4.48 | |

With the assumed $l = 4\,mm$, buckling is not a problem in any case, but compressive failure is hard to avoid. It may be possible to exceed the elastic limit and go into plastic compressive yield if ductility is high enough to ensure no rupture. <u>But</u> this means no reusability.