

This exercise is about comparing the performance of chemical versus electrical thrusters for a specific mission. The mission is a heliocentric transfer from near Earth to near Mars. The initial mass  $M_0=20,000$  kg starts “near” Earth, but outside its SOI, and ends “near” Mars, but again, outside its SOI.

(a) Chemical

Assume a two-impulse heliocentric Hohmann transfer, and calculate the total  $\Delta V$  required. No staging is assumed. Assume a jet velocity  $c=4500$  m/s (specific impulse  $I=459$  s, typical of a LOX-LH rocket), and calculate the final mass delivered to near Mars. Of this, assume 5% of the initial mass (1000 kg) is structure+engine, and the rest is “payload”; calculate this payload. Finally, calculate the transfer duration.

(b) Electrical

(b1) One peculiarity of low thrust trajectories is that their required  $\Delta V$  is typically greater than that of impulsive maneuvers. The trajectory is a gradual spiral, and using some reasonable approximations, one finds that for a mission like this,

$$\Delta V = v_{c1} - v_{c2} = \sqrt{\frac{\mu_s}{r_1}} - \sqrt{\frac{\mu_s}{r_2}}$$

This seems backwards (initial minus final), but it is not; it reflects the fact that the vehicle gains twice as much potential energy than the kinetic energy it loses in the climb. Calculate this  $\Delta V$ .

In designing the low-thrust electrical propulsion system, there are two free parameters:

- The initial acceleration  $a_0$ , or, equivalently, the thrust duration  $\Delta t$ , which is basically the whole transfer duration.
- The jet velocity  $c$ , or specific impulse  $I$  to use, since that can be adjusted by design.

Assume the mass of the power and propulsion system per unit power is 10 kg/kW. The efficiency of the thruster and power system is taken as  $\eta=0.7$ . Notice that the Notes assumed  $\eta=1$ ; with this efficiency included, the payload fraction expression is modified to

$$\frac{m_{\text{pay}}}{m_{\text{tot}}} = e^{-\frac{\Delta V}{c}} - \left(\frac{\alpha_e a_0 \Delta V}{2\eta}\right) \left(\frac{c}{\Delta V}\right)$$

(b2) Since the mission duration is more or less free, select a few initial accelerations, in the range from  $a_0=10^{-4} \text{ m/s}^2$  to  $6 \times 10^{-4} \text{ m/s}^2$ . For each  $a_0$ , there is an optimum jet velocity  $c$  that maximizes the payload ratio given above. Calculate this optimum  $c$  for each case, and the corresponding payload ratio. For consistency with the chemical case, subtract 5% of the initial mass as being “inert mass”, and calculate the final “payload” in each case.

(b3) One argument frequently used against low thrust systems is that the very low acceleration implies very long mission times. However, since the acceleration acts all the time, this is not necessarily so, and it must depend on the amount of electrical power available per unit initial mass. To quantify these statements, calculate the transfer time for each of the cases considered in (b2). Do this by dividing the propellant mass used by the mass flow rate. The mass flow rate is in turn the thrust  $F=M_0a_0$ , and the propellant mass follows from the rocket equation. In addition, calculate the implied specific power  $P/M_0$ . Here,  $P = \frac{Fc}{2\eta}$ .

For reference, modern GEO communication satellites already have specific powers approaching 10 W/kg, even though their power system is not designed for primary propulsion, but rather for their communications mission.

Present your results for Part (b) in tabular or graphical form. A useful format could be a set of graphs of specific impulse, initial acceleration, transfer duration and payload mass, all vs. specific power. Compare the payload and the duration to those for the chemical case.

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