16.333: Lecture #2

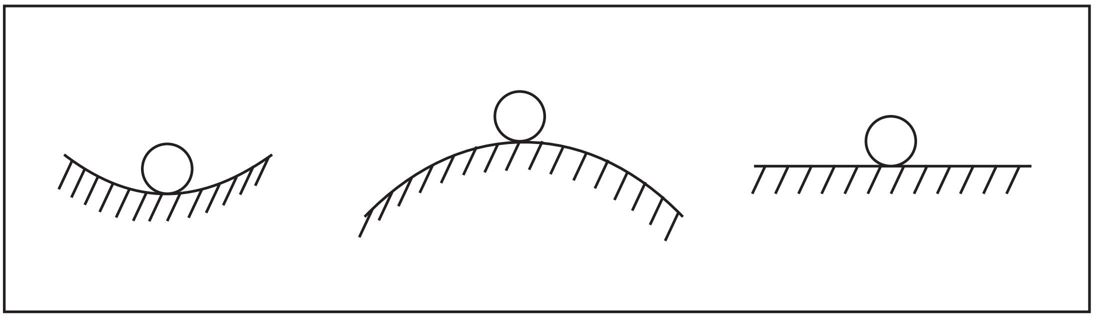
Static Stability

Aircraft Static Stability (longitudinal)

Wing/Tail contributions

Static Stability

- **Static stability** is all about the **initial tendency** of a body to return to its equilibrium state after being disturbed



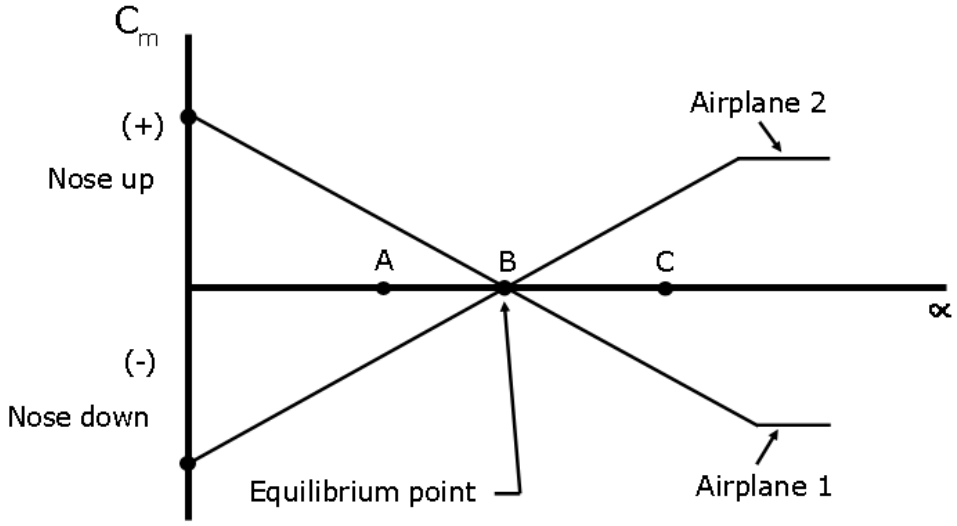
- To have a statically stable equilibrium point, the vehicle must develop a restoring force/moment to bring it back to the eq. condition

- Later on we will also deal with **dynamic stability**, which is concerned with the time history of the motion after the disturbance
  - Can be SS but not DS, but to be DS, must be SS

  ⇒ SS is a necessary, but not sufficient condition for DS

- To investigate the static stability of an aircraft, can analyze response to a disturbance in the angle of attack
  - At eq. pt., expect moment about c.g. to be zero $C_{M_{cg}} = 0$
  - If then perturb α up, need a restoring moment that pushes nose back down (negative)

- Classic analysis:



  - Eq at point B
  - A/C 1 is statically stable

- Conditions for static stability

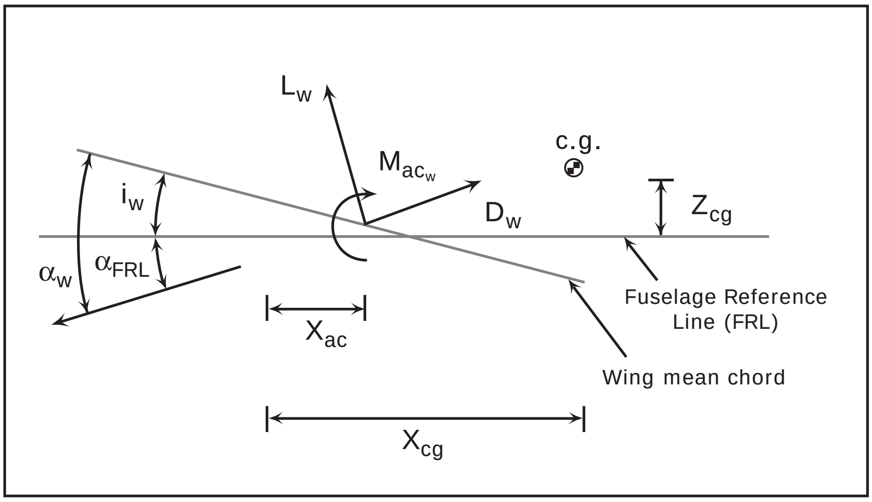
$$C_M = 0; \qquad \frac{\partial C_M}{\partial \alpha} \equiv C_{M_\alpha} < 0$$

note that this requires $C_M|_{\alpha_0} > 0$

- Since $C_L = C_{L_\alpha}(\alpha - \alpha_0)$ with $C_{L_\alpha} > 0$, then an equivalent condition for SS is that

$$\frac{\partial C_M}{\partial C_L} < 0$$

Basic Aerodynamics



- Take reference point for the wing to be the **aerodynamic center** (roughly the 1/4 chord point)[1]
- Consider wing contribution to the pitching moment about the c.g.
- Assume that wing incidence is i_w so that, if $\alpha_w = \alpha_{FRL} + i_w$, then

$$\alpha_{FRL} = \alpha_w - i_w$$

  – With x_k measured from the leading edge, the moment is:

$$\begin{aligned} M_{cg} &= (L_w \cos\alpha_{FRL} + D_w \sin\alpha_{FRL})(x_{cg} - x_{ac}) \\ &\quad + (L_w \sin\alpha_{FRL} - D_w \cos\alpha_{FRL})(z_{cg}) + M_{ac_w} \end{aligned}$$

  – Assuming that $\alpha_{FRL} \ll 1$,

$$\begin{aligned} M_{cg} &\approx (L_w + D_w\alpha_{AFRL})(x_{cg} - x_{ac}) \\ &\quad + (L_w\alpha_{FRL} - D_w)(z_{cg}) + M_{ac_w} \end{aligned}$$

  – But the second term contributes very little (drop)

[1] The aerodynamic center (AC) is the point on the wing about which the coefficient of pitching moment is constant. On all airfoils the ac very close to the 25% chord point (+/- 2%) in subsonic flow.

- Non-dimensionalize:

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S} \qquad C_M = \frac{M}{\frac{1}{2}\rho V^2 S\bar{c}}$$

- Gives:

$$C_{M_{cg}} = (C_{L_w} + C_{D_w}\alpha_{FRL})(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}) + C_{M_{ac}}$$

- Define:
  - $x_{cg} = h\bar{c}$ (leading edge to c.g.)
  - $x_{ac} = h_{\bar{n}}\bar{c}$ (leading edge to AC)

- Then

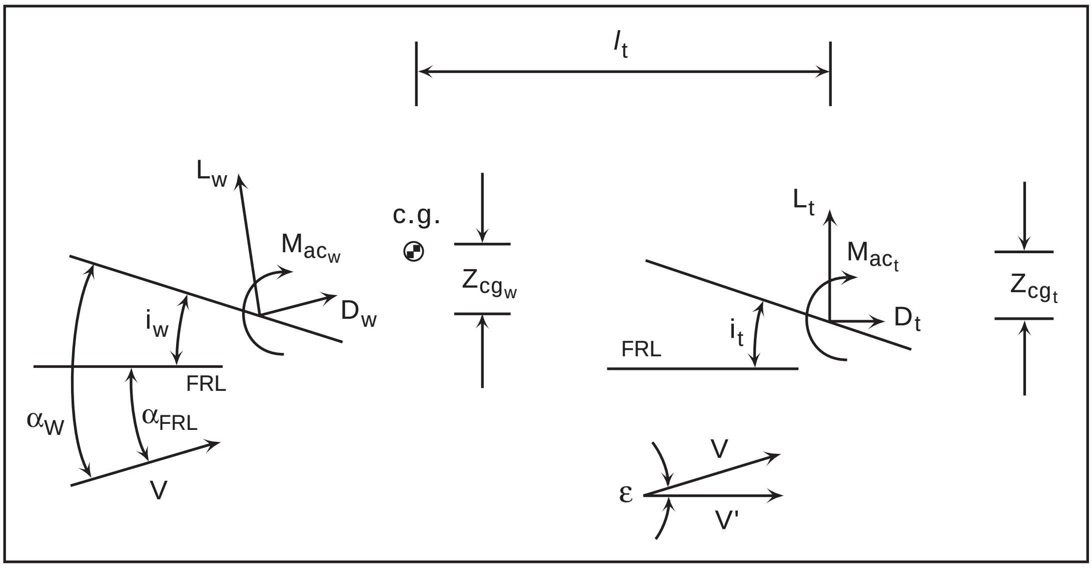
$$\begin{aligned} C_{M_{cg}} &= (C_{L_w} + C_{D_w}\alpha_{FRL})(h - h_{\bar{n}}) + C_{M_{ac}} \\ &\approx (C_{L_w})(h - h_{\bar{n}}) + C_{M_{ac}} \\ &= C_{L_{\alpha_w}}(\alpha_w - \alpha_{w_0})(h - h_{\bar{n}}) + C_{M_{ac}} \end{aligned}$$

- Result is interesting, but the key part is how this helps us analyze the static stability:

$$\frac{\partial C_{M_{cg}}}{\partial C_{L_w}} = (h - h_{\bar{n}}) > 0$$

since c.g. typically further back that AC
  - Why most planes have a second lifting surface (front or back)

Contribution of the Tail



- Some lift provided, but moment is the key part
- Key items:

1. Angle of attack $\alpha_t = \alpha_{FRL} + i_t - \epsilon$, ϵ is wing **downwash**

2. Lift $L = L_w + L_t$ with $L_w \gg L_t$ and

$$C_L = C_{L_w} + \eta \frac{S_t}{S} C_{L_t}$$

$\eta = (1/2\rho V_t^2)/(1/2\rho V_w^2) \approx$0.8–1.2 depending on location of tail

3. Downwash usually approximated as

$$\epsilon = \epsilon_0 + \frac{d\epsilon}{d\alpha}\alpha_w$$

where ϵ_0 is the downwash at α_0. For a wing with an elliptic distribution

$$\epsilon \approx \frac{2C_{L_w}}{\pi AR} \Rightarrow \frac{d\epsilon}{d\alpha} = \frac{2C_{L_{\alpha w}}}{\pi AR}$$

- Pitching moment contribution: L_t and D_t are $\perp$, $\|$ to V' not V
  - So they are at angle $\bar{\alpha} = \alpha_{FRL} - \epsilon$ to FRL, so must rotate and then apply moment arms l_t and z_t.

$$\begin{aligned} M_t = \quad & -l_t \left[L_t \cos\bar{\alpha} + D_t \sin\bar{\alpha} \right] \\ & -z_t \left[D_t \cos\bar{\alpha} - L_t \sin\bar{\alpha} \right] + M_{ac_t} \end{aligned}$$

- First term largest by far. Assume that $\bar{\alpha} \ll 1$, so that $M_t \approx -l_t L_t$

$$L_t = \frac{1}{2}\rho V_t^2 S_t C_{L_t}$$

$$\begin{aligned} C_{M_t} &= \frac{M_t}{\frac{1}{2}\rho V^2 S\bar{c}} = -\frac{l_t}{\bar{c}} \frac{\frac{1}{2}\rho V_t^2 S_t C_{L_t}}{\frac{1}{2}\rho V^2 S} \\ &= -\frac{l_t S_t}{S\bar{c}} \eta C_{L_t} \end{aligned}$$

- Define the horizontal tail volume ratio $V_H = \frac{l_t S_t}{S\bar{c}}$, so that

$$C_{M_t} = -V_H \eta C_{L_t}$$

- Note: angle of attack of the tail $\alpha_t = \alpha_w - i_w + i_t - \epsilon$, so that

$$C_{L_t} = C_{L_{\alpha_t}} \alpha_t = C_{L_{\alpha_t}}(\alpha_w - i_w + i_t - \epsilon)$$

  where $\epsilon = \epsilon_0 + \epsilon_\alpha \alpha_w$

- So that

$$\begin{aligned} C_{M_t} &= -V_H \eta C_{L_{\alpha_t}}(\alpha_w - i_w + i_t - (\epsilon_0 + \epsilon_\alpha \alpha_w)) \\ &= V_H \eta C_{L_{\alpha_t}}(\epsilon_0 + i_w - i_t - \alpha_w(1 - \epsilon_\alpha)) \end{aligned}$$

- More compact form:

$$
\begin{aligned}
C_{M_t} &= C_{M_{0_t}} + C_{M_{\alpha_t}}\alpha_w \\
\Rightarrow C_{M_{0_t}} &= V_H \eta C_{L_{\alpha_t}}(\epsilon_0 + i_w - i_t) \\
\Rightarrow C_{M_{\alpha_t}} &= -V_H \eta C_{L_{\alpha_t}}(1 - \epsilon_\alpha)
\end{aligned}
$$

where we can chose V_H by selecting l_t, S_t and i_t

- Write wing form as $C_{M_w} = C_{M_{0_w}} + C_{M_{\alpha_w}}\alpha_w$

$$
\begin{aligned}
\Rightarrow C_{M_{0_w}} &= C_{M_{ac}} - C_{L_{\alpha_w}}\alpha_{w_0}(h - h_{\bar{n}}) \\
\Rightarrow C_{M_{\alpha_w}} &= C_{L_{\alpha_w}}(h - h_{\bar{n}})
\end{aligned}
$$

- And total is:

$$
\begin{aligned}
C_{M_{cg}} &= C_{M_0} + C_{M_\alpha}\alpha_w \\
\Rightarrow C_{M_0} &= C_{M_{ac}} - C_{L_{\alpha_w}}\alpha_{w_0}(h - h_{\bar{n}}) + V_H \eta C_{L_{\alpha_t}}(\epsilon_0 + i_w - i_t) \\
\Rightarrow C_{M_\alpha} &= C_{L_{\alpha_w}}(h - h_{\bar{n}}) - V_H \eta C_{L_{\alpha_t}}(1 - \epsilon_\alpha)
\end{aligned}
$$

- For static stability need $C_M = 0$ and $C_{M_\alpha} < 0$
  - To ensure that $C_M = 0$ for reasonable value of α, need $C_{M_0} > 0$ $\Rightarrow$ use i_t to trim the aircraft

- For $C_{M_\alpha} < 0$, consider setup for case that makes $C_{M_\alpha} = 0$
  - Note that this is a discussion of the aircraft cg location ("find h")
  - But tail location currently given relative to c.g. (l_t behind it), which is buried in V_H $\Rightarrow$ need to define it differently

- Define $l_t = \bar{c}(h_t - h)$; h_t measured from the wing leading edge, then

$$V_H = \frac{l_t S_t}{\bar{c} S} = \frac{S_t}{S}(h_t - h)$$

  which gives

$$\begin{aligned} C_{M_\alpha} &= C_{L_{\alpha w}}(h - h_{\bar{n}}) - \frac{S_t}{S}(h_t - h)\eta C_{L_{\alpha_t}}(1 - \epsilon_\alpha) \\ &= h(C_{L_{\alpha w}} + \eta\frac{S_t}{S}C_{L_{\alpha_t}}(1 - \epsilon_\alpha)) - C_{L_{\alpha w}} h_{\bar{n}} - h_t \eta \frac{S_t}{S} C_{L_{\alpha_t}}(1 - \epsilon_\alpha) \end{aligned}$$

- A bit messy, but note that if $L_T = L_w + L_t$, then

$$C_{L_T} = C_{L_w} + \eta\frac{S_t}{S}C_{L_t}$$

  so that

$$\begin{aligned} C_{L_T} &= C_{L_{\alpha w}}(\alpha_w - \alpha_{w_0}) + \eta\frac{S_t}{S}C_{L_{\alpha_t}}(-\epsilon_0 - i_w + i_t + \alpha_w(1 - \epsilon_\alpha)) \\ &= C_{L_{0_T}} + C_{L_{\alpha_T}}\alpha_w \end{aligned}$$

  with $C_{L_{\alpha_T}} = C_{L_{\alpha w}} + \eta\frac{S_t}{S}C_{L_{\alpha_t}}(1 - \epsilon_\alpha)$

- Now have that

$$C_{M_\alpha} = hC_{L_{\alpha_T}} - C_{L_{\alpha_w}} h_{\bar{n}} - h_t \eta \frac{S_t}{S} C_{L_{\alpha_t}} (1 - \epsilon_\alpha)$$

- Solve for the case with $C_{M_\alpha} = 0$, which gives

$$h = h_{\bar{n}} \left(\frac{C_{L_{\alpha_w}}}{C_{L_{\alpha_T}}} \right) + \eta \frac{S_t}{S} h_t \frac{C_{L_{\alpha_t}}}{C_{L_{\alpha_T}}} (1 - \epsilon_\alpha)$$

- Note that with $\gamma = \frac{C_{L_{\alpha_T}}}{C_{L_{\alpha_w}}} \approx 1$, then

$$h = \frac{h_{\bar{n}} + (\gamma - 1) h_t}{\gamma} \equiv h_{NP}$$

which is called the **stick fixed neutral point**

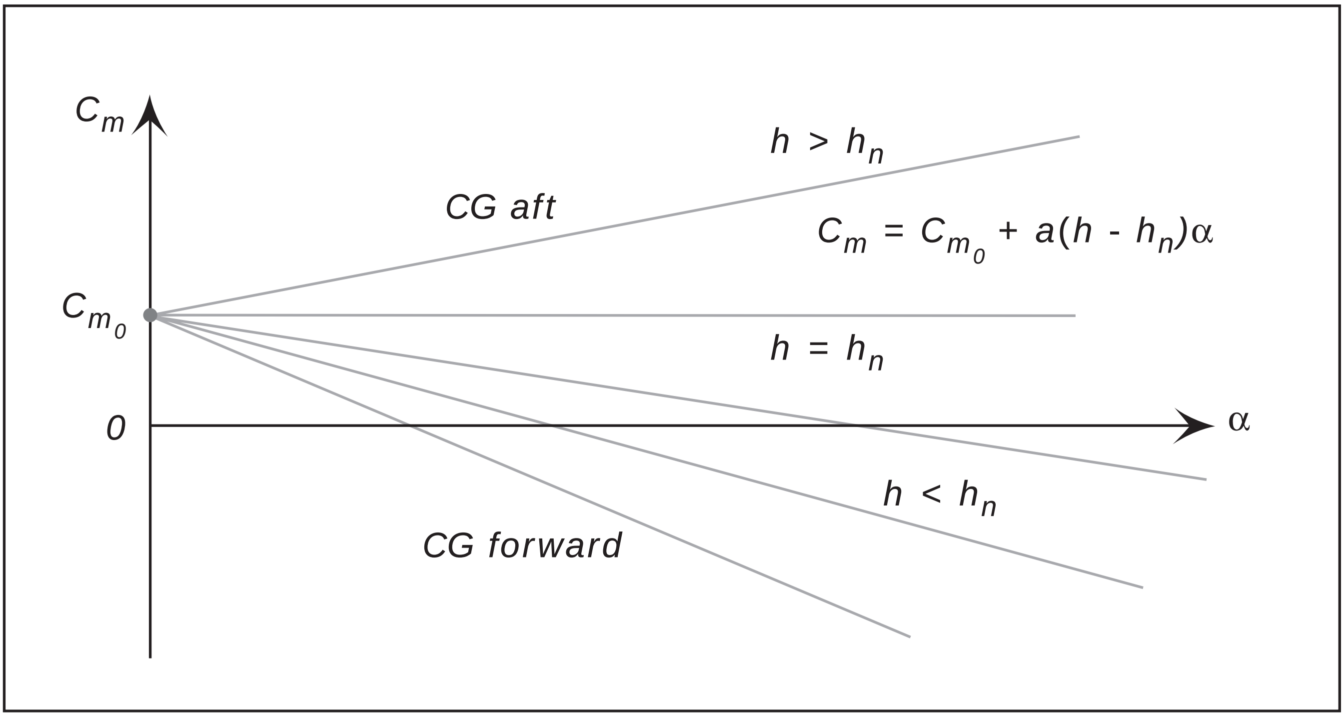
- Can rewrite as

$$\begin{aligned} C_{M_\alpha} &= C_{L_{\alpha_T}} \left(h - h_{\bar{n}} \left(\frac{C_{L_{\alpha_w}}}{C_{L_{\alpha_T}}} \right) - \eta \frac{S_t}{S} h_t \frac{C_{L_{\alpha_t}}}{C_{L_{\alpha_T}}} (1 - \epsilon_\alpha) \right) \\ &= C_{L_{\alpha_T}} (h - h_{NP}) \end{aligned}$$

which gives the pitching moment about the c.g. as a function of the location of the c.g. with respect to the stick fixed neutral point

- For static stability, c.g. must be in front of NP ($h_{NP} > h$)

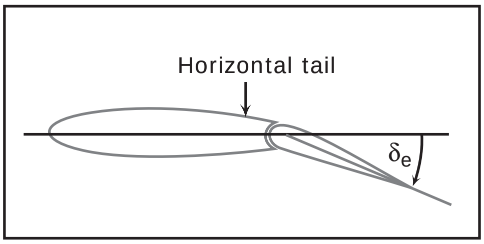
- Summary plot: $(a = C_{L_{\alpha_T}})$ and $(h_n = h_{NP})$



- Observations:
  - If c.g. at h_{NP}, then $C_{M_\alpha} = 0$
  - If c.g. aft of h_{NP}, then $C_{M_\alpha} > 0$ (statically unstable)
  - What is the problem with the c.g. being too far forward?

Control Effects

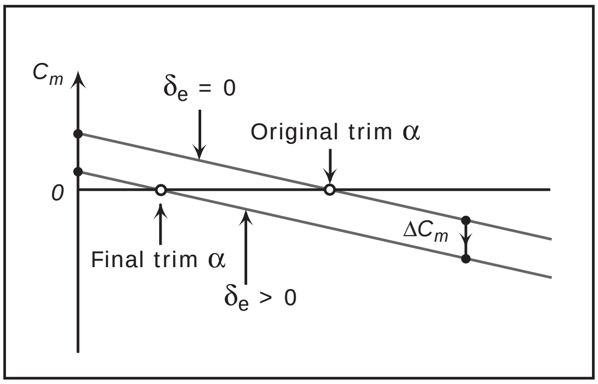
- Can use elevators to provide incremental lift and moments
  - Use this to trim aircraft at different flight settings, i.e. $C_M = 0$



- Deflecting elevator gives:

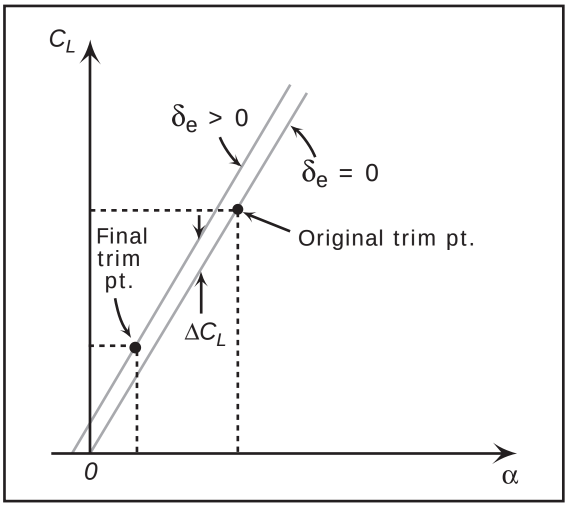
$$\Delta C_L = \frac{dC_L}{d\delta_e}\delta_e = C_{L_{\delta_e}}\delta_e, \text{ with } C_{L_{\delta_e}} > 0$$

$$\Rightarrow C_L = C_{L_\alpha}(\alpha - \alpha_0) + C_{L_{\delta_e}}\delta_e$$



$$\Delta C_m = \frac{dC_m}{d\delta_e}\delta_e = C_{m_{\delta_e}}\delta_e, \text{ with } C_{m_{\delta_e}} < 0$$

$$\Rightarrow C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta_e}}\delta_e$$



- For trim, need $C_m = 0$ and $C_{L_{\text{TRIM}}}$

$$\begin{bmatrix} C_{m_\alpha} & C_{m_{\delta_e}} \\ C_{L_\alpha} & C_{L_{\delta_e}} \end{bmatrix} \begin{bmatrix} \alpha_{TRIM} \\ (\delta_e)_{\text{TRIM}} \end{bmatrix} = \begin{bmatrix} -C_{m_0} \\ C_{L_{\text{TRIM}}} + C_{L_\alpha}\alpha_0 \end{bmatrix}$$

So elevator angle needed to trim:

$$(\delta_e)_{\text{TRIM}} = \frac{C_{L_\alpha}C_{m_0} + C_{m_\alpha}(C_{L_{\text{TRIM}}} + C_{L_\alpha}\alpha_0)}{C_{m_\alpha}C_{L_{\delta_e}} - C_{L_\alpha}C_{m_{\delta_e}}}$$

- Note that typically elevator down is taken as being positive
- Also, for level flight, $C_{L_{\text{TRIM}}} = W/(1/2\rho V^2 S)$, so expect $(\delta_e)_{\text{TRIM}}$ to change with speed.