16.323 Principles of Optimal Control
Spring 2008

16.323, #15

March 16th, 2006

16.323 Midterm #1

This is a closed-book exam, but you are allowed 1 page of notes (both sides). You have 1.5 hours. There are three **3** questions.

Hint: To maximize your score, clearly explain your approach before getting too bogged down in the equations.

1. The constrained minimum of the function

$$F(x,y) = x^3 + x^2y + xy^2 + y^3 - 10x^2 - 20xy + 105y^2 + 30x - 78y - 50$$

subject to the constraint $f(x,y) = 30y^2 - xy - 20y - x = 0$ is located at $x = 5$, $y = 1$. Estimate to first order the change in the value of F at the solution point if the constraint is changed from $f(x,y) = 0$ to $f(x,y) = 1$.

2. Consider the discrete-time LQR control problem for the system $x_{k+1} = 2x_k + u_k$

$$\min J = \frac{1}{4}x_N^2 + \frac{1}{2}\sum_{k=0}^{N-1}\left[3x_k^2 + u_k^2\right]$$

   (a) Is this system detectable in the cost and stabilizable?

   (b) What is the optimal feedback gain at $k = N - 1$

   (c) What is the steady state feedback gain for this discrete problem. Where is the closed-loop system pole when you use this steady state gain? Is it stable? Do these answers meet with your expectations given the result in part (a)?

3. Solve the following optimal control problem by dynamic programming on the indicated grid of values, and summarize the optimal control and state sequences.

$$\min J = x_3^2 + \sum_{k=0}^{2}[\ |x_k| + 2u_k^2]$$

subject to $x_{k+1} = x_k + u_k$ and $x_0 = 3$

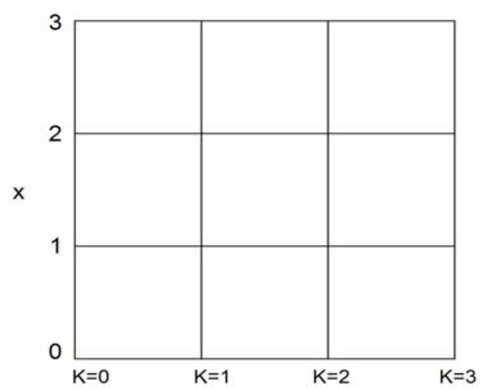

Figure 1: If you use this grid as part of your answer, make sure you put your name on this sheet and hand it in.