Lecture 15

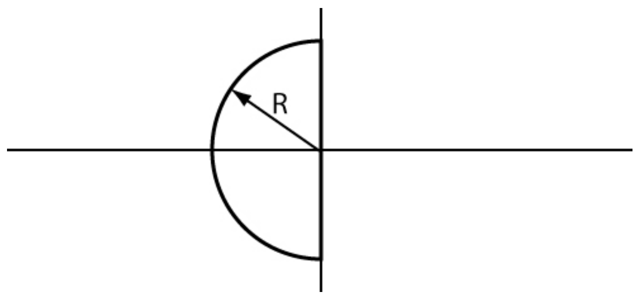
Last time: Compute the spectrum and integrate to get the mean squared value

$$\overline{y^2} = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} F(s)F(-s)S_{xx}(s)ds$$

Cauchy-Residue Theorem

$$\oint F(s)ds = 2\pi j \sum (\text{residue at enclosed poles})$$

Note that in the case of repeated roots of the denominator, a <u>pole of multiple order</u> contributes only a <u>single residue</u>.



To evaluate $\int_{-j\infty}^{j\infty} F(s)ds$ by integrating around a closed contour enclosing the entire left half plane, note that if $F(s) \to 0$ faster than $\frac{1}{s}$ for large s, the integral along the curved part of the contour is zero.

If $F(s) \sim \frac{k}{s^n}$ as $|s| \to \infty$, $\oint_{\text{semi-circle}} F(s)ds \le \frac{k}{R^n}\pi R = k\pi R^{-(n-1)} \to 0$ as $R \to \infty$ if $n > 1$

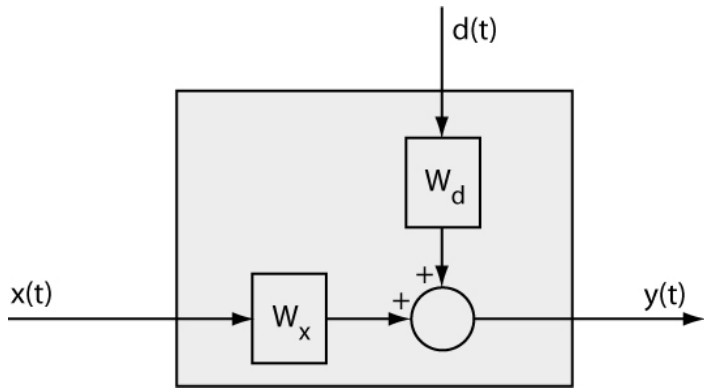
Integral tables

Applicable to rational functions; no predictor or smoother. Must factor the spectrum of the input into the following form.

$$I_n = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \frac{c(s)c(-s)}{d(s)d(-s)} ds$$

Refer to the handout "Tabulated Values of the Integral Form".
Roots of $c(s)$ and $d(s)$ in <u>left half plane</u> only. Should check the stability of the solution.

Application to the problem of System Identification



Record $x(t)$ and $y(t)$ and process that data.

$$y(t) = \int_0^\infty w_x(\tau_1)x(t-\tau_1)d\tau_1 + \int_0^\infty w_d(\tau_1)d(t-\tau_1)d\tau_1$$

$$R_{xy}(\tau) = E\left[x(t)y(t+\tau)\right] = \overline{x(t)y(t+\tau)}$$

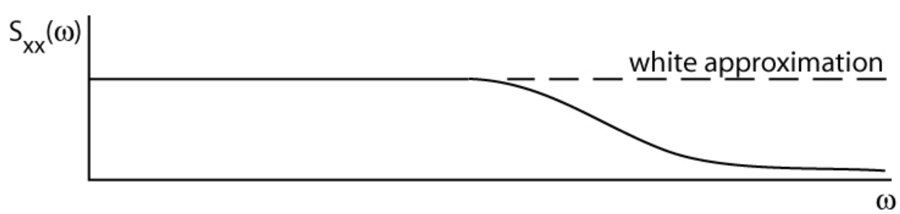
$$= \int_0^\infty w_x(\tau_1)\overline{x(t)y(t+\tau-\tau_1)}d\tau_1 + \int_0^\infty w_d(\tau_1)\overline{x(t)d(t+\tau-\tau_1)}d\tau_1$$

If $x(t)$, $d(t)$ are independent and at least $x(t)$ is zero mean, then $R_{xd}(\tau) = 0$.

$$R_{xy}(\tau) = \int_0^\infty w_x(\tau_1)R_{xx}(\tau-\tau_1)d\tau_1$$

If $x(t)$ is wide band relative to the system, approximate it as white.

$$R_{xx}(\tau) = S_x\delta(\tau)$$



$$R_{xy}(\tau) = \int_0^\infty w_x(\tau_1)S_x\delta(\tau-\tau_1)d\tau_1$$

$$= S_x w_x(\tau)$$