APPENDIX E

TABLE OF RANDOM-INPUT DESCRIBING FUNCTIONS (RIDFs)

In this table we employ the probability function (cf. Sec. 7.2) denoted by

$$PF(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$$

and its integral, the probability integral, denoted by

$$PI(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{v^2}{2}\right) dv$$

These functions are plotted in Fig. E.2-1.

This table is given in three sections:

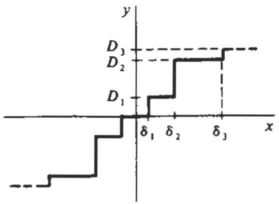
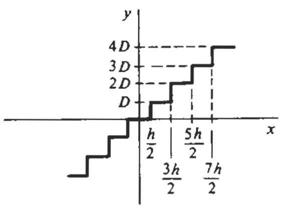
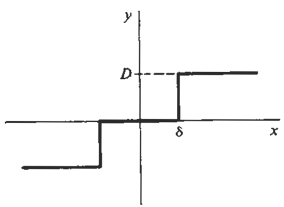
E.1 Gaussian-input RIDFs
E.2 Gaussian-plus-bias-input RIDFs
E.3 Gaussian-plus-bias-plus-sinusoid-input RIDFs

E.1 GAUSSIAN-INPUT RIDFs

$$x(t) = r(t) \qquad \text{an unbiased Gaussian process}$$

$$N_R(\sigma) = \frac{1}{\sqrt{2\pi}\sigma^3} \int_{-\infty}^{\infty} y(r) r \exp\left(-\frac{r^2}{2\sigma^2}\right) dr$$

TABLE OF RANDOM-INPUT DESCRIBING FUNCTIONS (RIDFs) *(Continued)*

Nonlinearity	Comments	$N_R(\sigma)$
 1. General odd quantizer	$D_0 = 0$ N is index of last quantizer level	$\frac{2}{\sigma}\sum_{i=1}^{N}(D_i - D_{i-1})PF\left(\frac{\delta_i}{\sigma}\right)$
 2. Uniform quantizer	N is index of last quantizer level See Fig. E.1-1	$\frac{2D}{\sigma}\sum_{i=1}^{N} PF\left(\frac{2i-1}{2}\frac{h}{\sigma}\right)$
 3. Relay with dead zone	See Fig. E.1-1	$2\frac{D}{\sigma}PF\left(\frac{\delta}{\sigma}\right)$

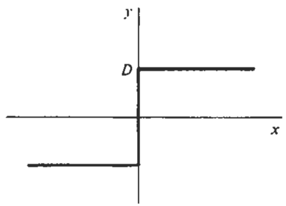
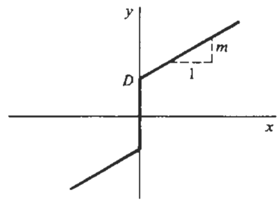
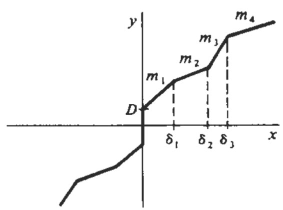
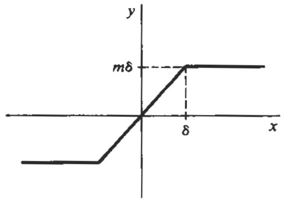
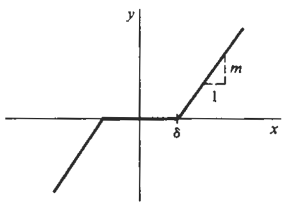
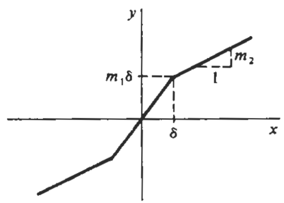
 4. Ideal relay		$\sqrt{\frac{2}{\pi}}\frac{D}{\sigma}$
 5. Preload		$\sqrt{\frac{2}{\pi}}\frac{D}{\sigma} + m$
 6. General piecewise-linear odd memoryless nonlinearity	N is index of last linear segment	$\sqrt{\frac{2}{\pi}}\frac{D}{\sigma} + 2\sum_{i=1}^{N-1}(m_i - m_{i+1})PI\left(\frac{\delta_i}{\sigma}\right) + 2m_N - m_1$

TABLE OF RANDOM-INPUT DESCRIBING FUNCTIONS (RIDFs) *(Continued)*

Nonlinearity	Comments	$N_R(\sigma)$
 7. Sharp saturation or limiter	See Fig. E.1-2	$m\left[2PI\left(\frac{\delta}{\sigma}\right) - 1\right]$
 8. Dead zone or threshold	See Fig. E.1-2	$2m\left[1 - PI\left(\frac{\delta}{\sigma}\right)\right]$
 9. Gain-changing nonlinearity		$m_1 + 2(m_2 - m_1)\left[1 - PI\left(\frac{\delta}{\sigma}\right)\right]$

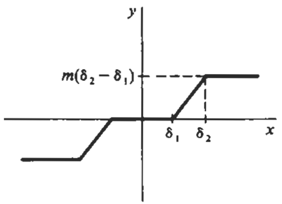
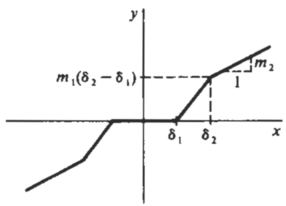
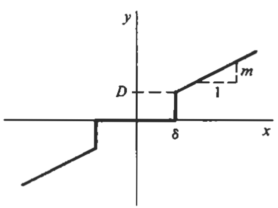
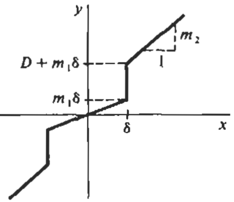
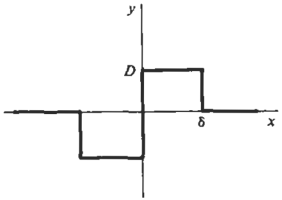
 10. Limiter with dead zone		$2m\left[PI\left(\frac{\delta_2}{\sigma}\right) - PI\left(\frac{\delta_1}{\sigma}\right)\right]$
 11. Gain-changing nonlinearity with dead zone		$2(m_1 - m_2)PI\left(\frac{\delta_2}{\sigma}\right) - 2m_1PI\left(\frac{\delta_1}{\sigma}\right) + 2m_2$
 12.		$2m\left[1 - PI\left(\frac{\delta}{\sigma}\right)\right] + 2\frac{D}{\sigma}PF\left(\frac{\delta}{\sigma}\right)$

TABLE OF RANDOM-INPUT DESCRIBING FUNCTIONS (RIDFs) *(Continued)*

Nonlinearity	Comments	$N_R(\sigma)$
 13.		$m_1 + 2(m_2 - m_1)\left[1 - PI\left(\frac{\delta}{\sigma}\right)\right] + 2\frac{D}{\sigma} PF\left(\frac{\delta}{\sigma}\right)$
 14.		$\sqrt{\frac{2}{\pi}}\frac{D}{\sigma}\left[1 - \sqrt{2\pi}\, PF\left(\frac{\delta}{\sigma}\right)\right]$
$y = c$ 15.		0
$y = x$ 16. Linear gain		1

$y = x\lvert x\rvert$ 17. Odd square law	See Fig. E.1-3	$\sqrt{\frac{2}{\pi}}\,2\sigma$
$y = x^3$ 18. Cubic characteristic	See Fig. E.1-3	$3\sigma^2$
$y = x^3\lvert x\rvert$ 19. Odd quartic characteristic	See Fig. E.1-3	$\sqrt{\frac{2}{\pi}}\,8\sigma^3$
$y = x^5$ 20. Quintic characteristic		$15\sigma^4$
$y = x^5\lvert x\rvert$ 21.		$\sqrt{\frac{2}{\pi}}\,48\sigma^5$
$y = x^7$ 22.		$105\sigma^6$

TABLE OF RANDOM-INPUT DESCRIBING FUNCTIONS (RIDFs) *(Continued)*

Nonlinearity	Comments	$N_R(\sigma)$
$y = x^7 \lvert x \rvert$ 23.		$\sqrt{\frac{2}{\pi}}\, 384\sigma^7$
$y = x^n$ 24.	$n = 3, 5, 7, \ldots$ See Sec. 7.2	$n(n-2)(n-4)\cdots(1)\sigma^{n-1}$
$y = x^{n-1} \lvert x \rvert$ 25.	$n = 2, 4, 6, \ldots$ See Sec. 7.2	$\sqrt{\frac{2}{\pi}}\, n(n-2)(n-4)\cdots(2)\sigma^{n-1}$
$y = \sqrt{x} \quad (x \geq 0)$ $= -\sqrt{-x} \quad (x < 0)$ 26. Odd square root	See Fig. E.1-3	$0.860\sigma^{-1/2}$
$y = x^{1/3}$ 27. Cube root characteristic		$0.830\sigma^{-2/3}$
$y = x^b \quad (x \geq 0)$ $= -(-x)^b \quad (x < 0)$ 28.	$\Gamma(x)$ is gamma function	$\sqrt{\frac{2}{\pi}}\, 2^{b/2}\Gamma\left(1 + \frac{b}{2}\right)\sigma^{b-1}$

$y = M \sin mx$ 29. Harmonic Nonlinearity	See Fig. E.1-4	$\sqrt{2\pi}\, MmPF(m\sigma) = Mme^{-m^2\sigma^2/2}$
$y = M \sinh mx$ 30.		$Mme^{m^2\sigma^2/2}$
$y = 1 - e^{-cx} \quad (x \geq 0)$ $\quad = -(1 - e^{cx}) \quad (x < 0)$ 31. Exponential saturation		$2ce^{c^2\sigma^2/2}[1 - PI(c\sigma)]$

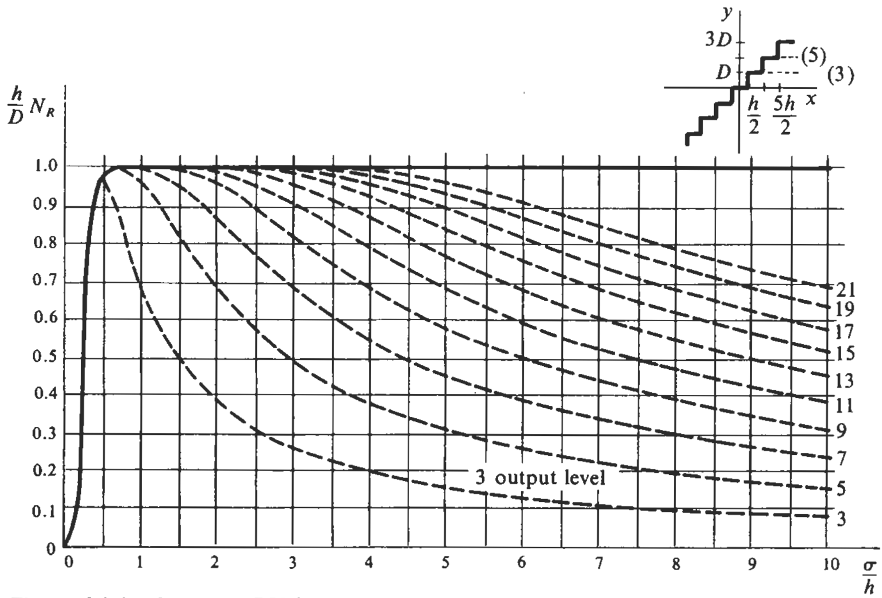


Figure E.1-1 *Quantizer RIDF.*

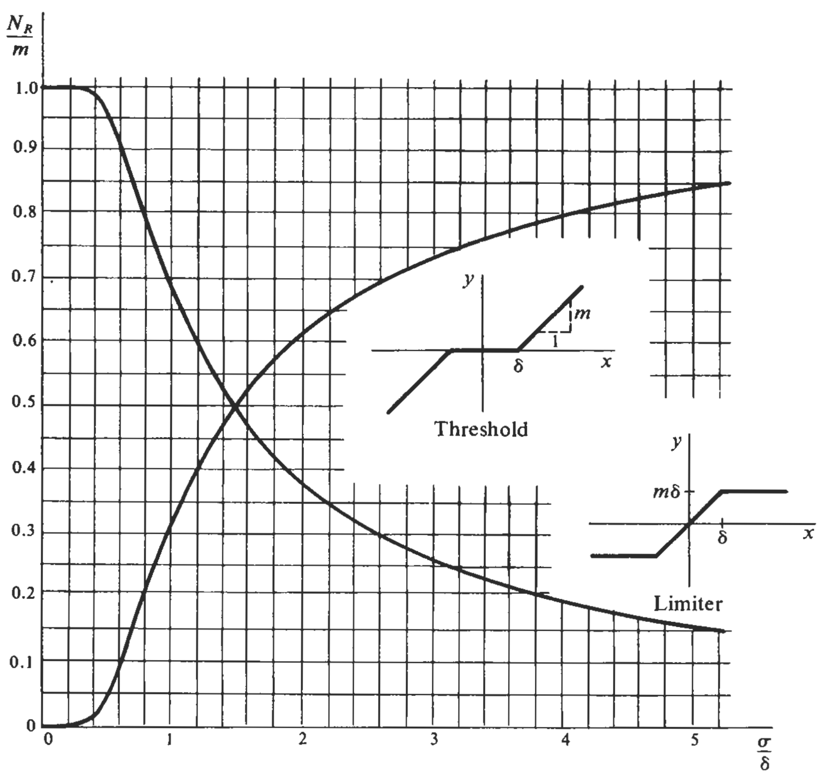


Figure E.1-2 *RIDFs for limiter and threshold characteristics.*

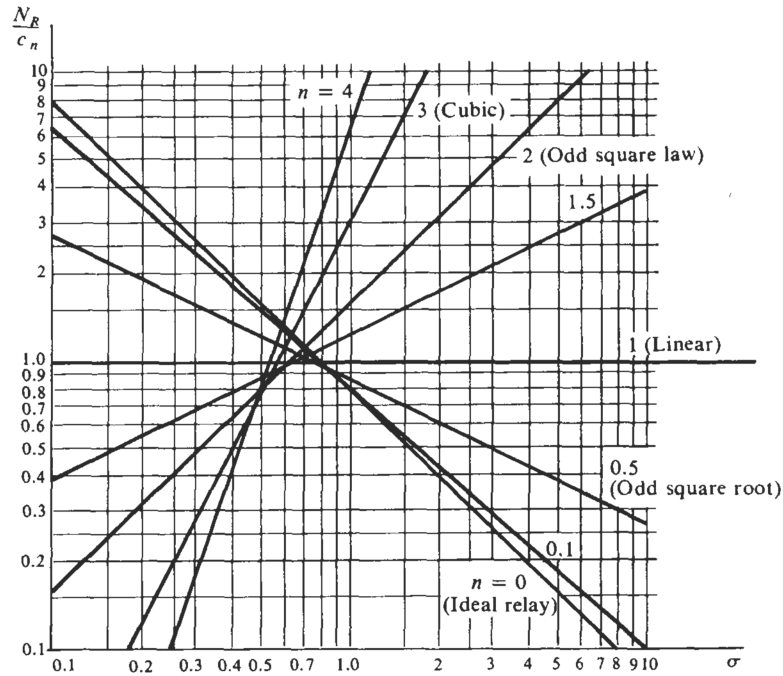


Figure E.1-3 *RIDF for the simple polynomial nonlinearity* $y = c_n x^n$ *(n odd) or* $y = c_n x^{n-1}|x|$ *(n even).*

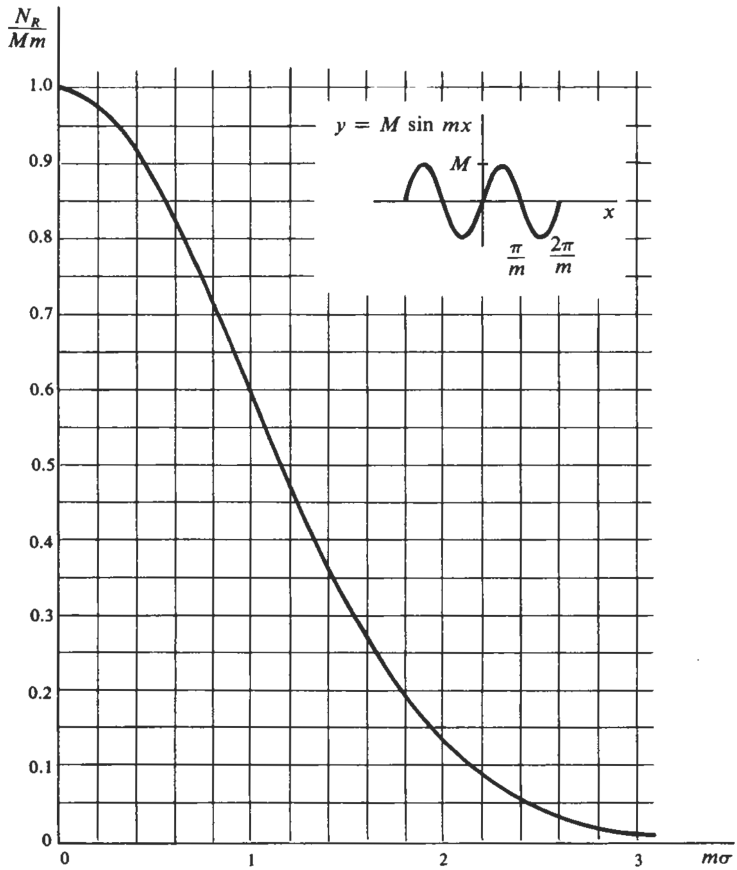


Figure E.1-4 *Harmonic nonlinearity RIDF.*

E.2 GAUSSIAN-PLUS-BIAS-INPUT RIDFs

$$x(t) = r(t) + B$$

The gain to the gaussian input component is given by:

$$N_R(\sigma,B) = \frac{1}{\sqrt{2\pi}\sigma^3}\int_{-\infty}^{\infty} y(r + B)r \exp\left(-\frac{r^2}{2\sigma^2}\right) dr$$

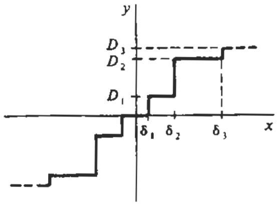
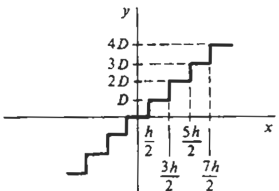
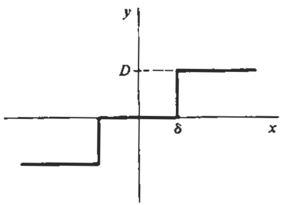
and the corresponding gain to the bias input component is:

$$N_B(\sigma,B) = \frac{1}{\sqrt{2\pi}\sigma B}\int_{-\infty}^{\infty} y(r + B) \exp\left(-\frac{r^2}{2\sigma^2}\right) dr$$

This section uses the additional function $G(x) = xPI(x) + PF(x)$.

The functions $PF(x)$, $PI(x)$, and $G(x)$ are plotted in Fig. E.2-1.

TABLE OF RANDOM-INPUT DESCRIBING FUNCTIONS (RIDFs) *(Continued)*

Nonlinearity	Comments	$N_R(\sigma,B)$ and $N_B(\sigma,B)$
 1. General odd quantizer	$D_0 = 0$ N is index of last quantizer level.	$N_R = \frac{1}{\sigma}\sum_{i=1}^{N}(D_i - D_{i-1})\left[PF\left(\frac{\delta_i + B}{\sigma}\right) + PF\left(\frac{\delta_i - B}{\sigma}\right)\right]$ $N_B = \frac{1}{B}\sum_{i=1}^{N}(D_i - D_{i-1})\left[PI\left(\frac{\delta_i + B}{\sigma}\right) - PI\left(\frac{\delta_i - B}{\sigma}\right)\right]$
 2. Uniform quantizer	N is index of last quantizer level.	$N_R = \frac{D}{\sigma}\sum_{i=1}^{N}\left[PF\left(\frac{2i-1}{2}\frac{h}{\sigma} + \frac{B}{\sigma}\right) + PF\left(\frac{2i-1}{2}\frac{h}{\sigma} - \frac{B}{\sigma}\right)\right]$ $N_B = \frac{D}{B}\sum_{i=1}^{N}\left[PI\left(\frac{2i-1}{2}\frac{h}{\sigma} + \frac{B}{\sigma}\right) - PI\left(\frac{2i-1}{2}\frac{h}{\sigma} - \frac{B}{\sigma}\right)\right]$
 3. Relay with dead zone		$N_R = \frac{D}{\sigma}\left[PF\left(\frac{\delta + B}{\sigma}\right) + PF\left(\frac{\delta - B}{\sigma}\right)\right]$ $N_B = \frac{D}{B}\left[PI\left(\frac{\delta + B}{\sigma}\right) - PI\left(\frac{\delta - B}{\sigma}\right)\right]$

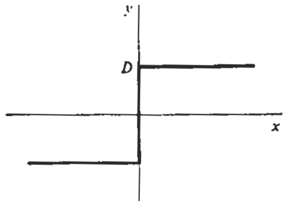
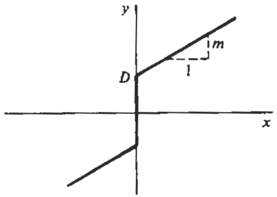
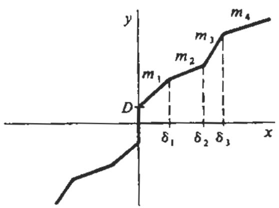
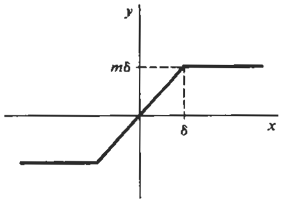
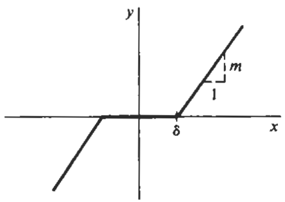
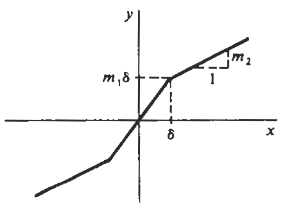
 4. Ideal relay		$N_R = 2\frac{D}{\sigma}PF\left(\frac{B}{\sigma}\right)$ $N_B = \frac{D}{B}\left[2PI\left(\frac{B}{\sigma}\right) - 1\right]$
 5. Preload		$N_R = 2\frac{D}{\sigma}PF\left(\frac{B}{\sigma}\right) + m$ $N_B = \frac{D}{B}\left[2PI\left(\frac{B}{\sigma}\right) - 1\right] + m$
 6. General piecewise-linear odd memoryless nonlinearity	N is index of last linear segment.	$N_R = 2\frac{D}{\sigma}PF\left(\frac{B}{\sigma}\right) + 2m_N - m_1 + \sum_{i=1}^{N-1}(m_i - m_{i+1})\left[PI\left(\frac{\delta_i + B}{\sigma}\right) + PI\left(\frac{\delta_i - B}{\sigma}\right)\right]$ $N_B = \frac{D}{B}\left[2PI\left(\frac{B}{\sigma}\right) - 1\right] + 2m_N - m_1 + \frac{\sigma}{B}\sum_{i=1}^{N-1}(m_i - m_{i+1})\left[G\left(\frac{\delta_i + B}{\sigma}\right) - G\left(\frac{\delta_i - B}{\sigma}\right)\right]$

TABLE OF RANDOM-INPUT DESCRIBING FUNCTIONS (RIDFs) *(Continued)*

<table>
<tr><th>Nonlinearity</th><th>Comments</th><th>$N_R(\sigma,B)$ and $N_B(\sigma,B)$</th></tr>
<tr><td>

7. Sharp saturation or limiter</td><td></td><td>$$N_R = m\left[PI\left(\frac{\delta + B}{\sigma}\right) + PI\left(\frac{\delta - B}{\sigma}\right) - 1\right]$$
$$N_B = m\left\{\frac{\sigma}{B}\left[G\left(\frac{\delta + B}{\sigma}\right) - G\left(\frac{\delta - B}{\sigma}\right)\right] - 1\right\}$$</td></tr>
<tr><td>

8. Dead zone or threshold</td><td></td><td>$$N_R = m\left[2 - PI\left(\frac{\delta + B}{\sigma}\right) - PI\left(\frac{\delta - B}{\sigma}\right)\right]$$
$$N_B = m\left\{2 - \frac{\sigma}{B}\left[G\left(\frac{\delta + B}{\sigma}\right) - G\left(\frac{\delta - B}{\sigma}\right)\right]\right\}$$</td></tr>
<tr><td>

9. Gain-changing nonlinearity</td><td></td><td>$$N_R = m_1 + (m_2 - m_1)\left[2 - PI\left(\frac{\delta + B}{\sigma}\right) - PI\left(\frac{\delta - B}{\sigma}\right)\right]$$
$$N_B = m_1 + (m_2 - m_1)\left\{2 - \frac{\sigma}{B}\left[G\left(\frac{\delta + B}{\sigma}\right) - G\left(\frac{\delta - B}{\sigma}\right)\right]\right\}$$</td></tr>
</table>

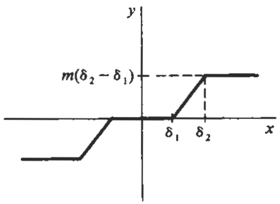
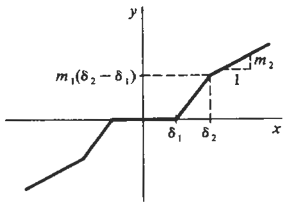
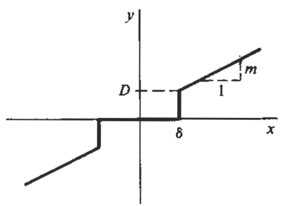
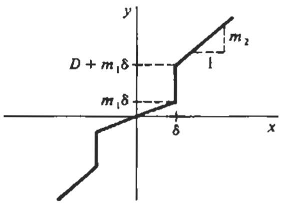
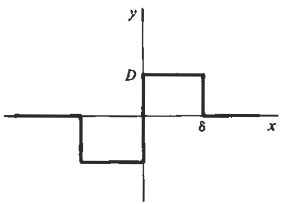
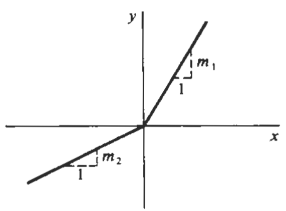
 10. Limiter with dead zone		$N_R = m\left[PI\left(\frac{\delta_2+B}{\sigma}\right) + PI\left(\frac{\delta_2-B}{\sigma}\right) - PI\left(\frac{\delta_1+B}{\sigma}\right) - PI\left(\frac{\delta_1-B}{\sigma}\right)\right]$ $N_B = m\frac{\sigma}{B}\left[G\left(\frac{\delta_2+B}{\sigma}\right) - G\left(\frac{\delta_2-B}{\sigma}\right) - G\left(\frac{\delta_1+B}{\sigma}\right) + G\left(\frac{\delta_1-B}{\sigma}\right)\right]$
 11. Gain-changing nonlinearity with dead zone		$N_R = 2m_2 - m_1\left[PI\left(\frac{\delta_1+B}{\sigma}\right) + PI\left(\frac{\delta_1-B}{\sigma}\right)\right] +$ $(m_1-m_2)\left[PI\left(\frac{\delta_2+B}{\sigma}\right) + PI\left(\frac{\delta_2-B}{\sigma}\right)\right]$ $N_B = 2m_2 + \frac{\sigma}{B}\left\{-m_1\left[G\left(\frac{\delta_1+B}{\sigma}\right) - G\left(\frac{\delta_1-B}{\sigma}\right)\right] +\right.$ $\left.(m_1-m_2)\left[G\left(\frac{\delta_2+B}{\sigma}\right) - G\left(\frac{\delta_2-B}{\sigma}\right)\right]\right\}$
 12.		$N_R = m\left[2 - PI\left(\frac{\delta+B}{\sigma}\right) - PI\left(\frac{\delta-B}{\sigma}\right)\right] + \frac{D}{\sigma}\left[PF\left(\frac{\delta+B}{\sigma}\right) + PF\left(\frac{\delta-B}{\sigma}\right)\right]$ $N_B = m\left\{2 - \frac{\sigma}{B}\left[G\left(\frac{\delta+B}{\sigma}\right) - G\left(\frac{\delta-B}{\sigma}\right)\right]\right\} + \frac{D}{B}\left[PI\left(\frac{\delta+B}{\sigma}\right) - PI\left(\frac{\delta-B}{\sigma}\right)\right]$

TABLE OF RANDOM-INPUT DESCRIBING FUNCTIONS (RIDFs) *(Continued)*

Nonlinearity	Comments	$N_R(\sigma,B)$ and $N_B(\sigma,B)$
 13.		$N_R = m_1 + \frac{D}{\sigma}\left[PF\left(\frac{\delta + B}{\sigma}\right) + PF\left(\frac{\delta - B}{\sigma}\right)\right] + (m_2 - m_1)\left[2 - PI\left(\frac{\delta + B}{\sigma}\right) - PI\left(\frac{\delta - B}{\sigma}\right)\right]$ $N_B = m_1 + \frac{D}{B}\left[PI\left(\frac{\delta + B}{\sigma}\right) - PI\left(\frac{\delta - B}{\sigma}\right)\right] + (m_2 - m_1)\left\{2 - \frac{\sigma}{B}\left[G\left(\frac{\delta + B}{\sigma}\right) - G\left(\frac{\delta - B}{\sigma}\right)\right]\right\}$
 14.		$N_R = \frac{D}{\sigma}\left[2PF\left(\frac{B}{\sigma}\right) - PF\left(\frac{\delta + B}{\sigma}\right) - PF\left(\frac{\delta - B}{\sigma}\right)\right]$ $N_B = \frac{D}{B}\left[2PI\left(\frac{B}{\sigma}\right) - PI\left(\frac{\delta + B}{\sigma}\right) + PI\left(\frac{\delta - B}{\sigma}\right) - 1\right]$
$y = c$ 15.		$N_R = 0$ $N_B = \frac{c}{B}$
$y = x$ 16. Linear gain		$N_R = 1$ $N_B = 1$

$y = x\|x\|$ 17. Odd-square law		$N_R = 4\sigma PF\left(\frac{B}{\sigma}\right) + 2B\left[2PI\left(\frac{B}{\sigma}\right) - 1\right]$ $N_B = 2\sigma PF\left(\frac{B}{\sigma}\right) + B\left[1 + \left(\frac{\sigma}{B}\right)^2\right]\left[2PI\left(\frac{B}{\sigma}\right) - 1\right]$
$y = x^3$ 18. Cubic characteristic		$N_R = 3\sigma^2 + 3B^2$ $N_B = 3\sigma^2 + B^2$
$y = x^5$ 20. Quintic characteristic		$N_R = 15\sigma^4 + 30\sigma^2 B^2 + 5B^4$ $N_B = 15\sigma^4 + 10\sigma^2 B^2 + B^4$
$y = x^7$ 22.		$N_R = 105\sigma^6 + 315\sigma^4 B^2 + 105\sigma^2 B^4 + 7B^6$ $N_B = 105\sigma^6 + 105\sigma^4 B^2 + 21\sigma^2 B^4 + B^6$
$y = x^n$ 24.	$n = 3, 5, 7, \ldots$ See Sec. 7.2	$N_R = \sum_{k(\text{even})=0}^{n-1} \frac{n!}{k!\,(n-k)!}\sigma^{n-k-1}B^k(1)(3)\cdots(n-k)$ $N_B = B^{n-1} + \sum_{k(\text{odd})=1}^{n-2} \frac{n!}{k!\,(n-k)!}\sigma^{n-k}B^{k-1}(1)(3)\cdots(n-k-1)$

Nonlinearity	**Comments**	$N_R(\sigma,B)$ **and** $N_B(\sigma,B)$
$y = M \sin mx$ 29. Harmonic nonlinearity		$N_R = Mm \cos mB\, e^{-m^2\sigma^2/2}$ $N_B = \frac{M}{B} \sin mB\, e^{-m^2\sigma^2/2}$
$y = M \sinh mx$ 30.		$N_R = Mm \cosh mB\, e^{m^2\sigma^2/2}$ $N_B = \frac{M}{B} \sinh mB\, e^{m^2\sigma^2/2}$
$y = 1 - e^{-cx} \quad (x \geq 0)$ $\quad = -(1 - e^{cx}) \quad (x < 0)$ 31. Exponential saturation		$N_R = \frac{2}{\sigma} PF\left(\frac{B}{\sigma}\right) + \frac{1}{\sigma} e^{c^2\sigma^2/2} \left\{ e^{cB} \left[c\sigma - c\sigma PI\left(c\sigma + \frac{B}{\sigma}\right) - PF\left(c\sigma + \frac{B}{\sigma}\right) \right] + e^{-cB} \left[c\sigma - c\sigma PI\left(c\sigma - \frac{B}{\sigma}\right) - PF\left(c\sigma - \frac{B}{\sigma}\right) \right] \right\}$ $N_B = \frac{1}{B}\left[2PI\left(\frac{B}{\sigma}\right) - 1\right] + \frac{1}{B} e^{c^2\sigma^2/2} \left\{ e^{cB}\left[1 - PI\left(c\sigma + \frac{B}{\sigma}\right)\right] - e^{-cB}\left[1 - PI\left(c\sigma - \frac{B}{\sigma}\right)\right] \right\}$
 51.		$N_R = m_2 + (m_1 - m_2) PI\left(\frac{B}{\sigma}\right)$ $N_B = m_2 + \frac{\sigma}{B}(m_1 - m_2) G\left(\frac{B}{\sigma}\right)$

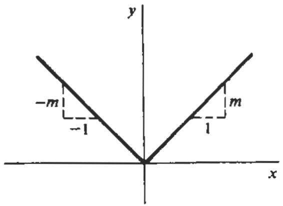
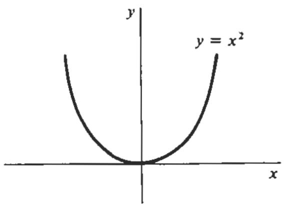
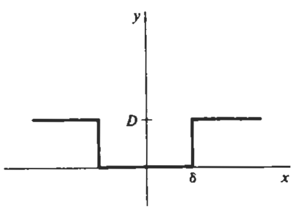
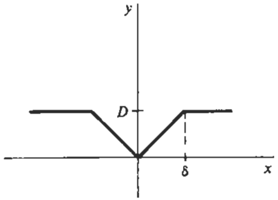
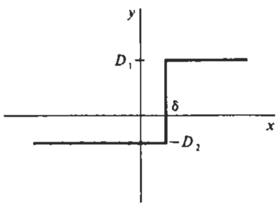
 52. Absolute value		$N_R = m\left[2PI\left(\frac{B}{\sigma}\right) - 1\right]$ $N_B = m\left[2\,\frac{\sigma}{B}\,G\left(\frac{B}{\sigma}\right) - 1\right]$
 53. Square-law		$N_R = 2B$ $N_B = \frac{1}{B}\,[\sigma^2 + B^2]$
 54.		$N_R = \frac{D}{\sigma}\left[-PF\left(\frac{\delta + B}{\sigma}\right) + PF\left(\frac{\delta - B}{\sigma}\right)\right]$ $N_B = \frac{D}{B}\left[2 - PI\left(\frac{\delta + B}{\sigma}\right) - PI\left(\frac{\delta - B}{\sigma}\right)\right]$

TABLE OF RANDOM-INPUT DESCRIBING FUNCTIONS (RIDFs) *(Continued)*

Nonlinearity	Comments	$N_R(\sigma,B)$ and $N_B(\sigma,B)$
 55.		$N_R = \frac{D}{\delta}\left[2PI\left(\frac{B}{\sigma}\right) - 1 - PI\left(\frac{\delta + B}{\sigma}\right) + PI\left(\frac{\delta - B}{\sigma}\right)\right]$ $N_B = \frac{D}{B}\left\{2 - \frac{B}{\delta} + \frac{\sigma}{\delta}\left[2G\left(\frac{B}{\sigma}\right) - G\left(\frac{\delta + B}{\sigma}\right) - G\left(\frac{\delta - B}{\sigma}\right)\right]\right\}$
 56. Biased ideal relay		$N_R = \frac{D_1 + D_2}{\sigma} PF\left(\frac{\delta - B}{\sigma}\right)$ $N_B = \frac{D_1}{B} - \frac{D_1 + D_2}{B} PI\left(\frac{\delta - B}{\sigma}\right)$

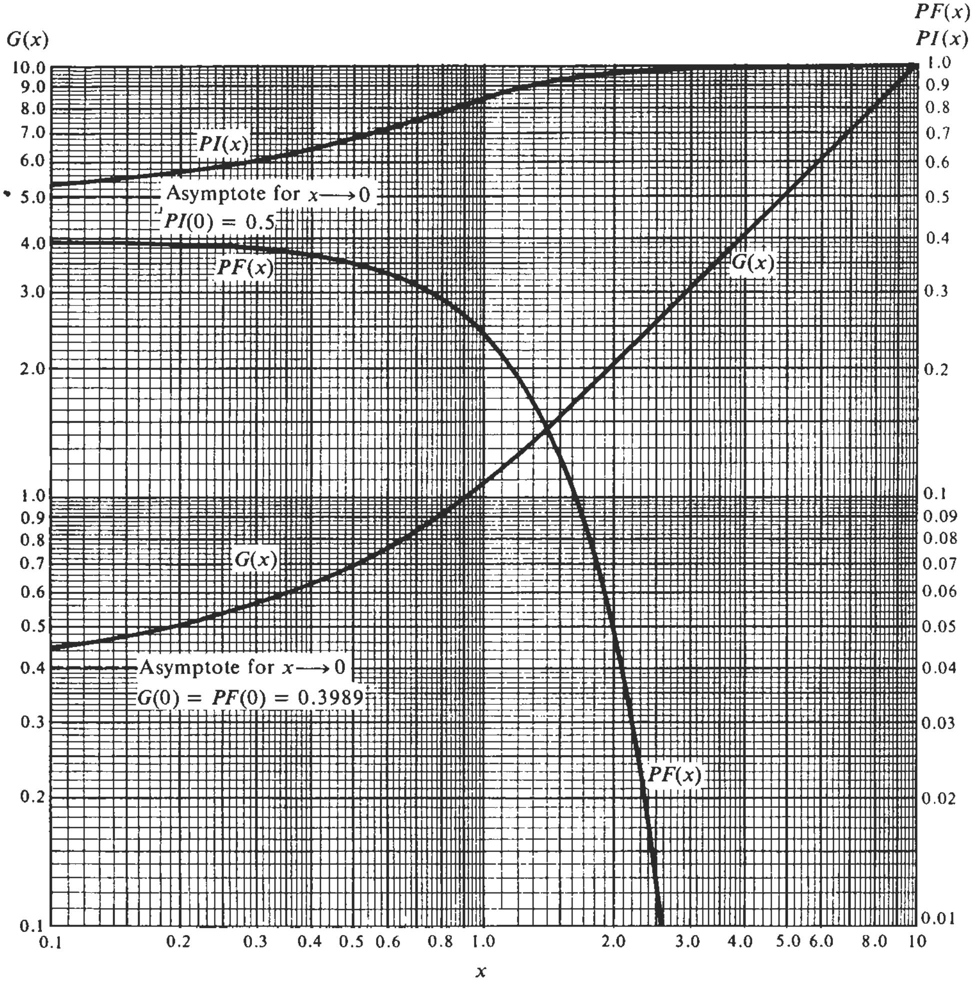


Figure E.2-1 *Graphs of PF(x), PI(x), and G(x).*

E.3 GAUSSIAN-PLUS-BIAS-PLUS-SINUSOID-INPUT RIDFs

$$x(t) = r(t) + B + A \sin (\omega t + \theta)$$

The gain to the gaussian input component is given by

$$N_R(\sigma,B,A) = \frac{1}{(2\pi)^{\frac{3}{2}}\sigma^3} \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dr\, y(r + B + A \sin \theta)\, r \exp \left(-\frac{r^2}{2\sigma^2}\right)$$

the gain to the bias input component is

$$N_B(\sigma,B,A) = \frac{1}{(2\pi)^{\frac{3}{2}}\sigma B} \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dr\, y(r + B + A \sin \theta) \exp \left(-\frac{r^2}{2\sigma^2}\right)$$

and the corresponding gain to the sinusoid input component is

$$N_A(\sigma,B,A) = \frac{2}{(2\pi)^{\frac{3}{2}}\sigma A} \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dr\, y(r + B + A \sin \theta) \sin \theta \exp \left(-\frac{r^2}{2\sigma^2}\right)$$

TABLE OF RANDOM-INPUT DESCRIBING FUNCTIONS (RIDFs) *(Continued)*

Nonlinearity	Comments	$N_R(\sigma,B,A)$, $N_B(\sigma,B,A)$, and $N_A(\sigma,B,A)$
$y = x^3$ 18. Cubic characteristic		$N_R = 3\sigma^2 + 3B^2 + \frac{3}{2}A^2$ $N_B = 3\sigma^2 + B^2 + \frac{3}{2}A^2$ $N_A = 3\sigma^2 + 3B^2 + \frac{3}{4}A^2$
$y = M \sin mx$ 29. Harmonic nonlinearity	J_0 and J_1 are the Bessel functions of orders 0 and 1, respectively.	$N_R = Mm \cos mB \exp\left(-\frac{m^2\sigma^2}{2}\right) J_0(mA)$ $N_B = \frac{M}{B} \sin mB \exp\left(-\frac{m^2\sigma^2}{2}\right) J_0(mA)$ $N_A = \frac{2M}{A} \cos mB \exp\left(-\frac{m^2\sigma^2}{2}\right) J_1(mA)$

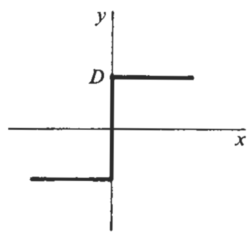


Figure E.3-1 Three-input RIDFs for the ideal-relay nonlinearity.

In 3 parts

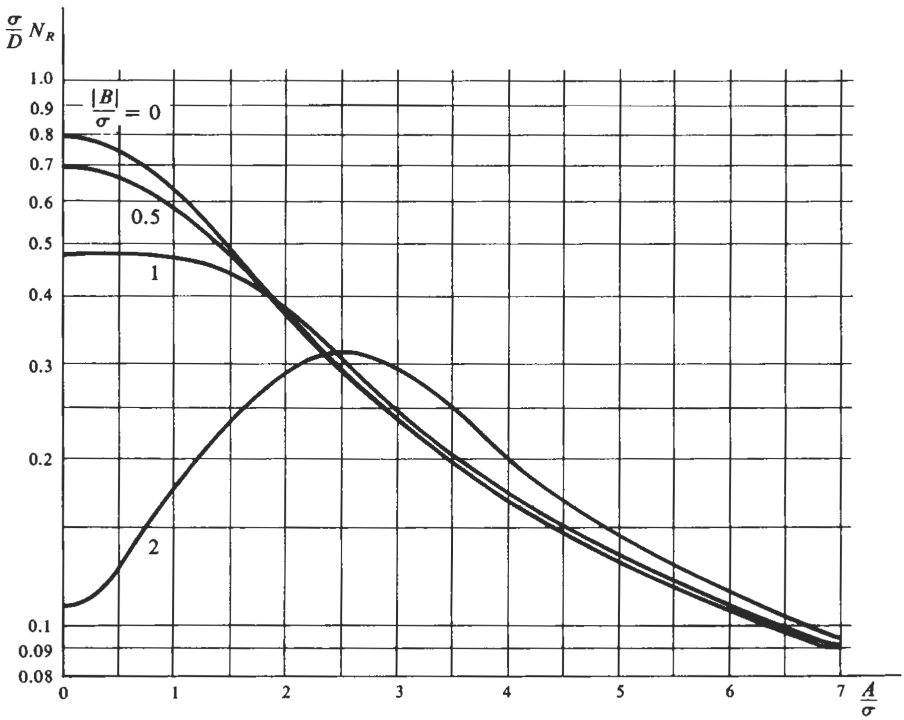


Figure E.3-1a Gain to the gaussian input component. (ideal relay)

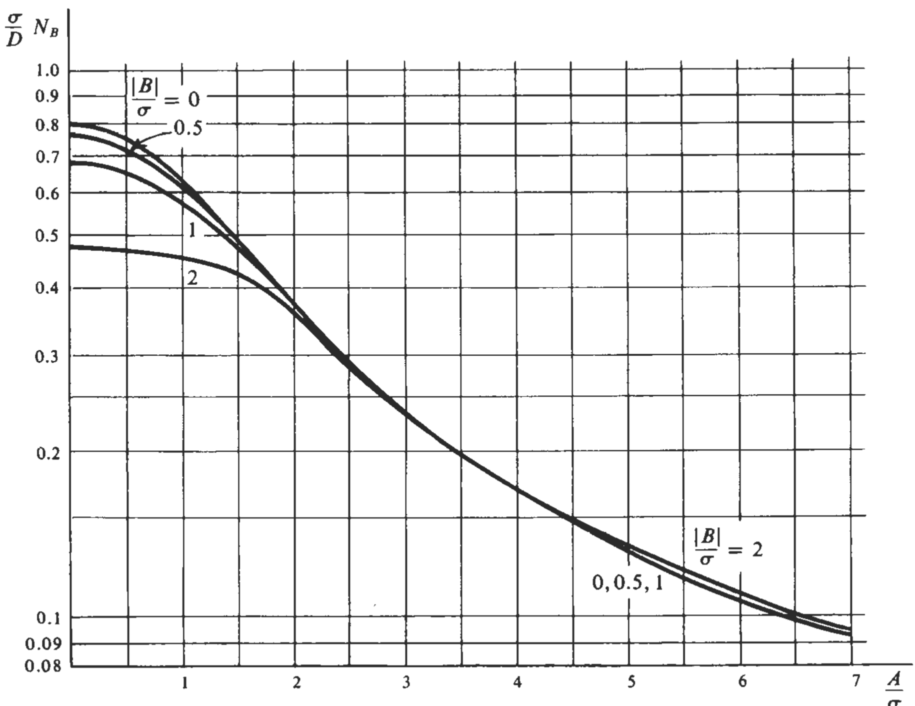


Figure E.3-1b *Gain to the bias input component. (ideal relay)*

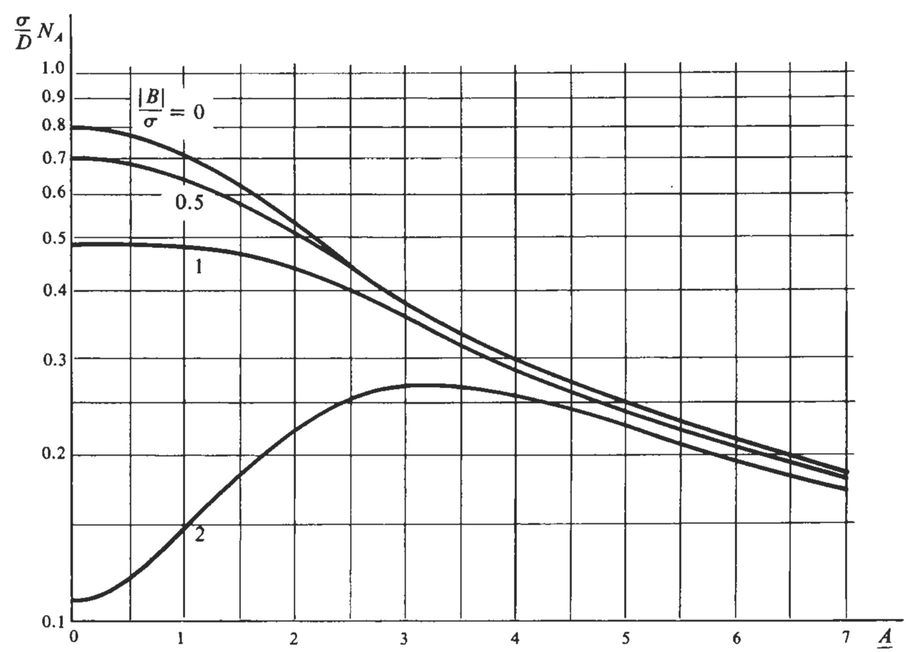


Figure E.3-1c *Gain to the sinusoid input component. (ideal relay)*

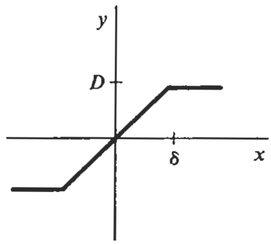


Figure E.3-2 Three-input RIDFs for the limiter nonlinearity.

In 11 parts

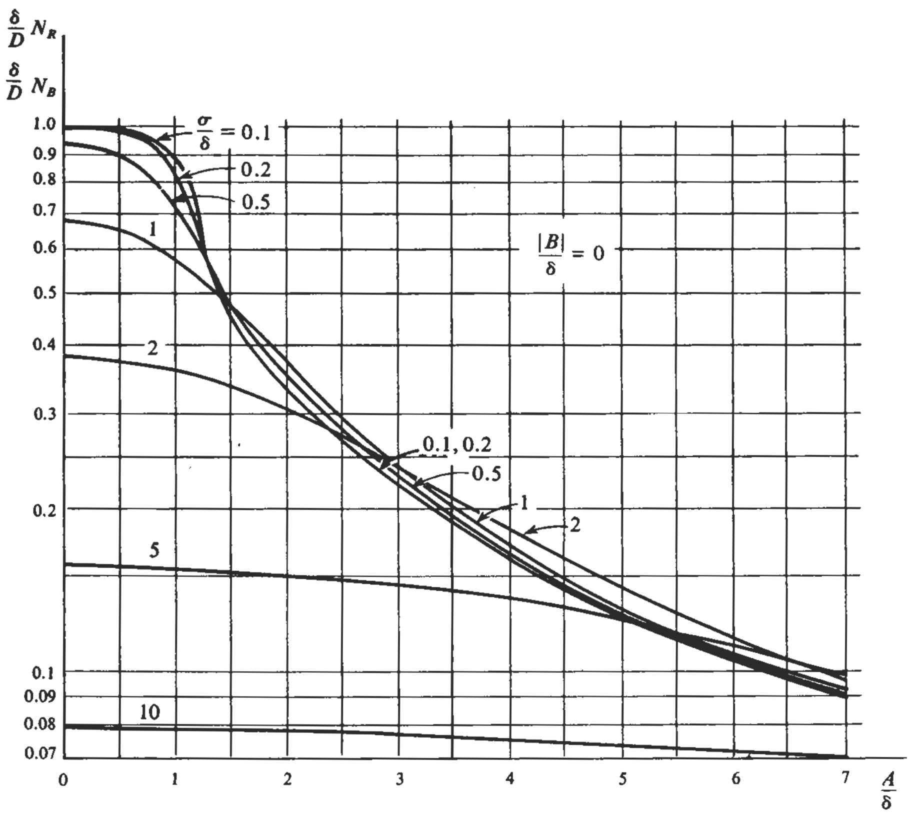


Figure E.3-2a Gain to the gaussian and bias input components. (limiter, $|B|/\delta = 0$)

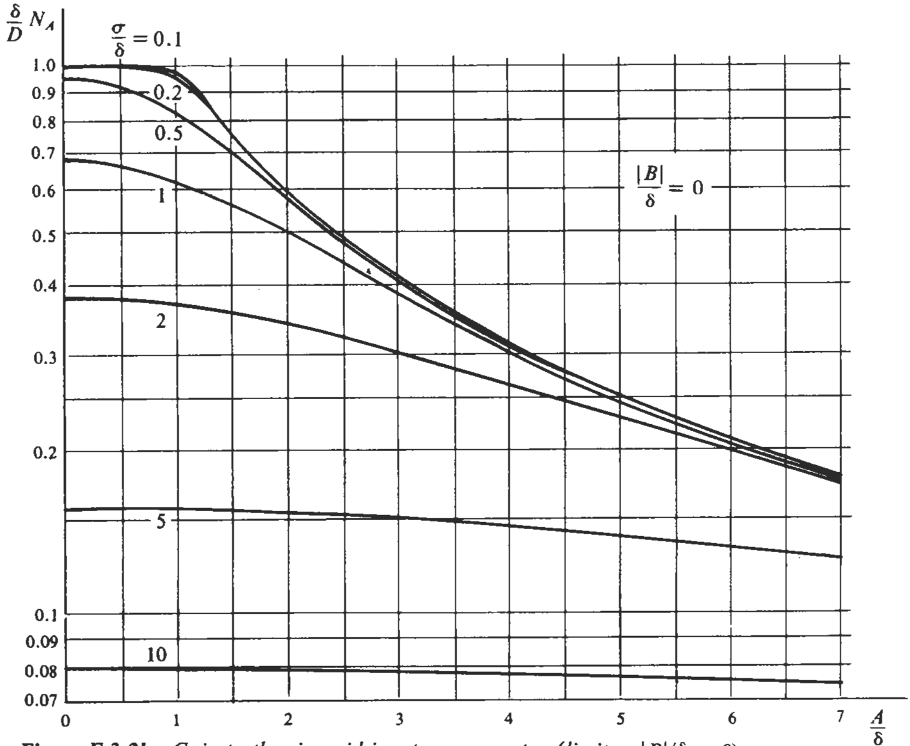


Figure E.3-2b *Gain to the sinusoid input component. (limiter,* $|B|/\delta = 0$*)*

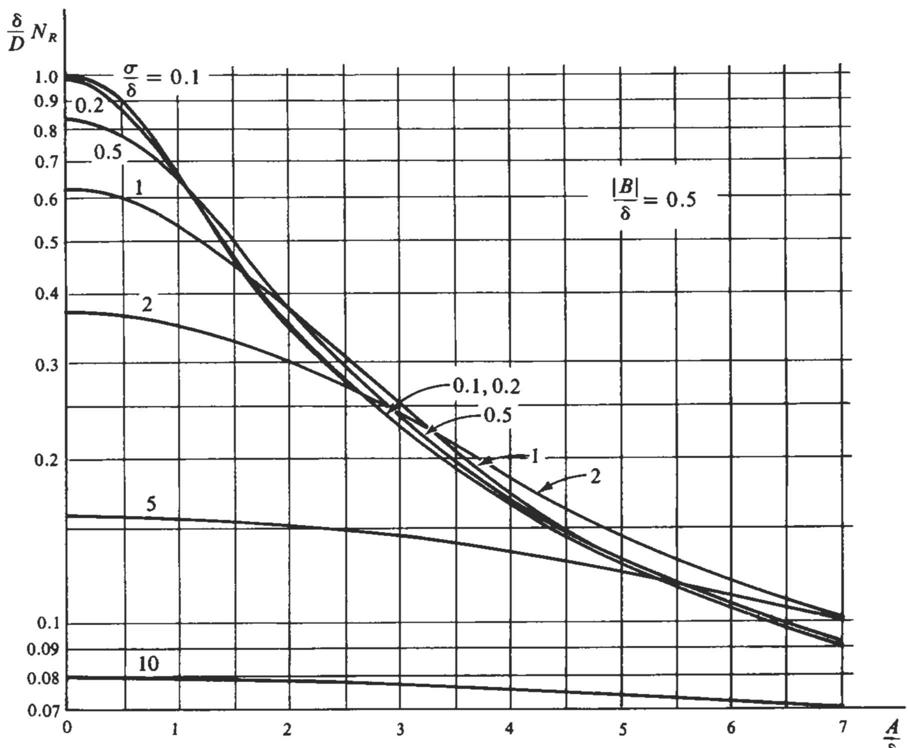


Figure E.3-2c *Gain to the gaussian input component. (limiter,* $|B|/\delta = 0.5$*)*

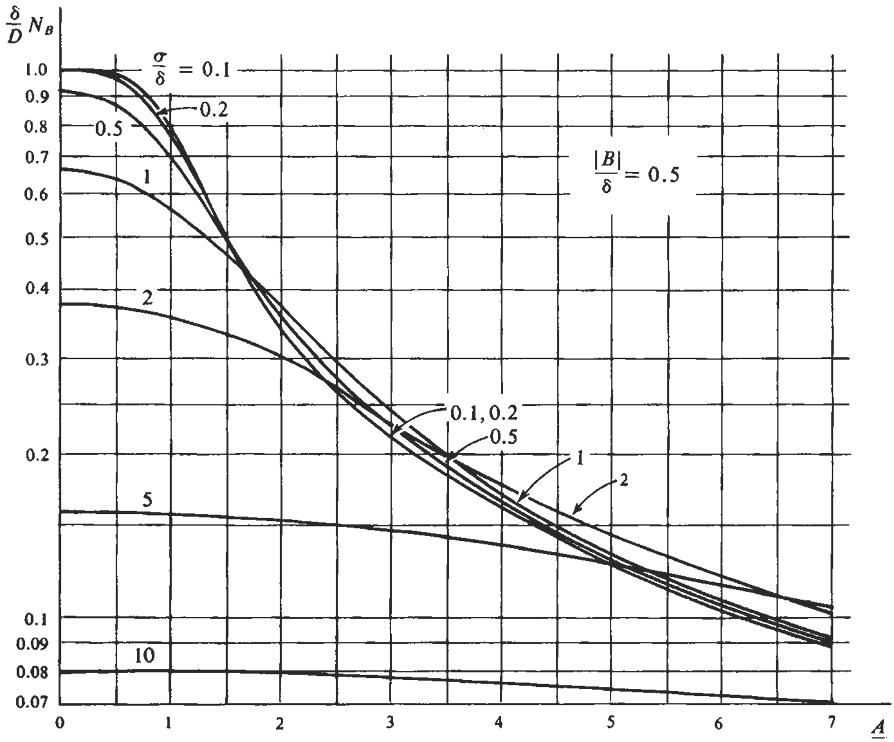


Figure E.3-2d *Gain to the bias input component. (limiter, $|B|/\delta = 0.5$)*

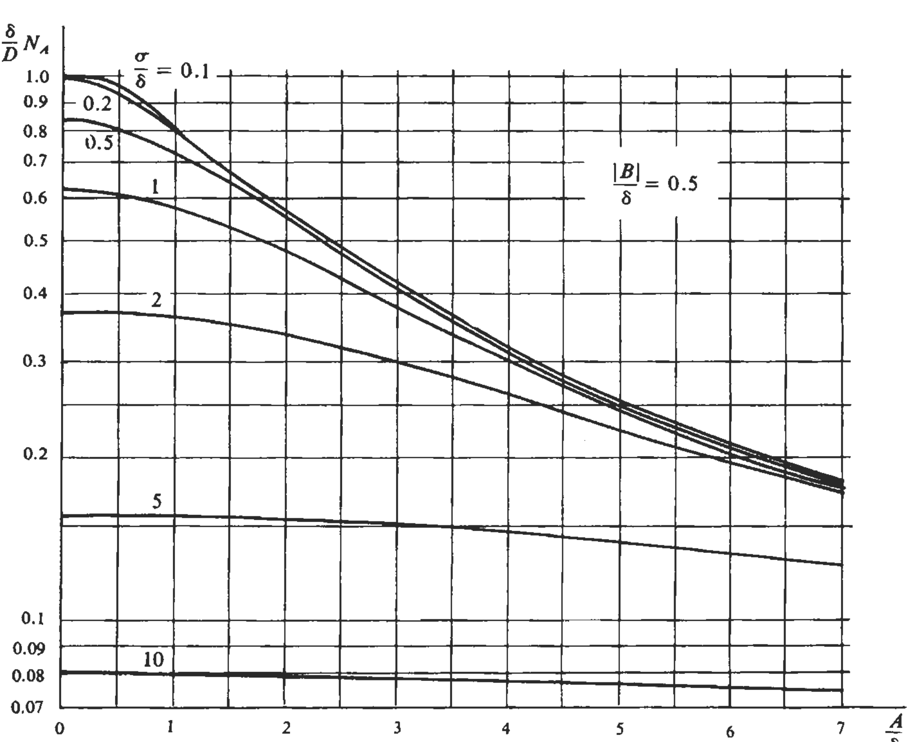


Figure E.3-2e *Gain to the sinusoid input component. (limiter, $|B|/\delta = 0.5$)*

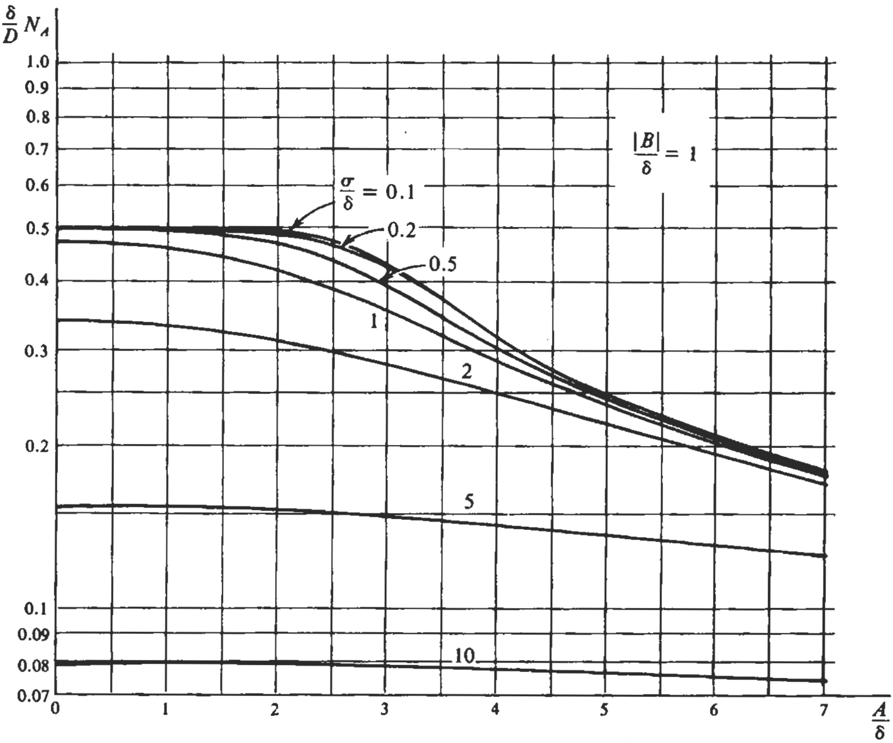


Figure E.3-2h *Gain to the sinusoid input component.* (*limiter*, $|B|/\delta = 1$)

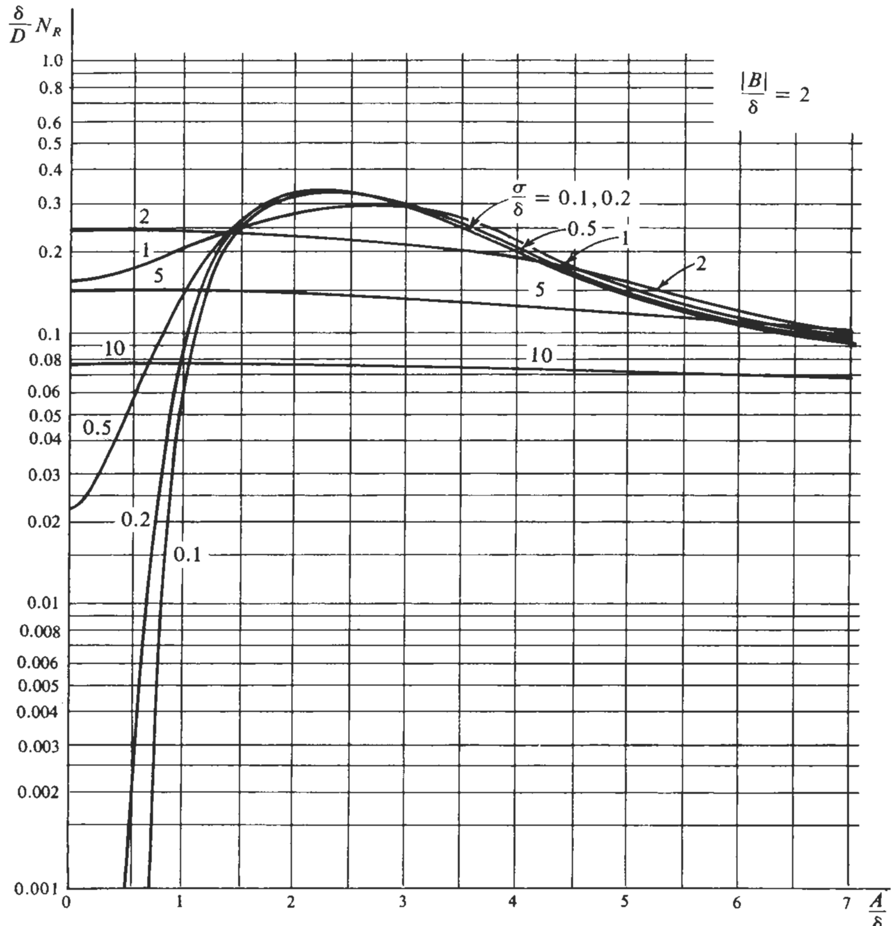


Figure E.3-2i *Gain to the gaussian input component. (limiter, $|B|/\delta = 2$)*

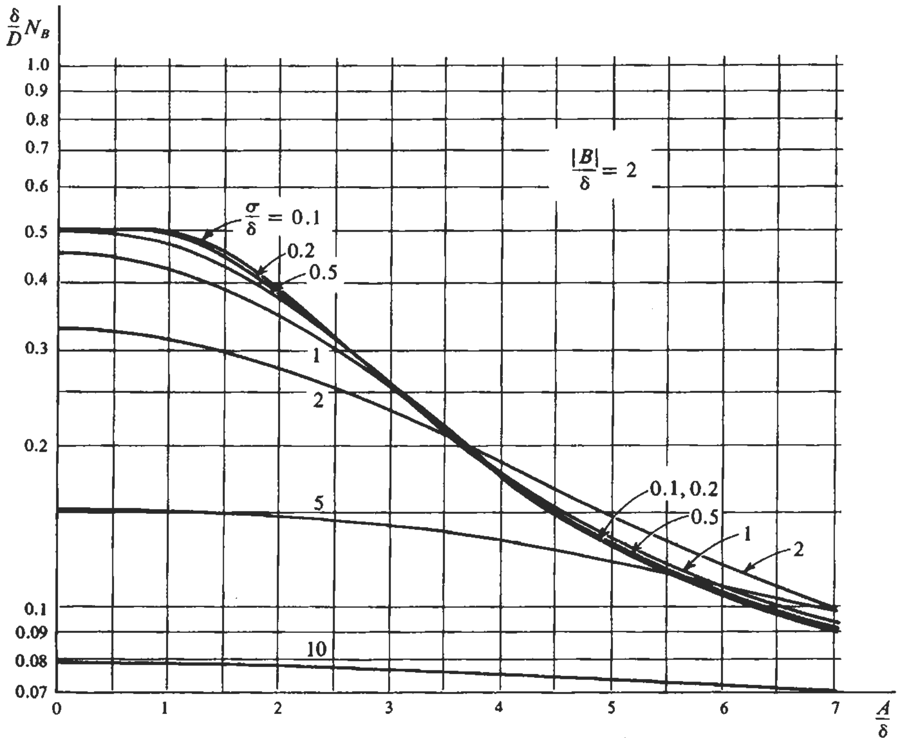


Figure E.3-2j *Gain to the bias input component.* (*limiter*, $|B|/\delta = 2$)

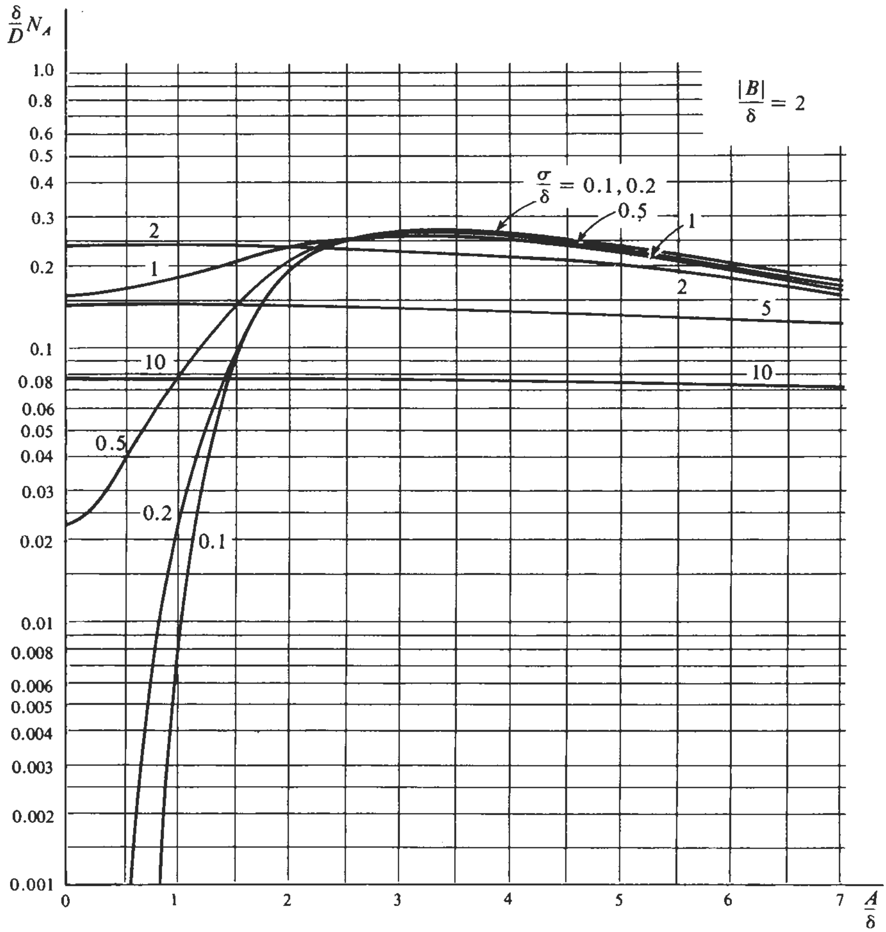


Figure E.3-2k *Gain to the sinusoid input component.* *(limiter, $|B|/\delta = 2$)*

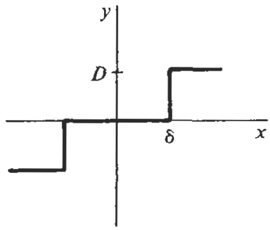


Figure E.3-3 *Three-input RIDFs for the relay with dead zone nonlinearity.*

In 11 parts

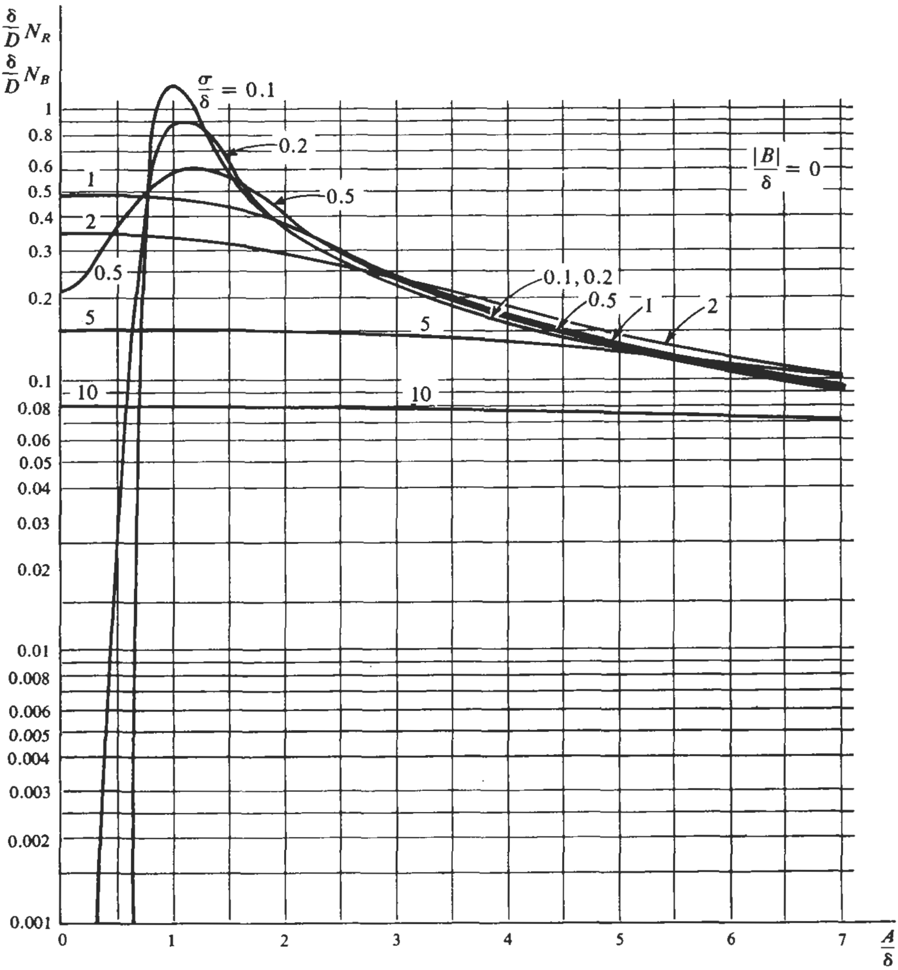


Figure E.3-3a *Gain to the gaussian and bias input components.* (*relay with dead zone,* $|B|/\delta = 0$)

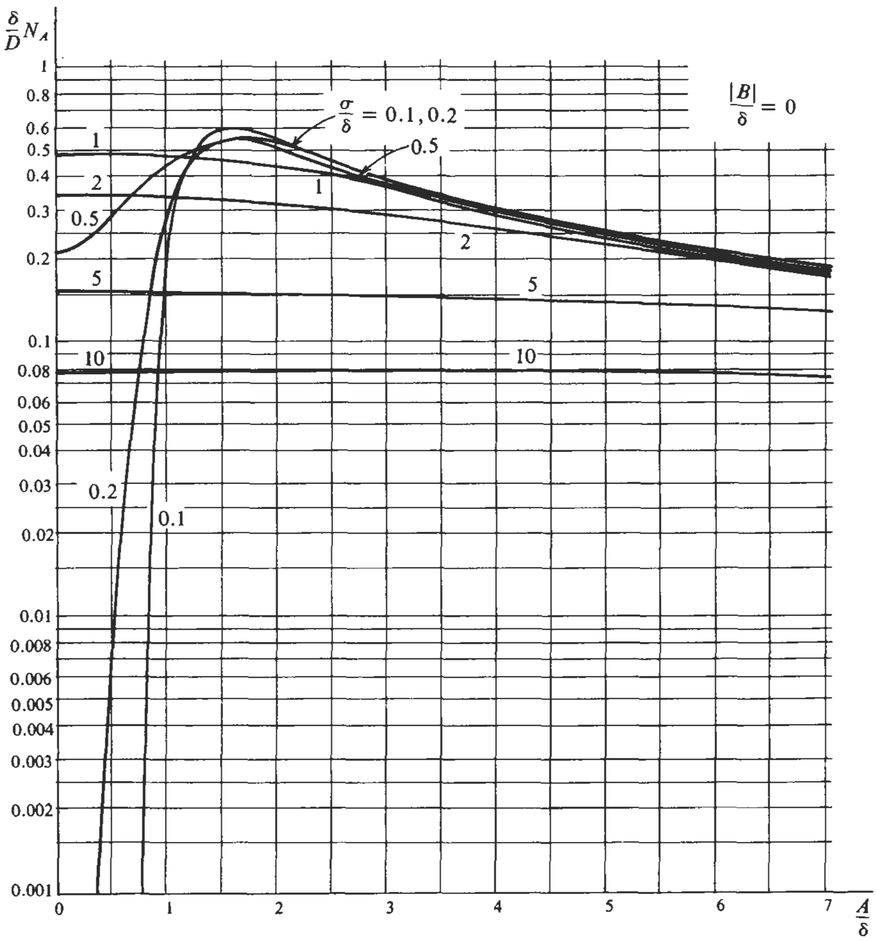


Figure E.3-3b Gain to the sinusoid input component. (relay with dead zone, $|B|/\delta = 0$)

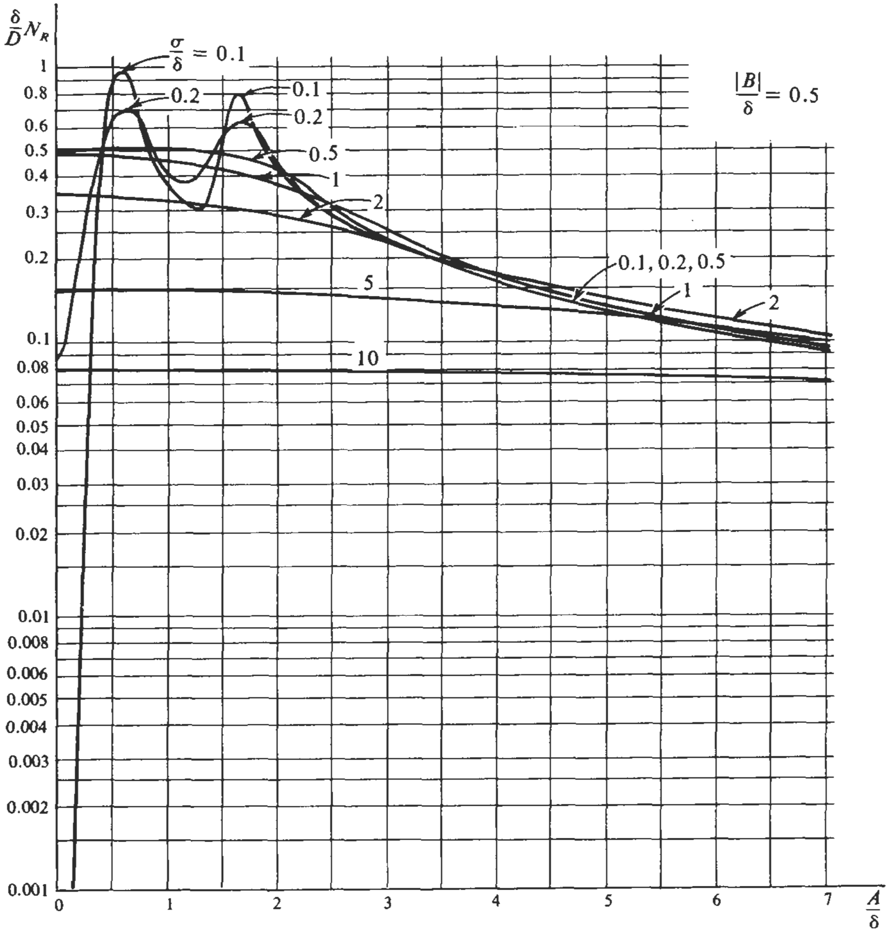


Figure E.3-3c *Gain to the gaussian input component. (relay with dead zone, $|B|/\delta = 0.5$)*

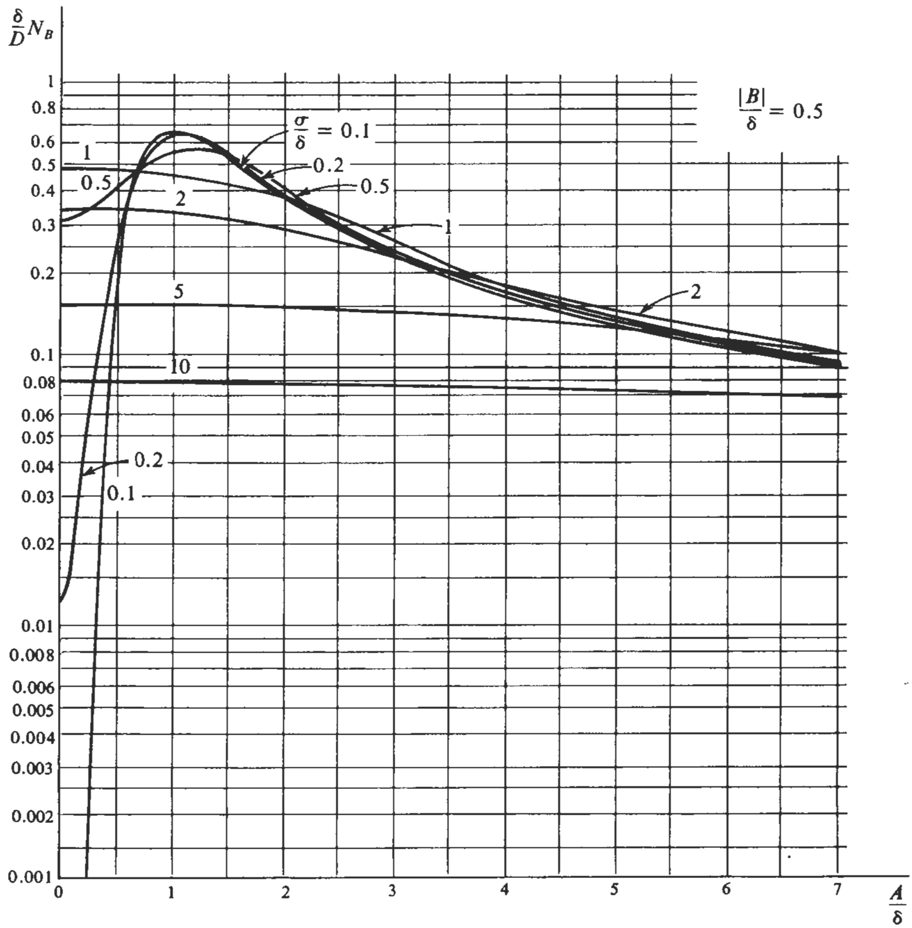


Figure E.3-3d *Gain to the bias input component.* *(relay with dead zone,* $|B|/\delta = 0.5$*)*

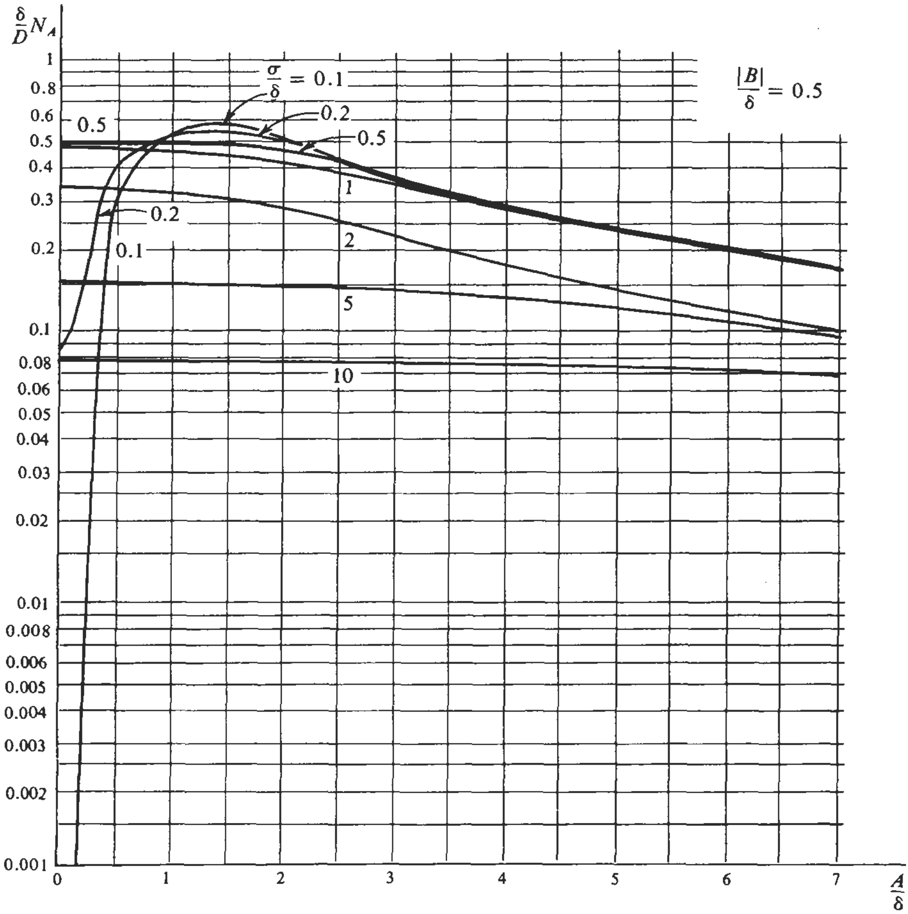


Figure E.3-3e *Gain to the sinusoid input component. (relay with dead zone,* $|B|/\delta = 0.5$*)*

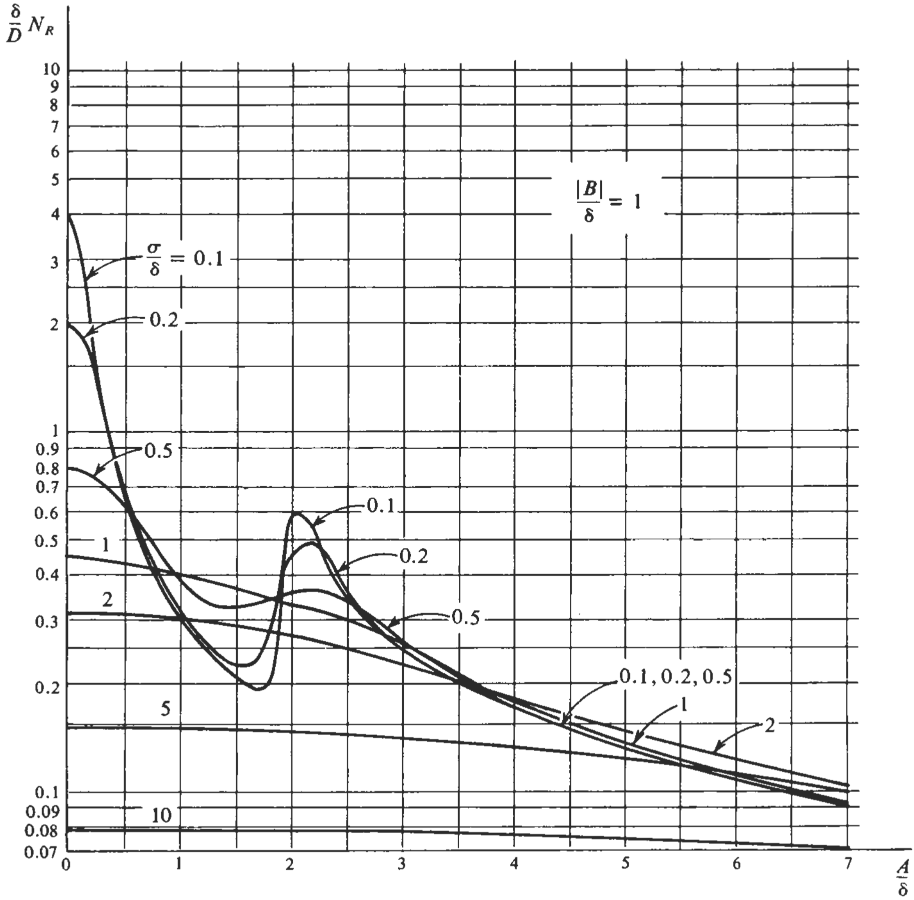


Figure E.3-3f *Gain to the gaussian input component. (relay with dead zone,* $|B|/\delta = 1$*)*

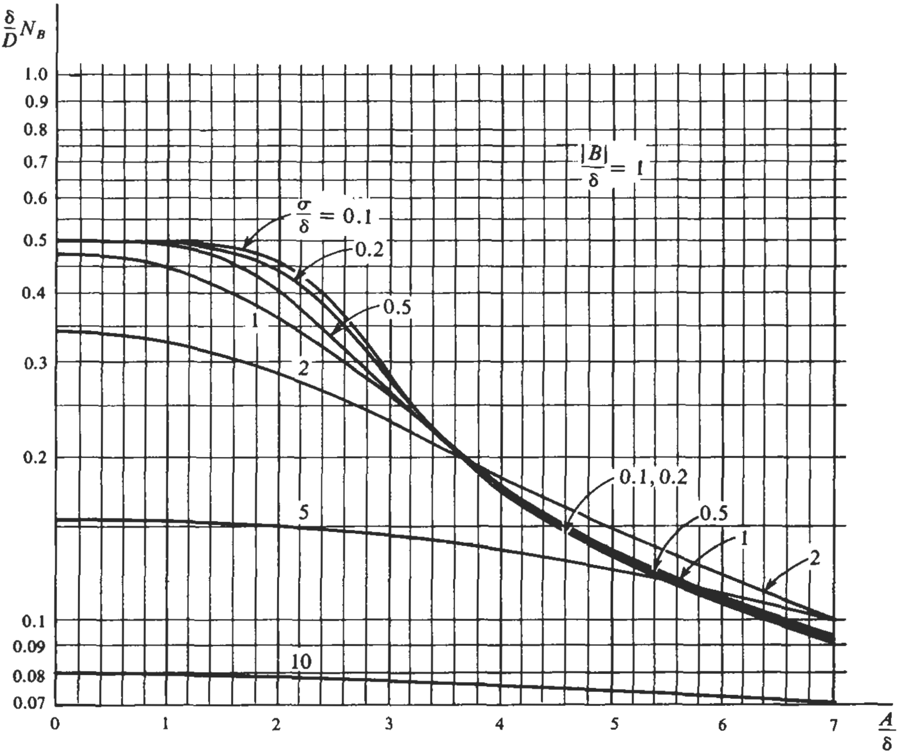


Figure E.3-3g *Gain to the bias input component. (relay with dead zone, $|B|/\delta = 1$)*

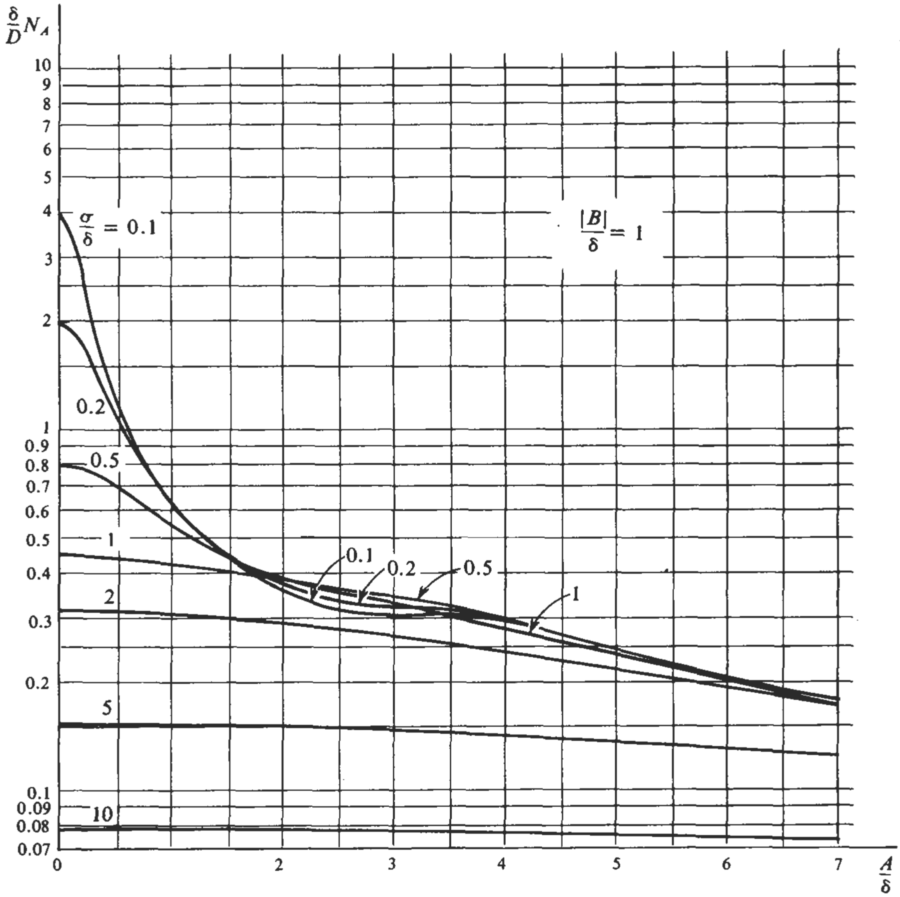


Figure E.3-3h *Gain to the sinusoid input component. (relay with dead zone, $|B|/\delta = 1$)*

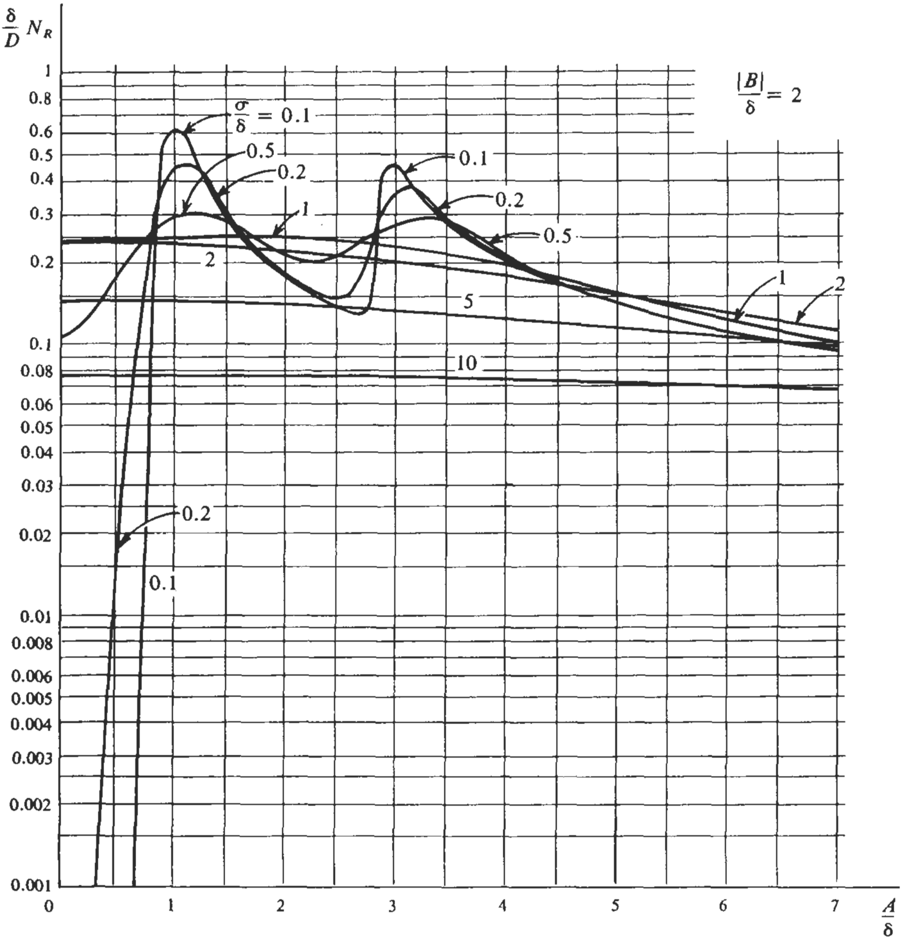


Figure E.3-3i *Gain to the gaussian input component. (relay with dead zone, $|B|/\delta = 2$)*

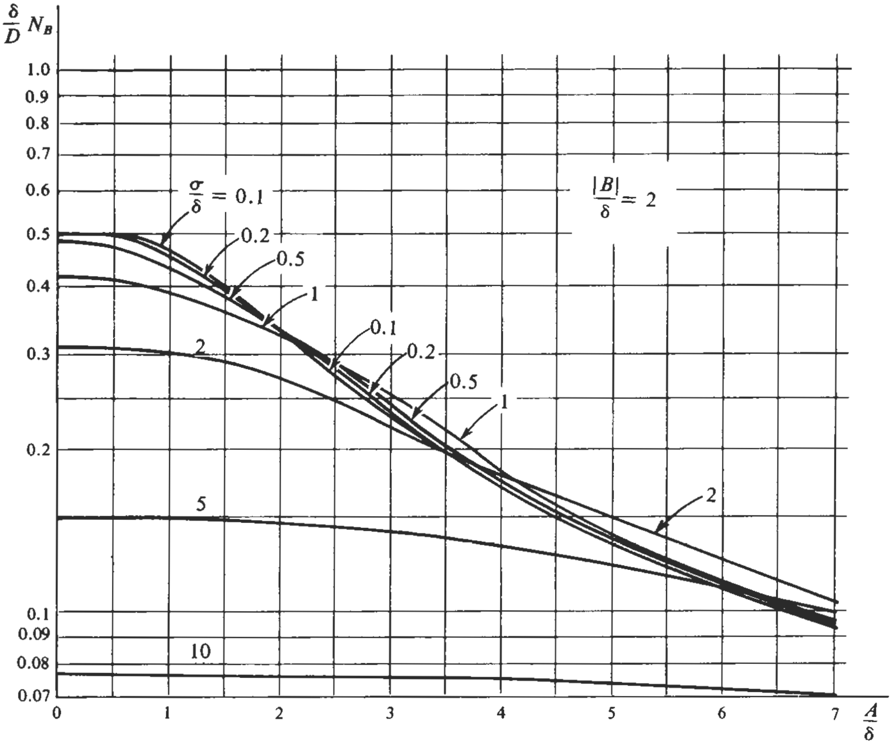


Figure E.3-3j *Gain to the bias input component. (relay with dead zone, $|B|/\delta = 2$)*

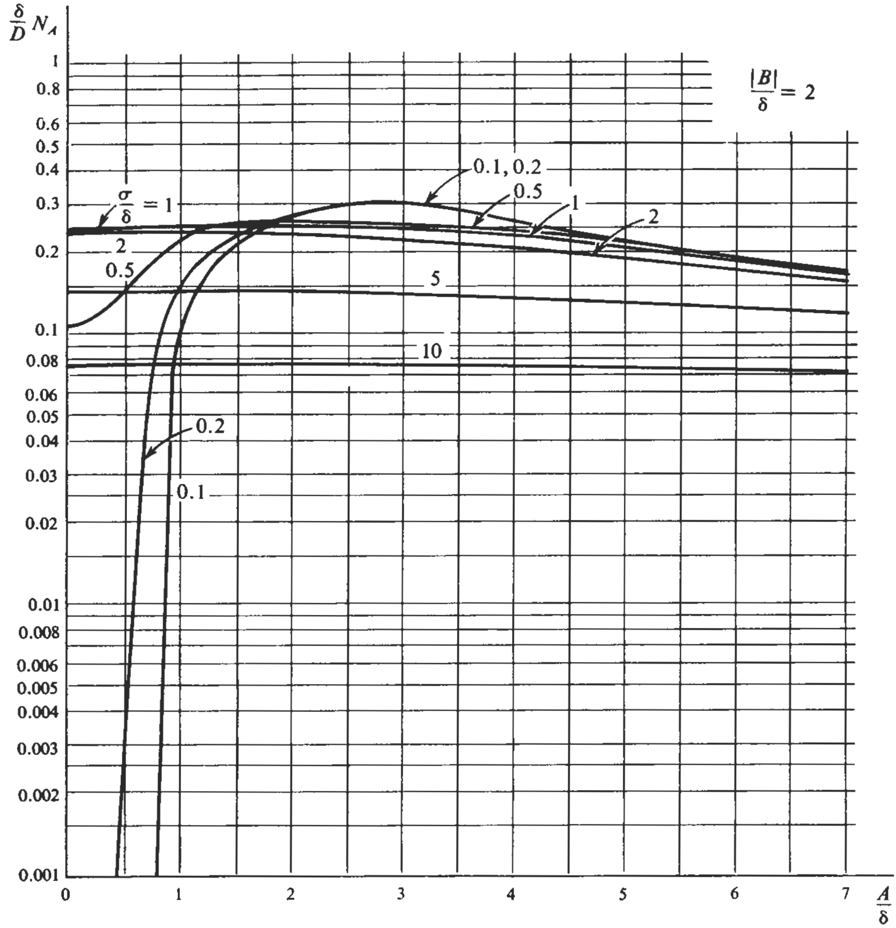


Figure E.3-3k *Gain to the sinusoid input component.* (*relay with dead zone,* $|B|/\delta = 2$)