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PULSE MODULATORS

INTRODUCTION

Much of the discussion on pulse modulation as applied to reaction jet control is unpublished. Some very interesting results have been discovered in company reports. They supplement the published works with discussions of modulation techniques and particularly with comparisons and trade-offs for specific applications. Fixed frequency pulse width systems are not in general used for this purpose.

Bang-bang systems require excessive bandwidths when stabilized with lead compensation in order to turn the thrustor off in as short a time as the valve will allow. Therefore, some form of pulse modulation scheme is found to give superior limit-cycle performance when disturbances are small. The thrustor is turned on for a short time and after it is off, information on whether additional thrust is needed can be fed back more leisurely, i.e., the feedback path bandwidth need only be wide enough to assure proper vehicle performance instead of valve performance.

One very simple mechanization which was developed for space vehicle attitude control application by Nicklas and Vivian [1962] is called derived rate. The thrustor on-time is limited by feeding back the valve command (usually the output of a Schmidt trigger) through a lag circuit. The time constant of the lag circuit and the trigger hysteresis determine the nominal on-time for typical limit cycle behavior.

The on-time can be more sharply controlled when disturbances become significant by feeding back a pulse to reset the trigger. This method was developed by Scott [1966].

Since the lag circuit approximately integrates the control force, it gives a measure of the velocity change and hence, introduces damping into the system. A derived rate controller can therefore be used without lead compensation (which obtains the velocity information by approximately differentiating the position signal). When the error signal is larger than the feedback from the trigger, the damping is not as effective and a lead circuit may be added for better damping.

Hinson and Sarles [1964] added a nonlinear feature to the derived rate modulator. By making the charge and discharge time constants of the feedback lag circuit different, a nonlinear modulation curve can be designed which improves the noise insensitivity and damping for low disturbance levels. Shigemoto [1967] and Jerkowsky [1968] have used describing functions successfully in analysing the derived rate modulators and adaptive features have been added to reduce valve activity by increasing pulse width in the presence of a steady disturbance by, for example, Nicklas [1966] and Shigemoto [1967].

A pulse-width pulse-frequency modulation technique (which must be used with lead compensation because it is basically destabilizing) has been used extensively on spacecraft. Its principle is described, for example, by Schaefer [1962], Melcher [1964], and in many Lockheed reports describing the Agena attitude control system; for example, Kunkle [1963]. Pulse-width and pulse-frequency systems [Farrenkopf et al, 1963] provide similar performance and may provide simpler mechanizations in a given situation.

Comparisons have been made of these systems by White [1964] and Wong [1966] for example, and optimum systems may vary with the requirements.

All of the foregoing reports deal with modulators for attitude control. A special problem arises for translation control when the vehicle is spinning. If lead compensation is used to obtain derivative information, Lange [1964] has shown that velocity information with respect to inertial space can be obtained with a simple mechanization by adding a correction term $\bar{\omega} \times \bar{p}$ to the lead compensated position signals, $\bar{p}$, obtained from the proof mass pickoff. If the calculated correction has an error in the angular velocity, $\bar{\omega}$, an error in the desired control is produced. Performance is relatively insensitive, however, and errors as large as 1 rad/sec are tolerable in a typical mechanization.

DESCRIPTION OF PULSE MODULATORS

Three types of modulators will be discussed:

(1) pulse frequency (or width)

(2) pulse-width pulse-frequency (PWPF)

(3) derived rate

Definitions:

T_c = time the control is on, pulse width

T_d = time the control is off

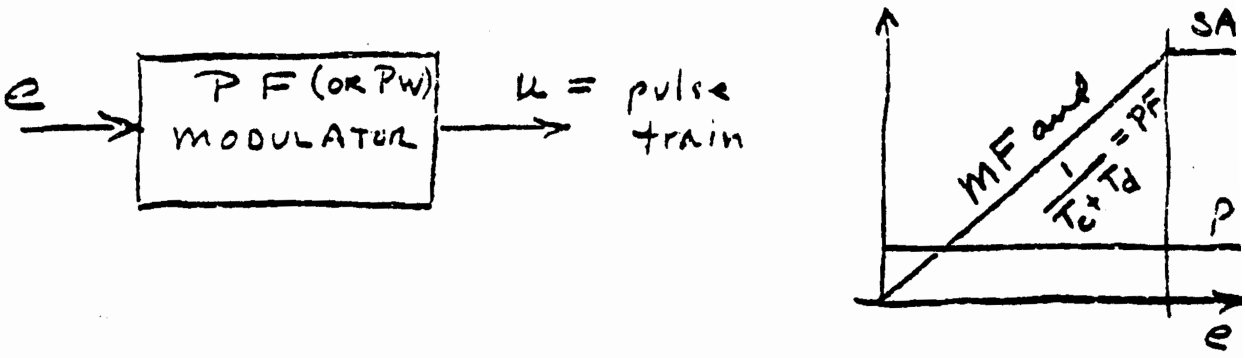
MF = modulation factor = $T_c/(T_c + T_d)$

A. Static Characteristics

With a constant input, the modulator has a behavior which is independent of the system in which it will be used. The pulse width and period are usually fast compared with the system dynamics so the input to the modulator (the error signal feedback) changes slowly and the static characteristics are a good indication of how the modulator will work in most cases.

1. Pulse Frequency (or width)

A modulator is designed so each pulse is of a fixed amplitude and duration (or fixed freq). Hence, the pulse width (frequency) is constant. The mod factor is therefore proportional to pulse frequency (width) which, in most mechanizations, is made proportional to the input. Typical applications for pulse frequency modulation are in pulse rebalanced inertial instruments where high accuracy is obtained by depending on the repeatability of pulse size. Dynamic range is obtained with a wide frequency range. This is frequently easier to achieve than a wide dynamic analog amplitude range on a linear controller or on sampled data or fixed-frequency, pulse-width systems when high accuracy is required. Pulse width modulation is used when information is available at a fixed rate as from a digital computer.

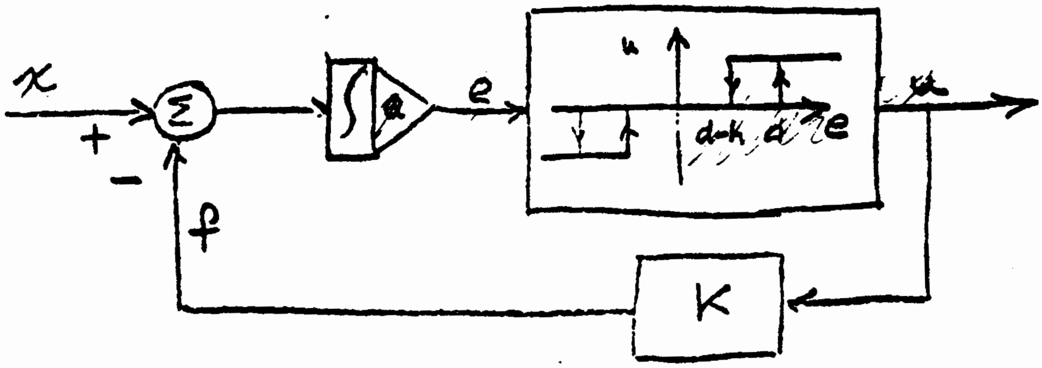


MODULATOR CHARACTERISTICS (for PW MODULATION INTERCHANGE PW and PF IN THE SKETCH AT THE RIGHT.)

2. Pulse-Width Pulse-Frequency (PWPF)

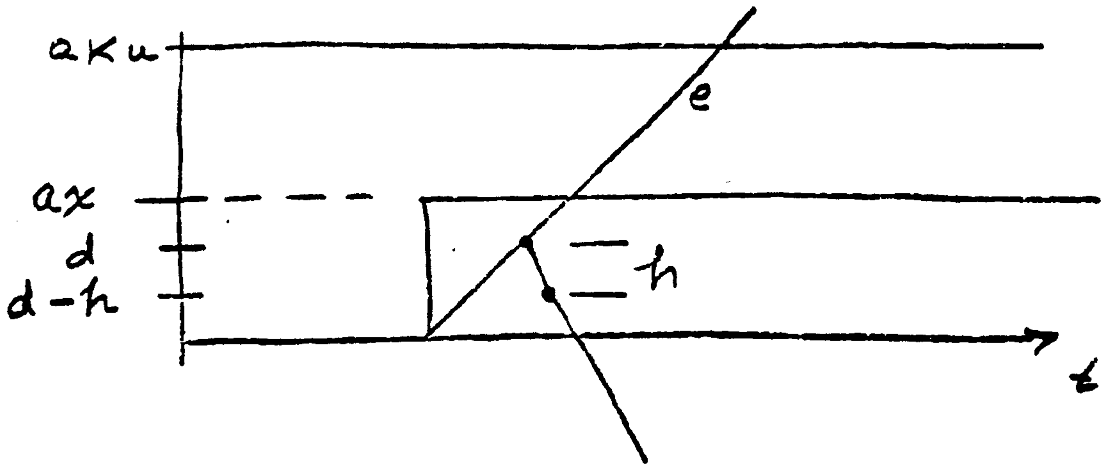
a. Linear

A linear PWPF provides a pseudo linear control when the PF is high compared with vehicle dynamics. It was used on the Agena Satellite and many others to operate gas jets. The jets operate best on-off. Closing positively prevents leakage and proportional operation is difficult to achieve due to hysteresis that is difficult to avoid in valve design. Frequently, a PWPF system saves power or helps overcome stiction in instrument and servo applications. In principle, the mechanization is



$$\dot{e} \;=\; = \; a(x - f) \;=\; a(x - Ku) \qquad (1)$$

To calculate the on and off times



$$h = x T_d a = (Ku - x) T_c a$$

$$T_d = h / x a \quad = \text{The off time}$$

$$T_c = \boxed{h / (Ku - x) a = PW} \qquad (2)$$

$$PF = \frac{1}{T_c + T_d} = \frac{a x (Ku - x)}{h (Ku - x + x)} = \frac{a x (1 - x/Ku)}{h}$$

$$\boxed{PF = \left(\frac{aKu}{h}\right)\left(\frac{x}{Ku}\right)\left(1 - \frac{x}{Ku}\right)} \qquad (3)$$

$$\boxed{MF = PWPF = \frac{x}{Ku}} \qquad (4)$$

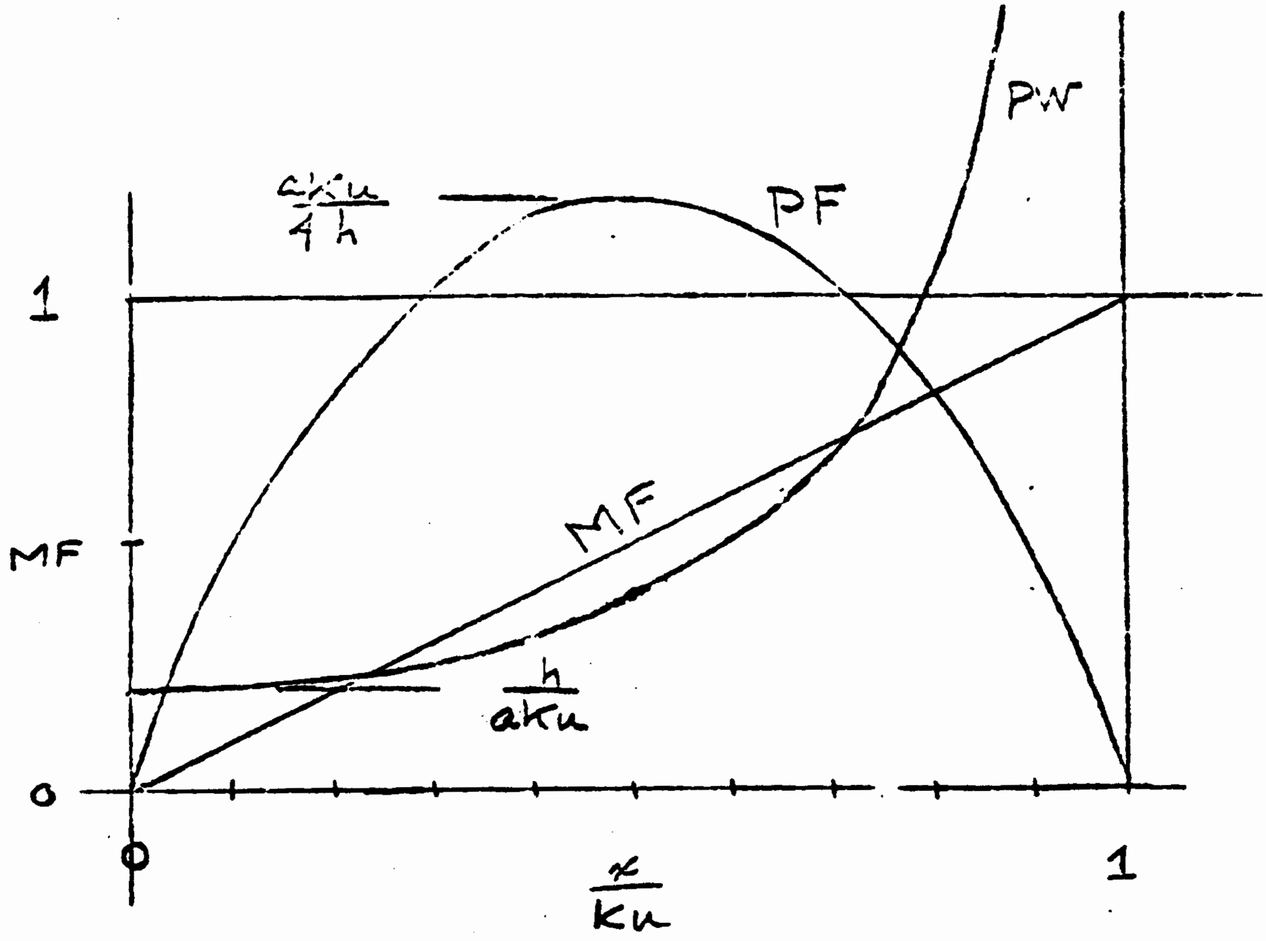


Fig. 1: PWPF CHARACTERISTICS FOR A SYMMETRICAL INTEGRATOR

The PF is a parabola with peak frequency at aKu/4h viz., a period which is four times the minimum pulse width. At this point, the input signal is one half the size of the feedback so the charge and discharge rates are the same and the pulse width is double its minimum value. Note that the symmetry of the expressions leads to a modulation of the off-time for x/Ku small. Typical time histories of the control u for various mod factors are sketched below in Fig. 2.

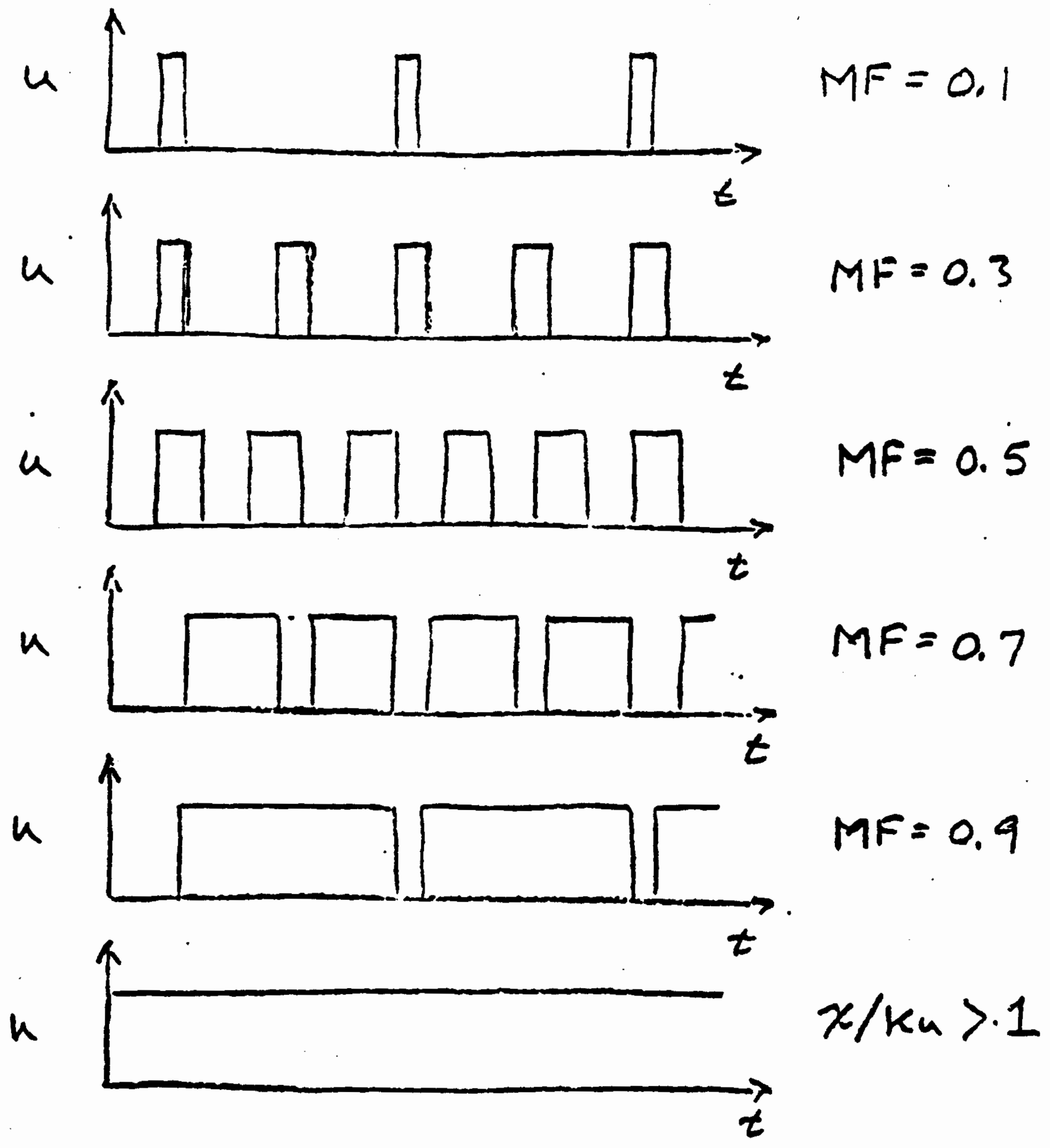


Fig. 2: TYPICAL TIME HISTORIES FOR PWPF MODULATOR (p = 1).

b. Nonlinear

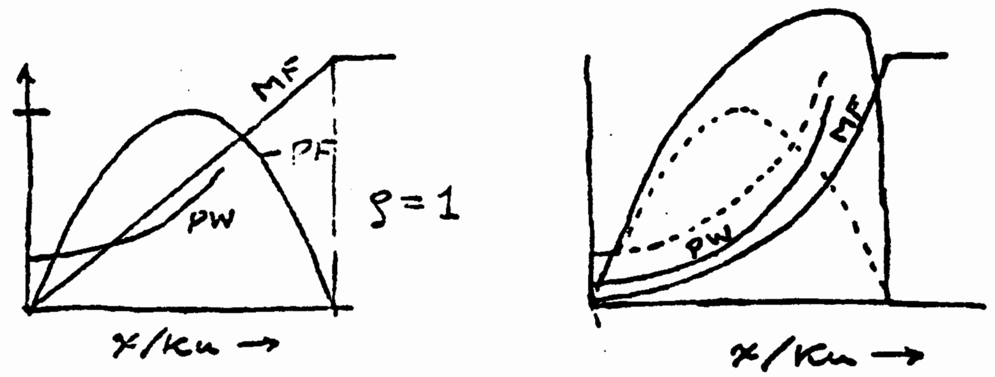
If the integrator gain is greater by ρ when feedback is applied, a nonlinear mod factor is obtained

$$T_d = h/xa \qquad (5)$$

$$T_c = h/\rho(Ku - x)a = PW$$

$$PF = \frac{ax(Ku - x)}{h\left(\frac{x}{\rho} + Ku - x\right)} = \frac{\frac{aKu}{h}\,\frac{x}{Ku}\left(1 - \frac{x}{Ku}\right)}{\left[1 - \frac{x}{Ku}\left(1 - \frac{1}{\rho}\right)\right]} \qquad (6)$$

Qualitatively this skews the PF toward the end of the range $x/Ku \to 1$ as shown below



A family of mod factor curves is given at top of page 9, Fig. 3, from

$$MF = \frac{x/Ku}{\rho\left[1 - \frac{x}{Ku}\left(1 - \frac{1}{\rho}\right)\right]} . \qquad (7)$$

For expansion around $x = 0$ and $x/Ku = 1$

$$\frac{dMF}{d(x/Ku)} = \frac{1}{\rho} \text{ and } \rho \text{ respectively.}$$

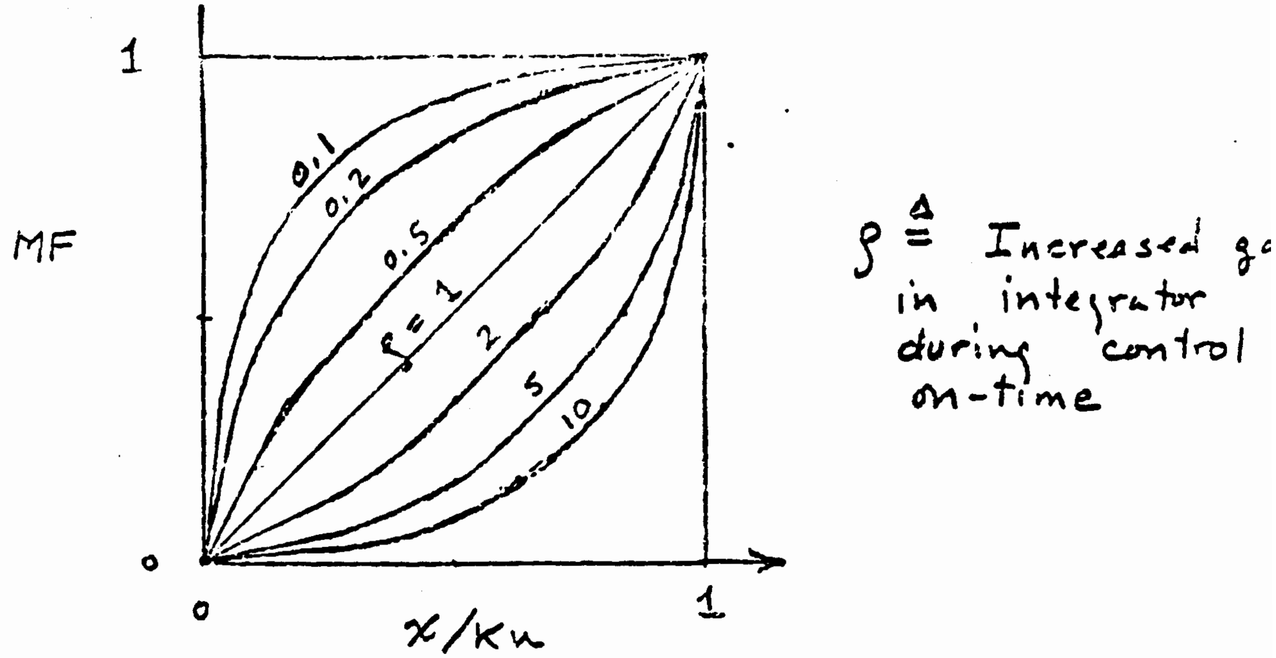


FIG. 3: MOD FACTORS FOR PWPF WITH UNSYMMETRICAL INTEGRATOR

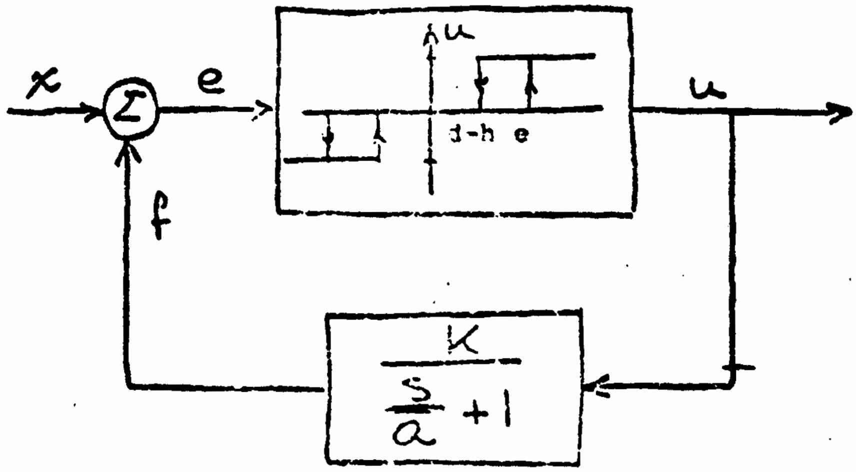
and at $x/Ku = 1/2$

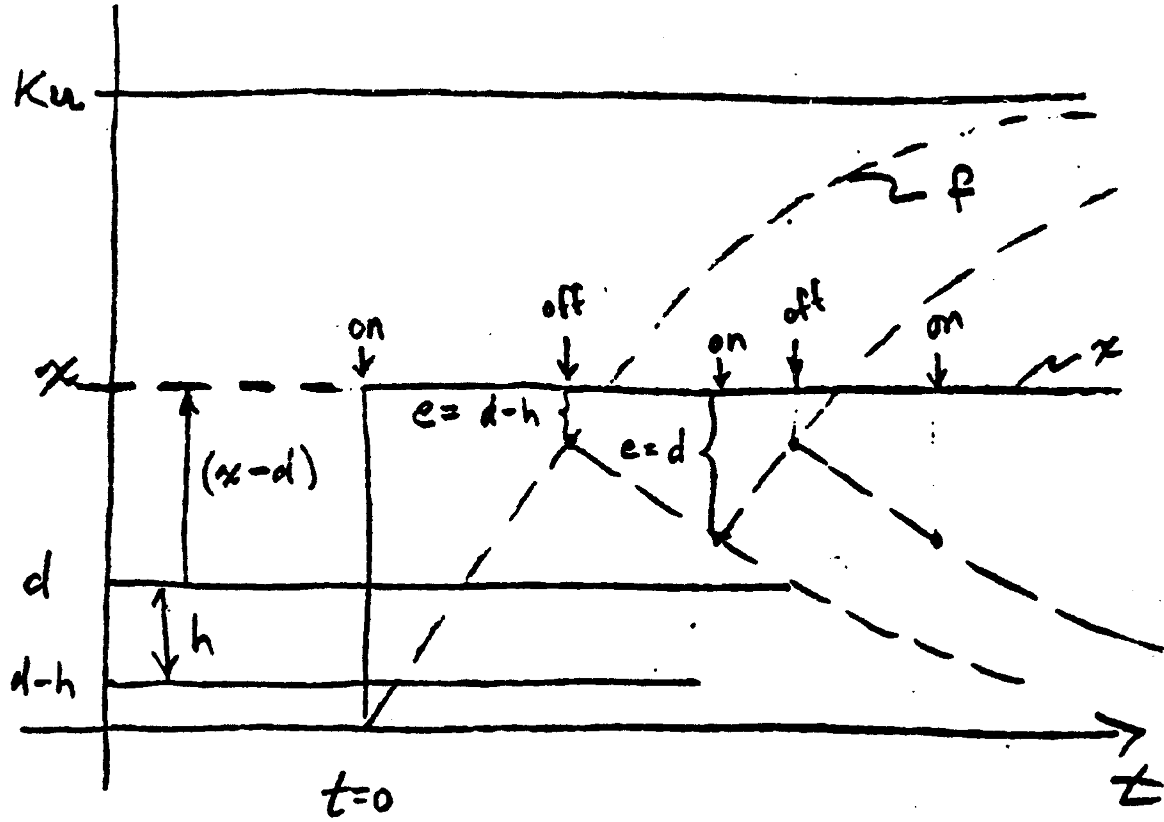
$$MF = \frac{1}{\rho + 1}$$

Values of $\rho > 1$ are desired when there is noise in x. The modulator response to this noise is smaller when the gain (slope of the mod factor) is less. Hence, there is less wasted control effort.

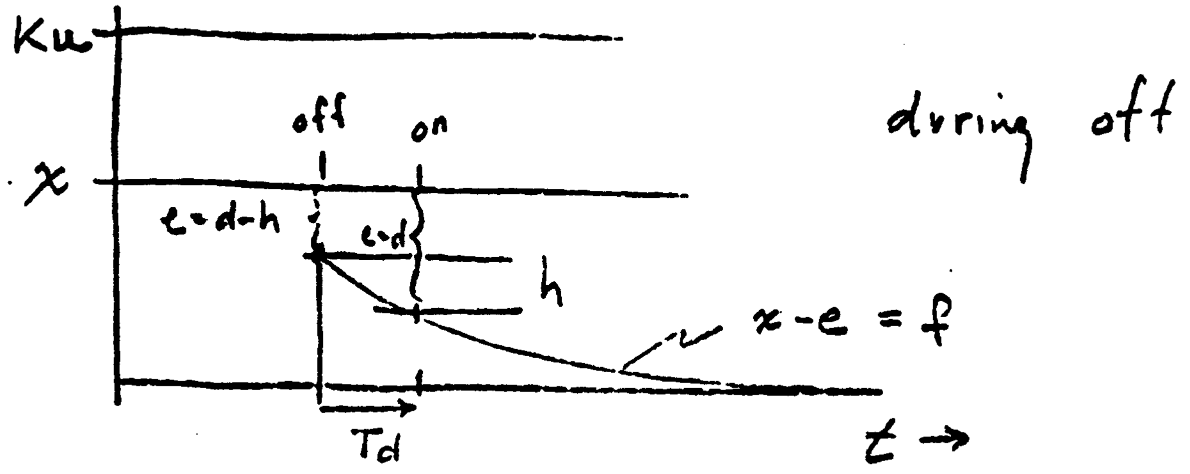
3. Derived Rate Modulator

A derived rate modulator provides damping, in addition to the other advantages of pulse modulated control. Hence, it provides excellent limit cycle behavior. It is used extensively for thrustor modulation in spacecraft application. The mechanization is, in principle:





Typical Modulator time history

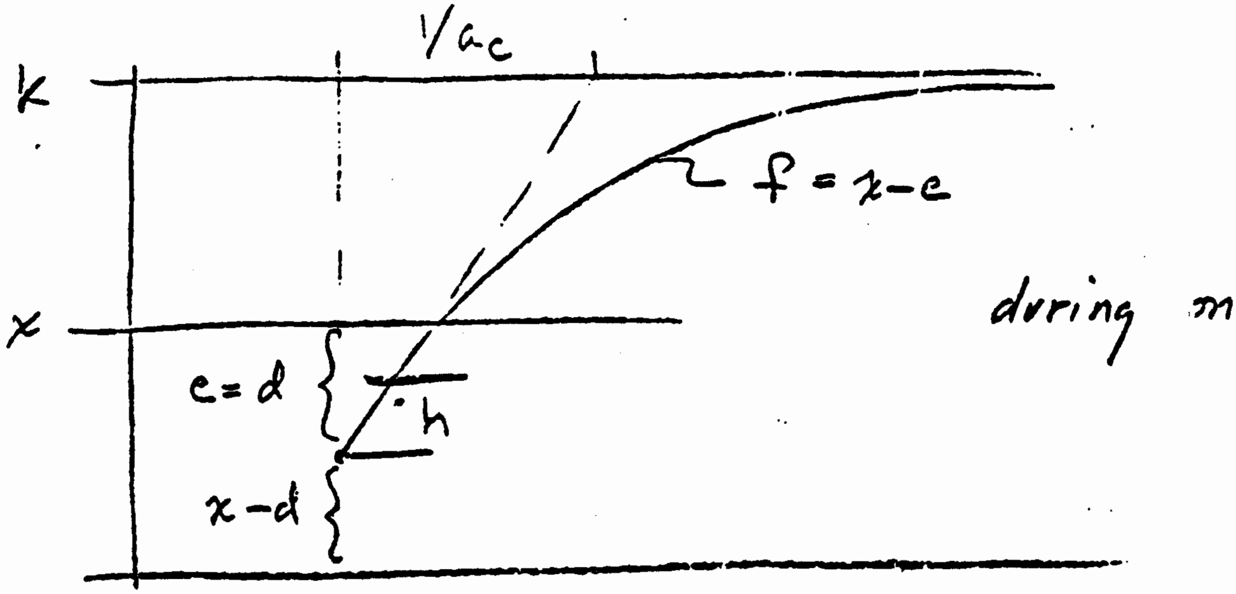


To calculate the off time, T_d, the change in f is h

$$[x - (d - h)]\,(1 - e^{-a_d T_d}) = h \qquad \text{OK}$$

from which

$$T_d = -\frac{1}{a_d}\log\left(\frac{x - d}{x - d + h}\right) = \frac{1}{a_d}\log\left(1 + \frac{h}{x - d}\right)$$



The on-time, T_c, is calculated in the same manner but the lag charges to Ku. The change in f is still h so

$$[Ku - (x - d)]\,[1 - e^{-a_c T_c}] = h \qquad 0.042$$

from which

$$T_c = \frac{1}{a_c}\log\left(1 + \frac{h}{Ku - (h) - (x - d)}\right) \tag{8}$$

where h is added and subtracted as needed to get the variables $Ku - h$ and $x - d$ identified. The $Ku - h$ corresponds to the range of $x - d$

which is the amount of input beyond the deadband.

The mod factor can now be calculated.

$$MF = \frac{T_c}{T_c + T_d} = \frac{1}{1 + T_d/T_c}$$

$$MF = \frac{1}{\left\{1 + \frac{a_c}{a_d}\,\frac{\log\left[1 + \left(\frac{h}{Ku - h}\right)\left(\frac{Ku - h}{x - d}\right)\right]}{\log\left[1 + \left(\frac{h}{Ku - h}\right)\left(\frac{1}{1 - \frac{x - d}{Ku - h}}\right)\right]}\right\}} \tag{9}$$

where the (x-d)/Ku-h) is the fraction of the range of modulation the input represents. Let

$$\frac{h}{Ku - h} = r$$

$$\frac{x - d}{Ku - h} = R$$

and

$$\frac{a_c}{a_d} = \rho$$

then (9')

$$MF = \frac{1}{1 + \rho\,\frac{\log(1 + r/R)}{\log[1 + r/(1-R)]}} .$$

In this form, the symmetry for small values of R and small values of $(1 - R)$ is apparent when $\rho = 1$ (for example, calculate $1 - MF$). A mod factor need only be known for $0 < R < \frac{1}{2}$. The rest is reflected about the point $R = MF = \frac{1}{2}$. The limits for $R \to 0$ and 1 are easily checked and correctly give $MF = 0$ and 1 respectively.

Effect of ρ is the same as for the PWPF but the variable r must be discussed first. There are two ways of linearizing the MF.

(a) Taylor series expansion about the center of symmetry $R = MF = \frac{1}{2}$ and $\rho = 1$

$$\left.\frac{dMF}{dR}\right|_{R=\frac{1}{2}} = \frac{2r}{(1 + 2r)\ \log(1 + 2r)}$$

so

$$MF = \frac{1}{2} + \frac{2r}{(1 + 2r)\ \log(1 + 2r)}\left(R - \frac{1}{2}\right) + ---$$

for

$$\text{small } r \Rightarrow \quad MF = R$$

Including the dependence on ρ

$$MF \cong \frac{1}{1 + \rho} + \frac{4\rho}{(1 + \rho)^2}\,\frac{2r}{(1 + 2r)\ \log(1 + 2r)}\ (R - \frac{1}{2}) + ---$$

which reduces to the previous expression, for $\rho = 1$. For small r

$$\log(1 + 2r) \approx 2r - (2r)^2/2 + ---$$

and

$$MF \sim \frac{1}{1 + \rho} + \frac{4\rho}{(1 + \rho)^2}\ (r - \tfrac{1}{2})\ .$$

This expression is useless near R = 0 and R = 1 even for small r.

(b) **Linearization before inverting the exponential. Assumption**

$$a_c T_c \text{ and } a_d T_d \ll 1$$

$$[Ku - (x - d)](a_c T_c) \cong h$$

and

$$[x - d + h](a_c T_c) \cong h$$

give
$$T_c = \frac{1}{a_c} \frac{h}{(Ku - h) - (x - d) + h}$$

$$T_d = \frac{1}{a_d} \frac{h}{(x - d) + h}$$

and

$$MF = \frac{1}{1 + \frac{a_c}{a_d}\left(\frac{(Ku - h) - (x - d) + h}{(x - d) + h}\right)}$$

$$= \frac{1}{1 + \rho \frac{1 - R + r}{R + r}} \text{ ,}$$

For r small and $\rho = 1$, $MF = R$ as expected. For good stability and precision of the switching levels, h is usually a significant fraction of the deadband, say 1/3. Hence, when tight control is used, r is small, say, of the order of 0.1 to 0.01 and smaller, then

$$MF \cong \frac{R/\rho}{1 - R(1 - 1/\rho)} \tag{7'}$$

which is the same expression obtained for the exact nonlinear expression for the PWPF modulator in Eq. (7), page 8.

The effect of r however, causes the derived rate modulator MF to have a significantly different shape as the exact expression, Eq. (7) involving the logarithms would suggest. There is a singularity in the slope at both $R = 0$ and $R = 1$ so that the behavior for $\rho = 1$ is

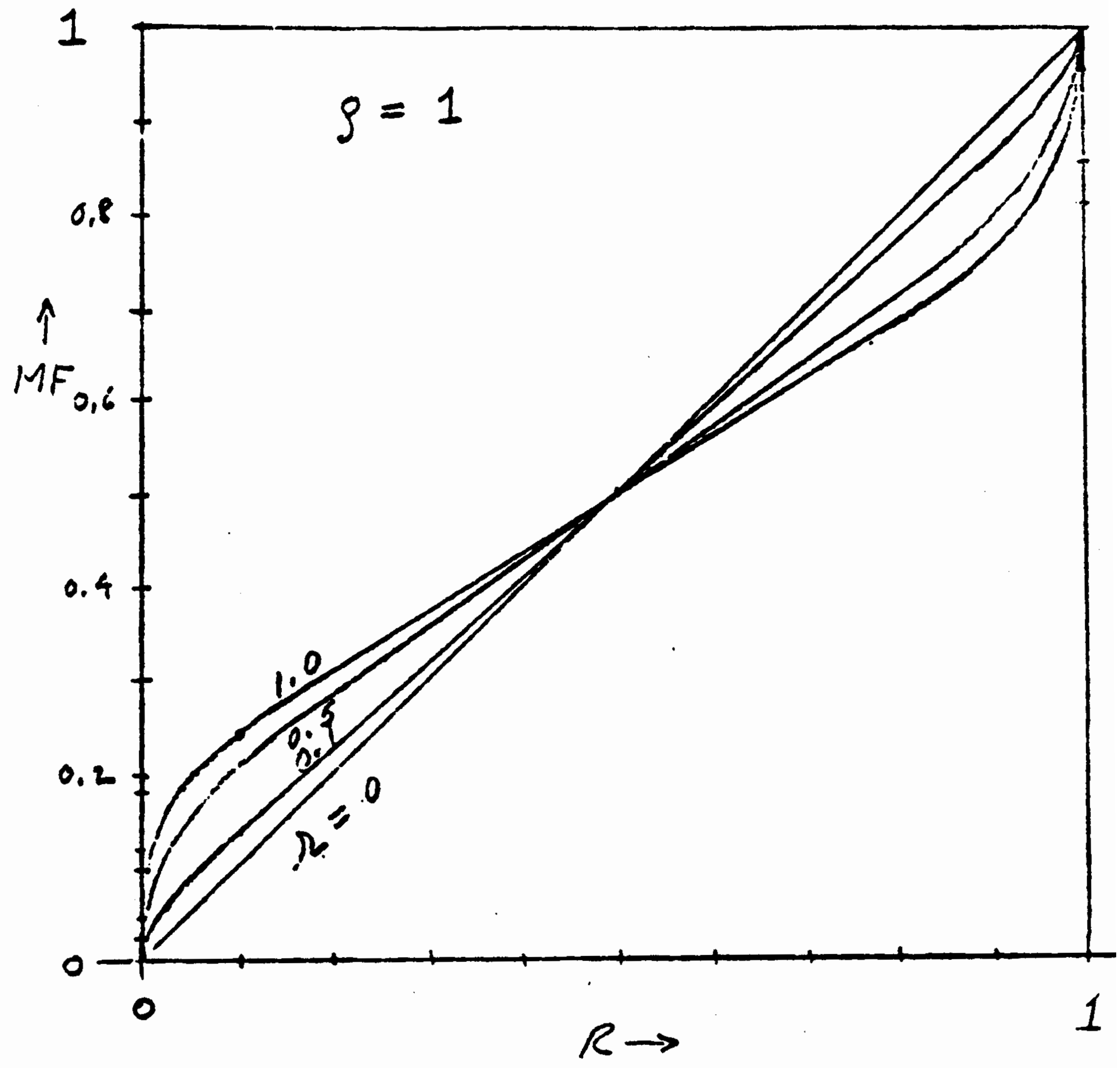


FIG. 4. DERIVED RATE MOD FACTOR DEPENDENCE ON RELATIVE SIZE OF HYSTERESIS AND MODULATION RANGE, Ku-h.

For practical purposes, r is usually small and the mod curves, Fig given for PWPF apply to the derived rate system as well.

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