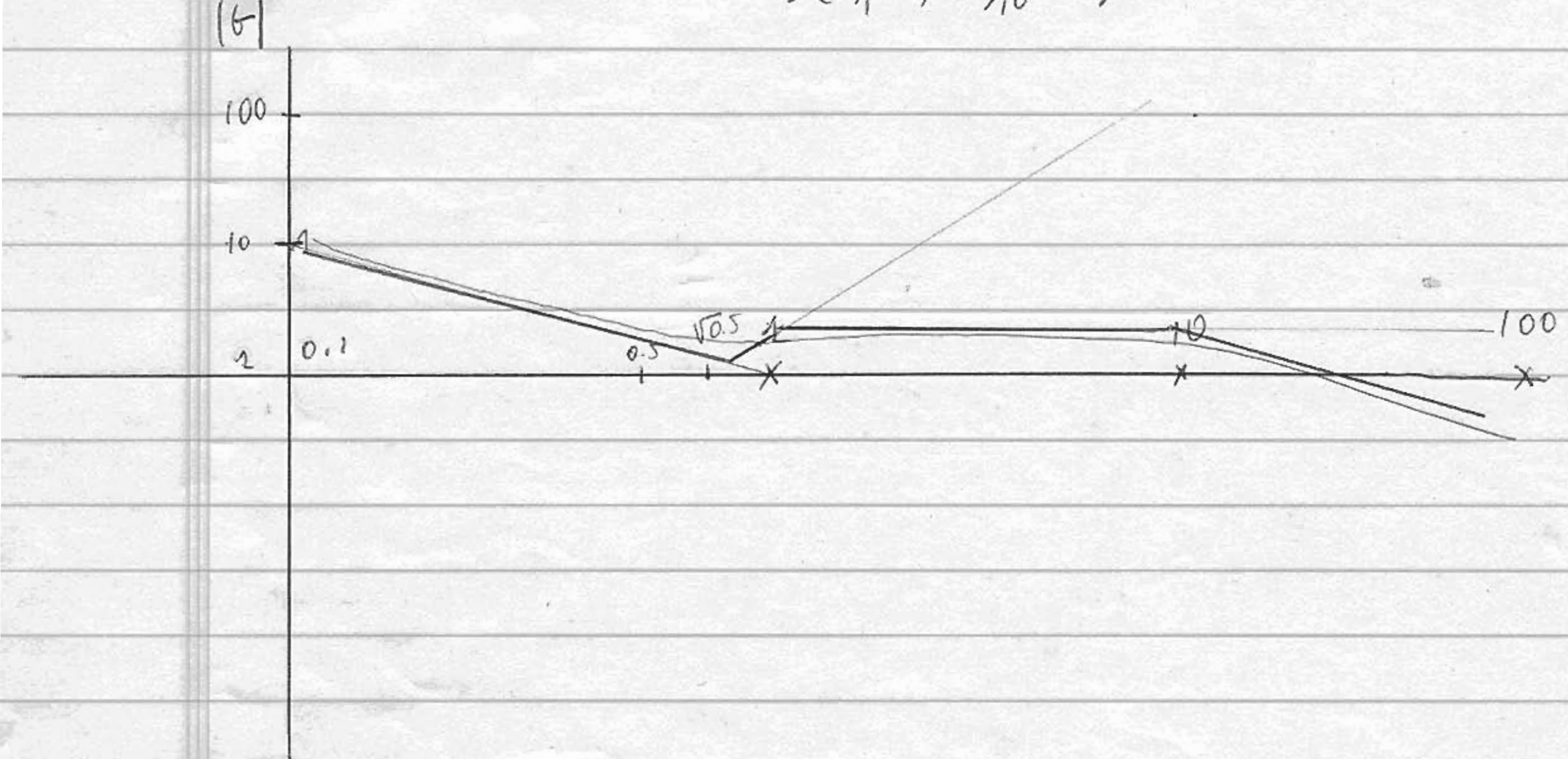
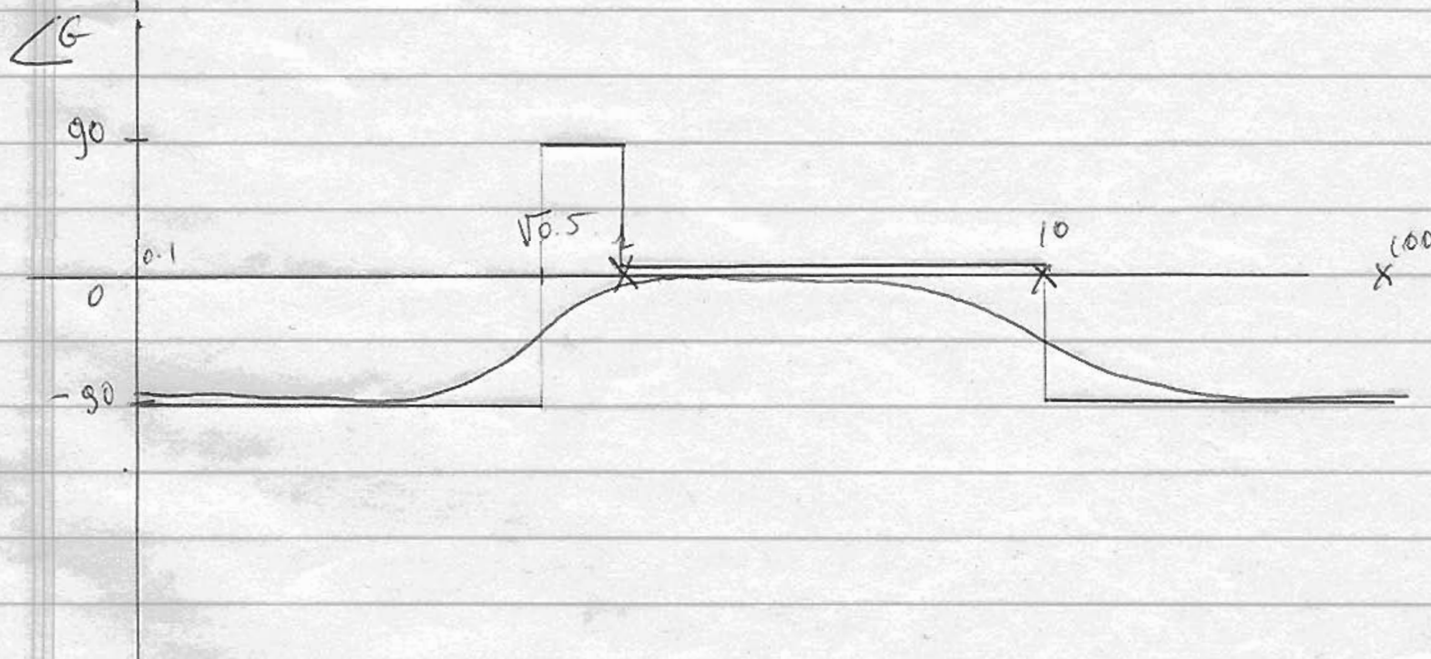
Quiz solutions

$$G(s) = \frac{\left(\left(s/\sqrt{0.5}\right)^2 + s/0.5 + 1\right)}{s\left(s/1+1\right)\left(s/10+1\right)}$$





2. $G(s) = \dfrac{sT}{1+sT}$.

a) We use the reduced variable:

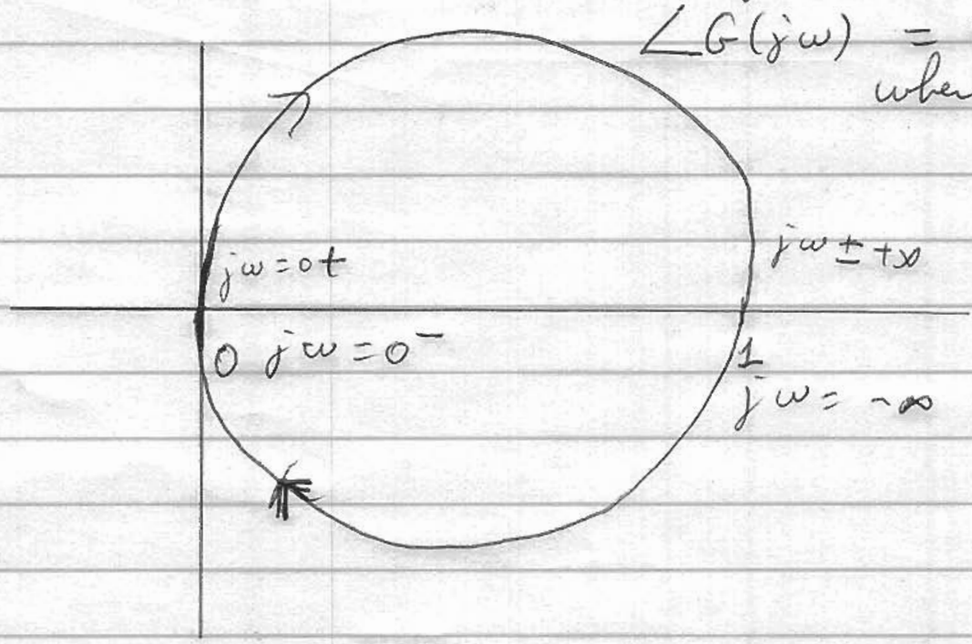
$s_1 = sT$

Then $G(s_1) = \dfrac{s}{1+s}$

$j\omega = 0 \quad G(j\omega) = 0$

$j\omega = +\infty \quad G(j\omega) = 1$

$\angle G(j\omega) = 90$ when $j\omega \sim 0$, $\omega > 0$



b) If it's a circle, the center is most likely $+\frac{1}{2}$.

Compute: $\left| G(j\omega) - \frac{1}{2} \right| = \left| \dfrac{s}{1+s} - \frac{1}{2} \right|$

$s = j\omega \qquad = \left| \dfrac{s - 1/2 - s/2}{1+s} \right|$

$= \left| \dfrac{s/2 - 1/2}{1+s} \right|$

$$= \left| \frac{1}{2}\left(\frac{j\omega - 1/2}{j\omega + 1/2}\right) \right| = 1/2$$

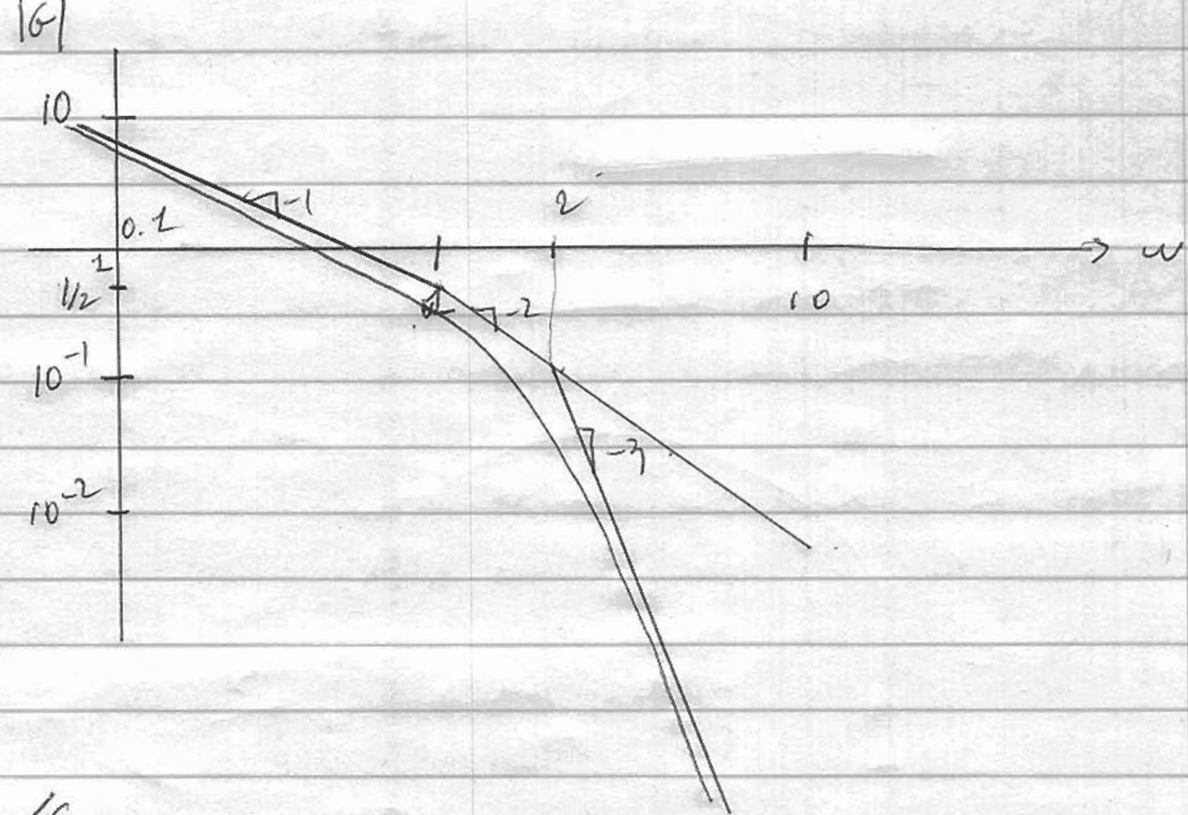
~~radius is const~~

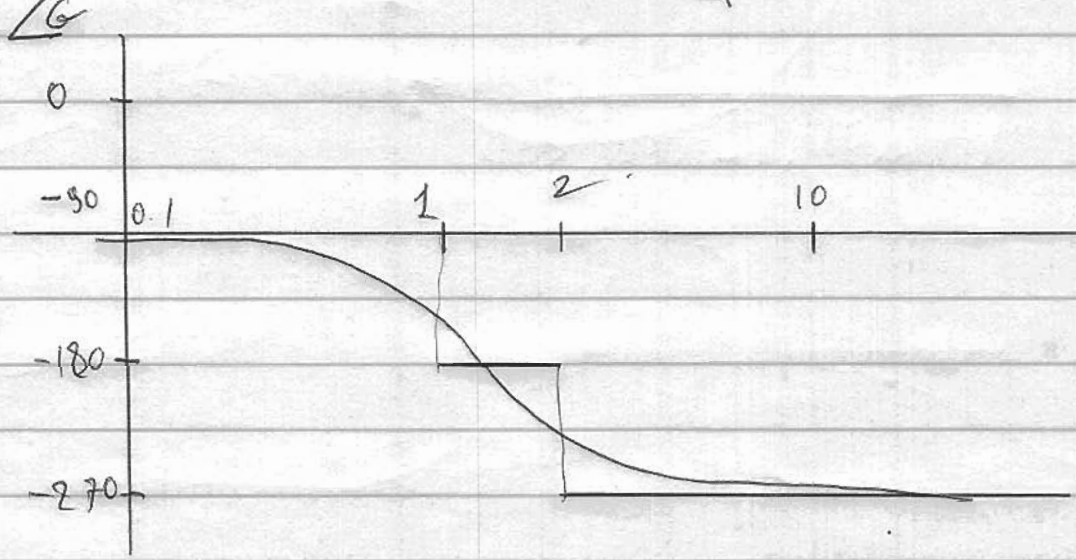
distance between $G(j\omega)$ and $1/2$ is constant, so it's a circle of radius $1/2$.

so c) is done too –

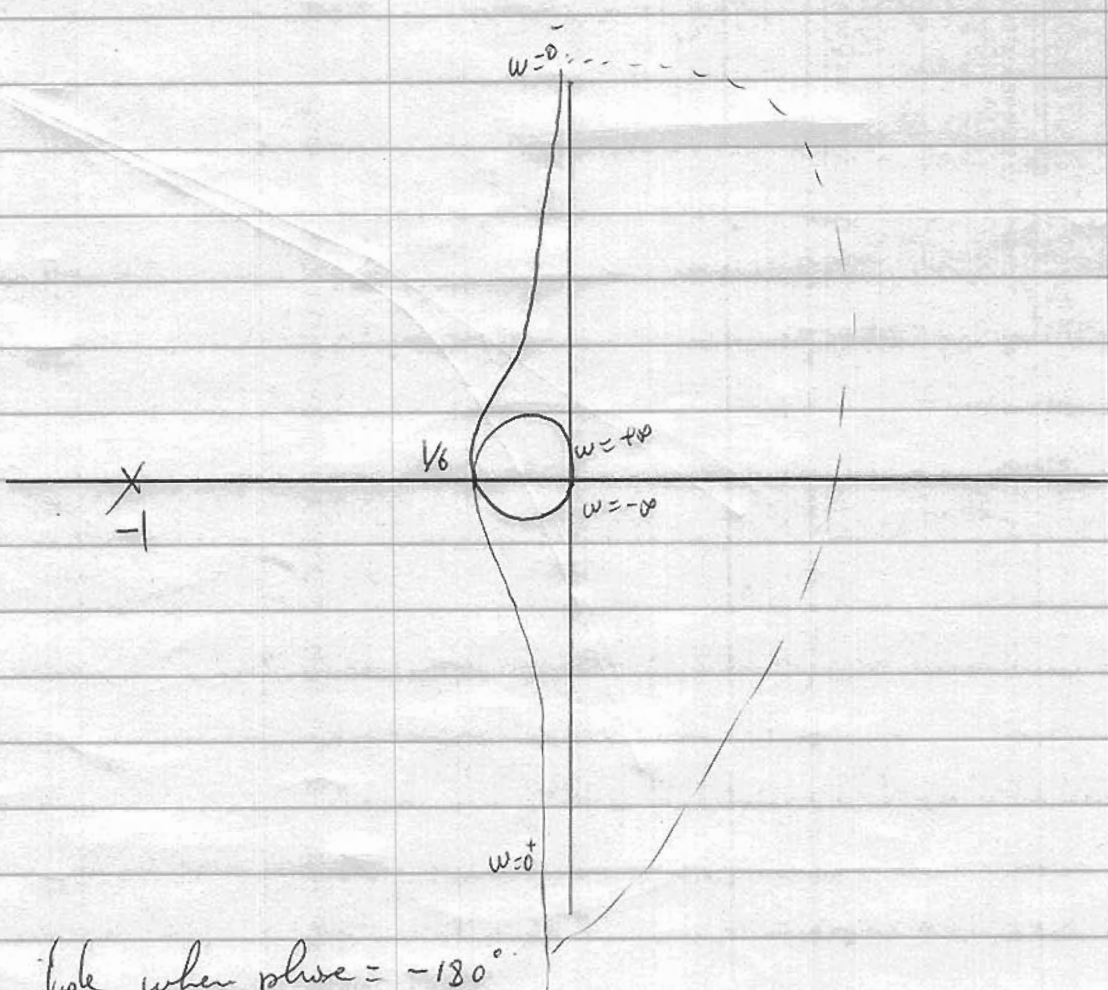
3) $$G(s) = \frac{K}{2(s)(s/1+1)(s/2+1)}$$

a) Assume $K = 1$.





b) Nyquist plot:



Magnitude when phase $= -180°$

$$\frac{1}{j\omega(j\omega+1)(j\omega+2)} = X \text{ real}$$

$$1 = X(j\omega)(j\omega+1)(j\omega+2)$$
$$= X(-\omega^2 + j\omega)(j\omega+2)$$
$$= X(-j\omega^3 - 2\omega^2 - \omega^2 + 2j\omega)$$
$$= X(-j\omega^3 - 3\omega^2 + 2j\omega)$$

So: $-\omega^3 + 2\omega = 0$ or: $\omega^2 = 2$ or $\omega = \sqrt{2}$.

and: $X(-3 \times 2) = 1$ or: $X = 1/6$.

c) The system goes unstable for $K=6$.

d) From the bode plot, the phase of the transfer function is $135°$ when w is about 0.5. pick $w=0.2$ for safety.

$$G(0.2) = \frac{1}{0.2j\,(0.2j+1)(0.2j+2)}$$

$$\sim \frac{5}{j\,(0.2j+1)(0.2j+2)},$$

phase $\sim -\pi/2 - 0.2 - 0.1 \sim$

-1.8 still $> 3\pi/4$.

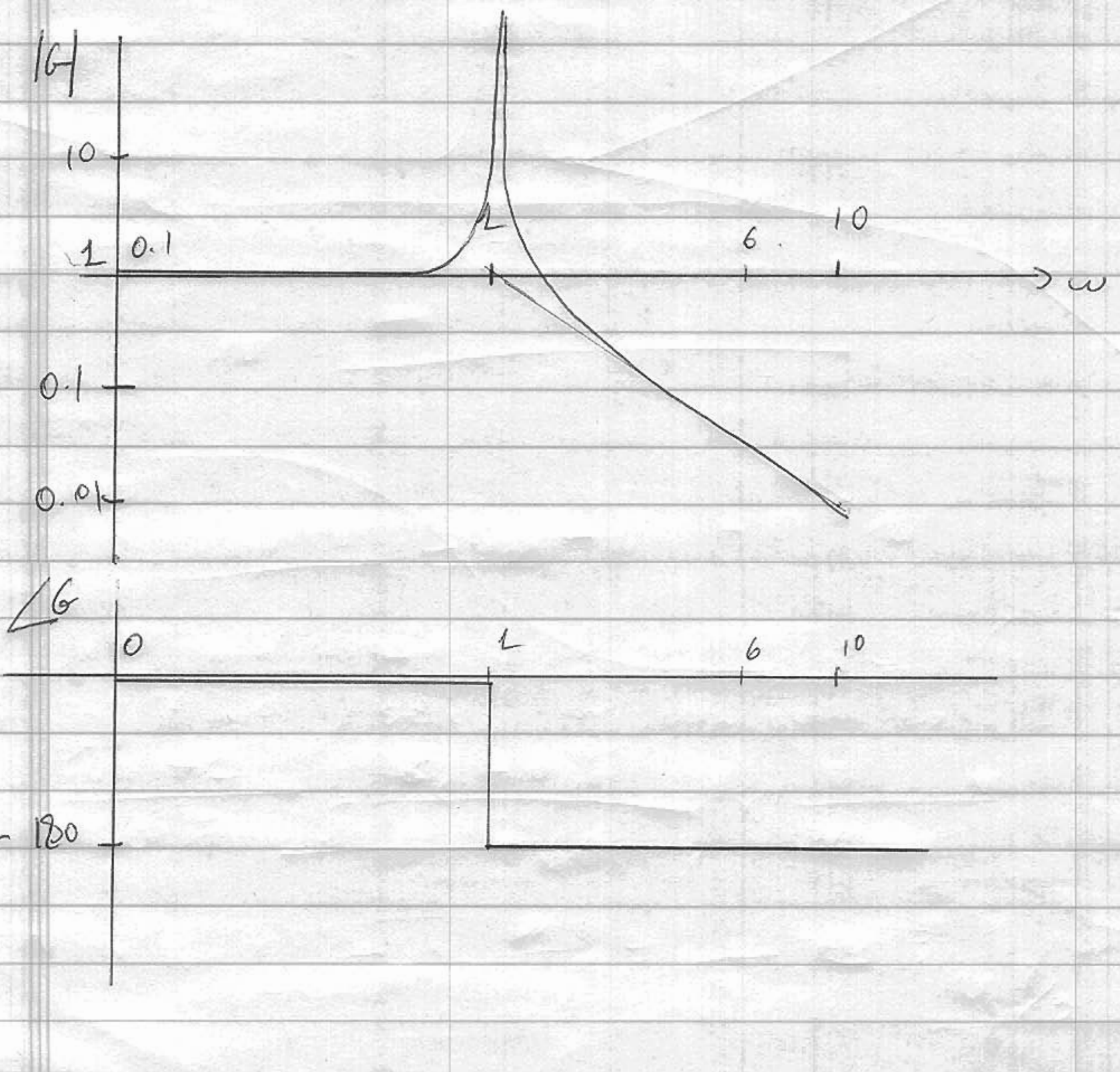
$$G = \frac{5}{1.1 \cdot 2.1} \sim 2.$$

choose $K \sim 1/2$. not the only answer!

e] the resulting BW is 0.2 rad/sec.
It's a shy controller!

4] $$G(s) = \frac{1}{s^2+1}$$

Let's do a Bode plot:



8) To get perfect tracking at DC, let's add an integrator (lag compensation)

eg:
$$K_{lag}(s) = \left(\frac{(s/0.1 + 1)}{s}\right)$$

Now our desired BW is above 6 rad/sec.
Let's pick a lead compensator that gives us 60° lead at least ~~over the~~ at that frequency (6 rad/sec)

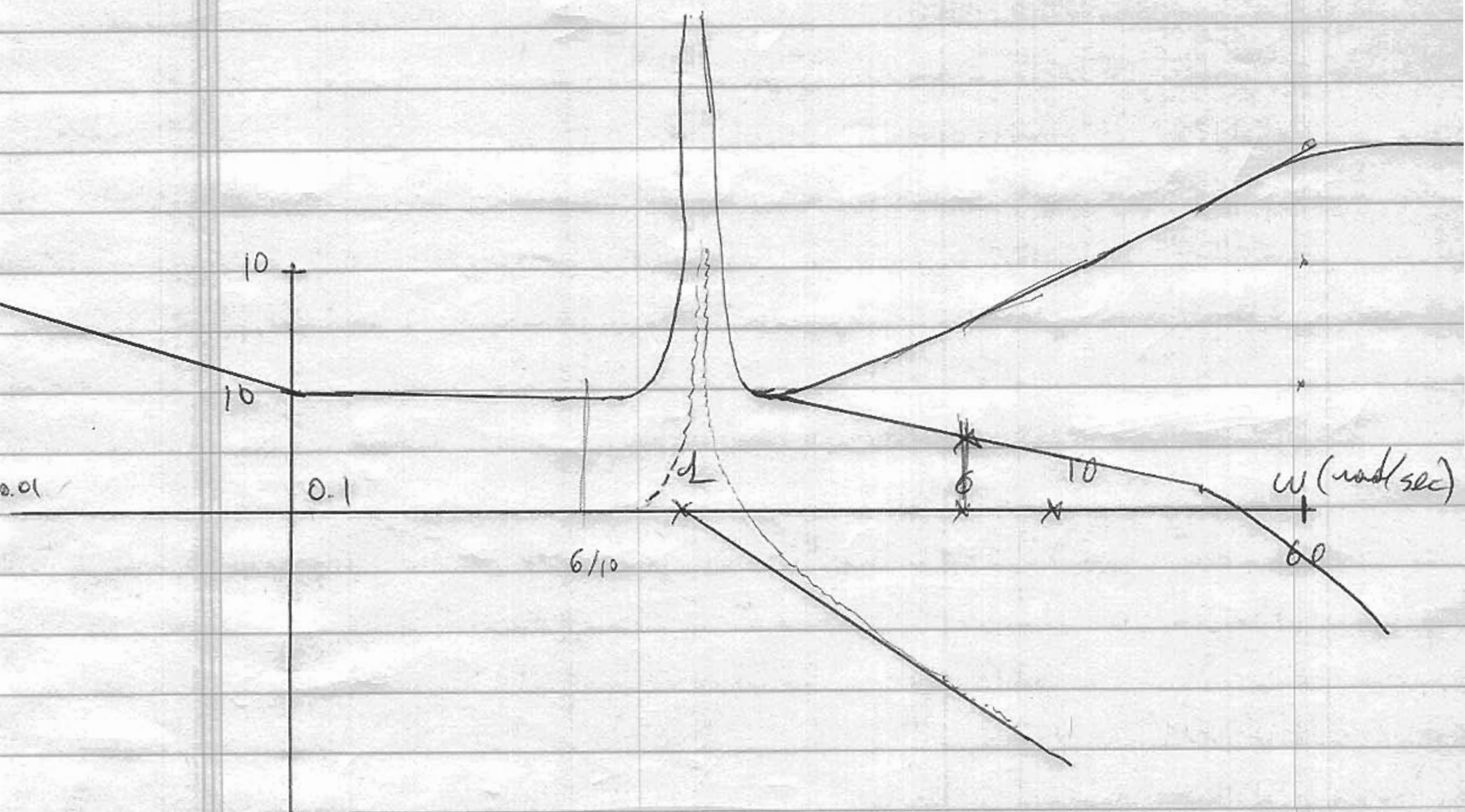
choose:

$$K_{lead} = \frac{s + 6/10}{s + 6 \times 10} \quad \text{(gives } 78^\circ \text{ lead).}$$

our compensation so far looks like:

$$K = K_0 \left(\frac{s/0.1 + 1}{s}\right)\left(\frac{s + 6/10}{s + 60}\right) \times 100$$

Bode plot:



We must now adjust the gain K_0

at 6 rad / sec, the loop gain is:

$$\left|\left(\frac{60j+1}{6j}\right)\left(\frac{6j+6/10}{6j+60}\right)\times\frac{1}{-36+1}\times 100\right|$$

$$\sim 100 \times 10 \times \frac{1}{10} \times \frac{\cancel{10}}{36} = \frac{100}{36} \simeq 3.$$ $\boxed{\text{So pick } K_0 \sim 1/3}$