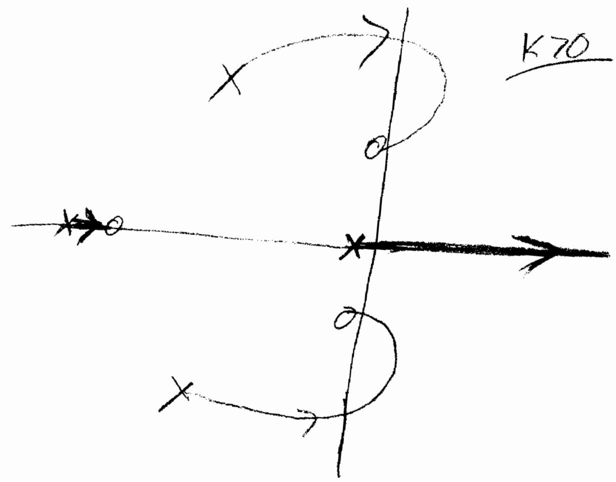
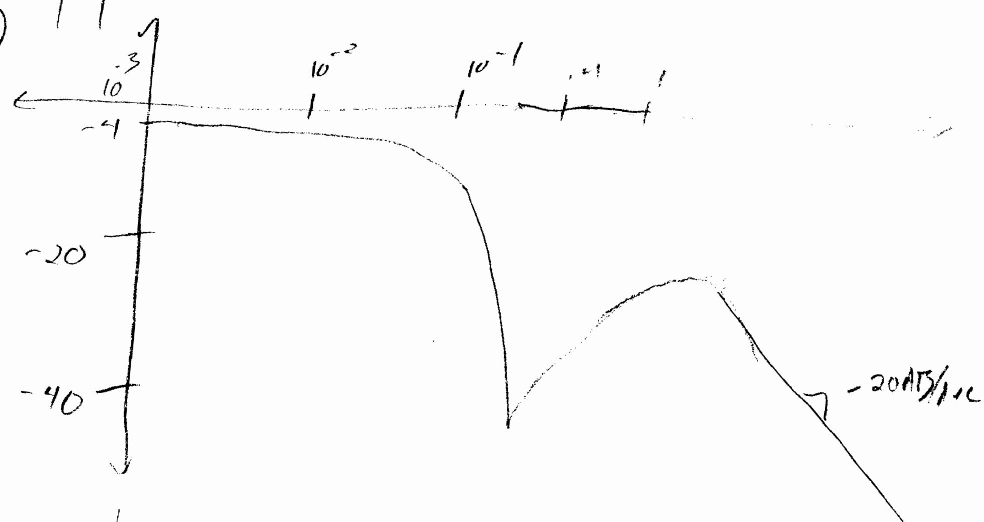
PSET 6

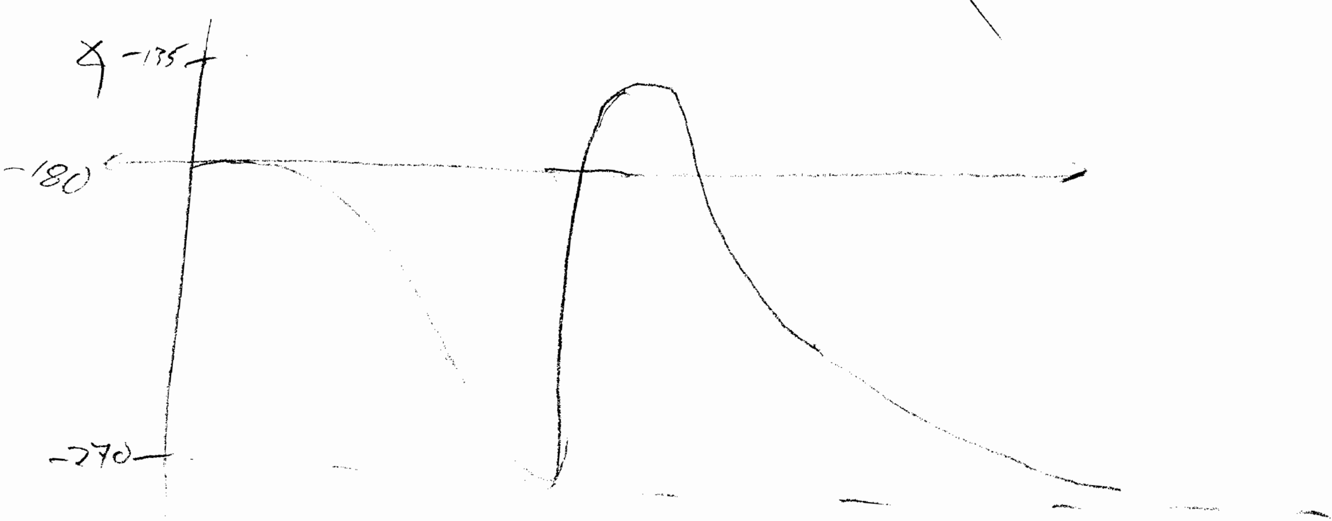
1.

a) $\frac{r(s)}{\delta T(s)} = \frac{-0.151(s+1.05)(s+0.0378 \pm 0.414j)}{(s+1.109)(s+0.0425)(s+0.646 \pm 0.731j)}$



b)





c) Bode plot of uncompensated system shown in part (b) PM

Here is one approach to designing a compensator

To get rid of notch, use notch filter of the form $G_{notch} = \dfrac{s^2 + 2\omega_n s + \omega_n^2}{s^2 + 2\zeta_w \omega_n s + \omega_n^2}$

with $\omega_n = 0.47$
$\zeta = 0.092$

Can recover phase with negative gain, this also ~~[illegible]~~ gives enough phase to allow addition of an integrator for disturbance rejection

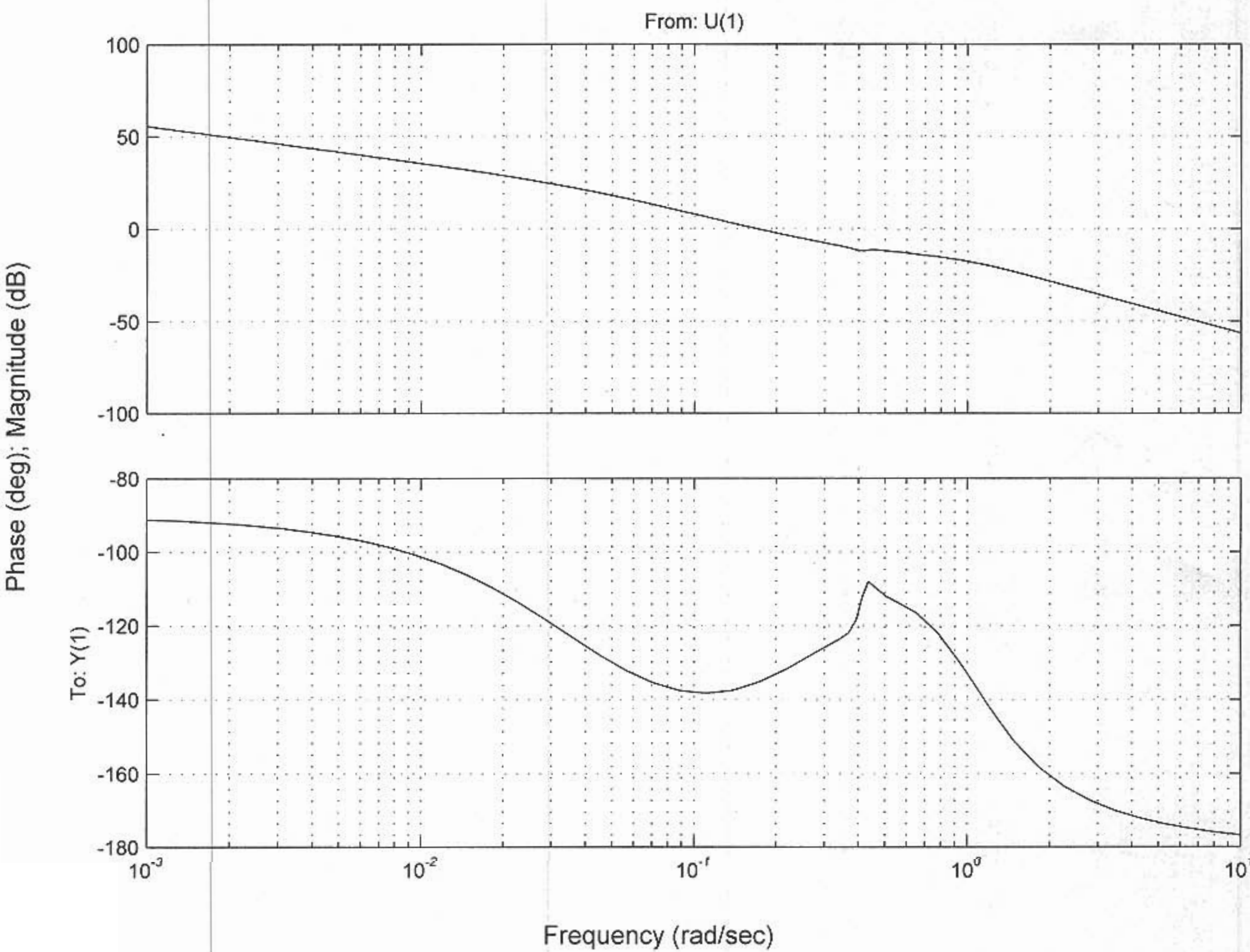
A Bode plot with $G_c = -\frac{1}{s} \cdot G_{notch}$ is shown in figure C1

I also added a lead network with zero at -0.168 and pole at -0.239 to set phase margin of $\sim 55^\circ$ at $\omega_c = 0.2$ rad/s
Note this is well below the max ~~[illegible]~~ recommended ~~bandwidth~~ bandwidth of ~ 3 rad/s

A Bode plot and step response are shown in Figures C2 and C3

Pm

Figure C1



From: U(1)
100
50
0
-50
-100
Phase (deg); Magnitude (dB)
To: Y(1)
-80
-100
-120
-140
-160
-180
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
Frequency (rad/sec)

Pm

Figure C2

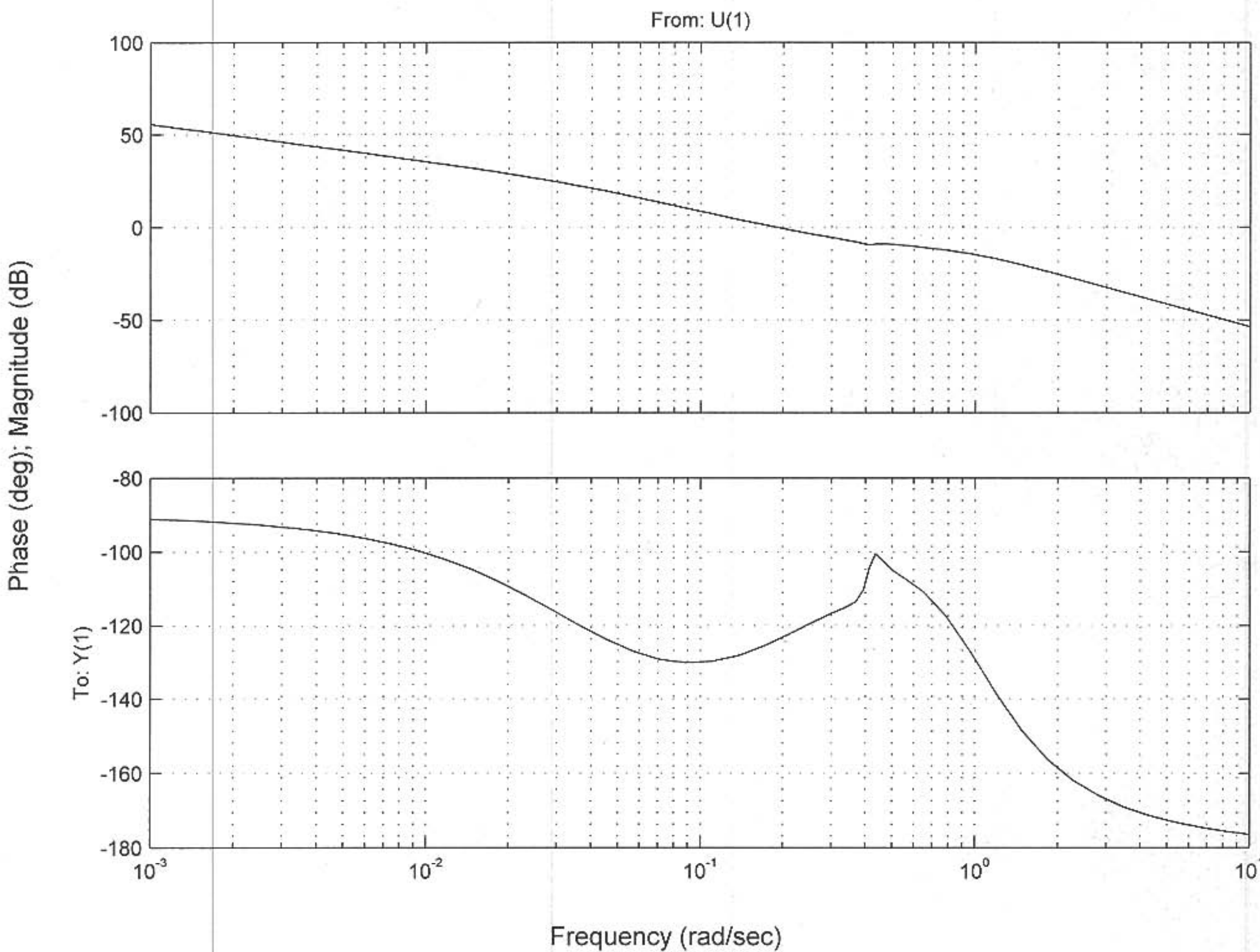
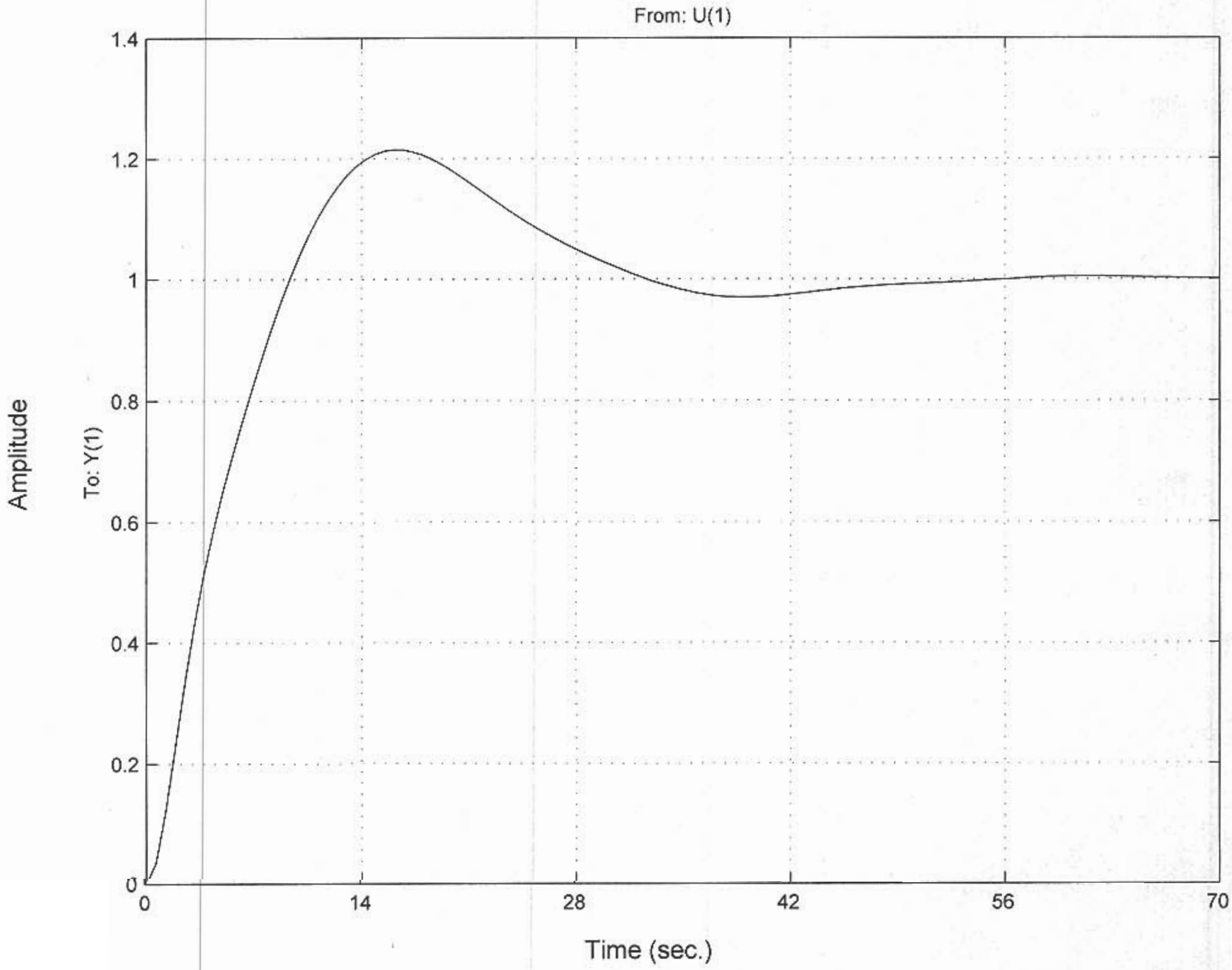

From: U(1)
Phase (deg); Magnitude (dB)
To: Y(1)
100
50
0
-50
-100
-80
-100
-120
-140
-160
-180
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
Frequency (rad/sec)

Figure C3



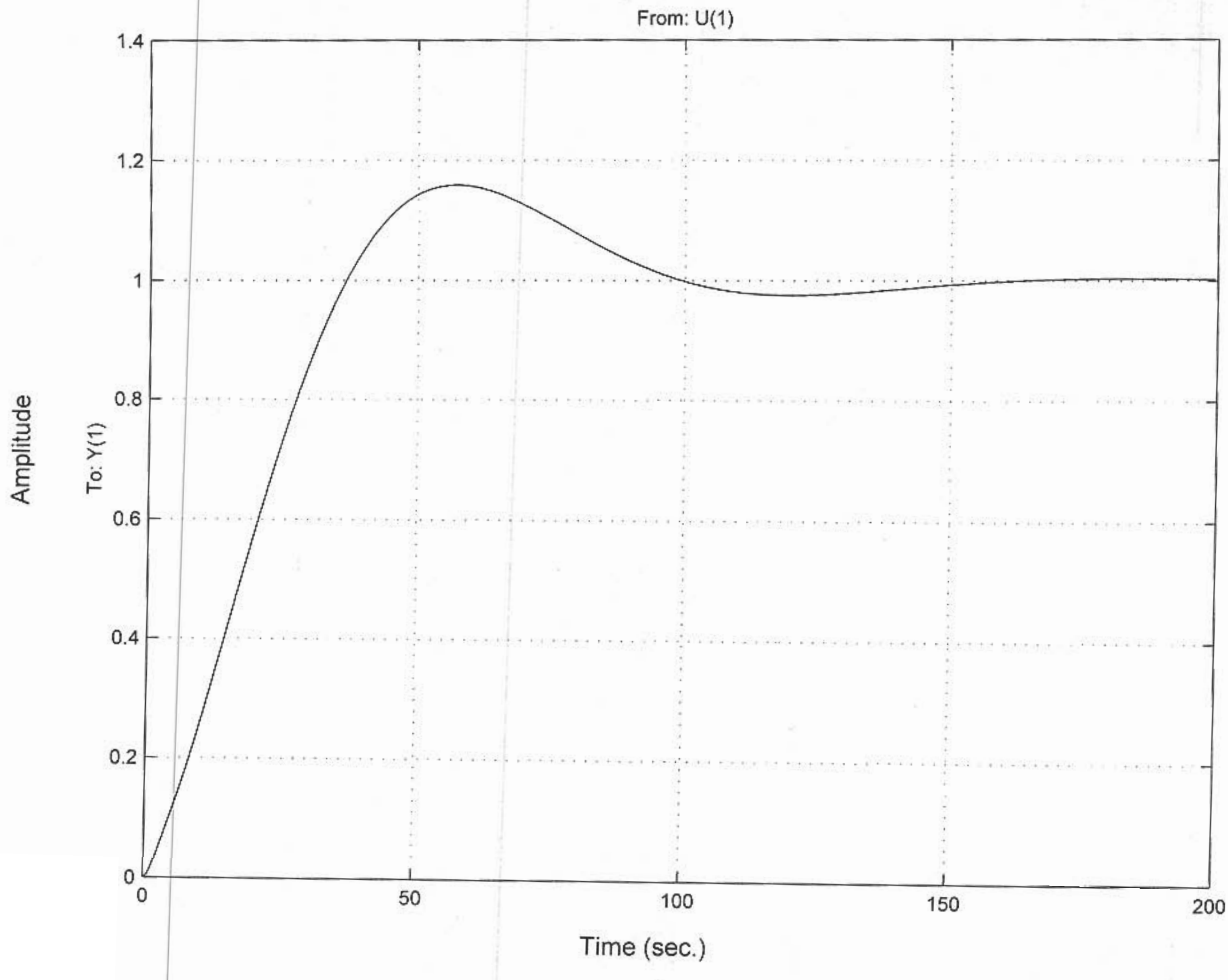
From: U(1)
To: Y(1)
Amplitude
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
14
28
42
56
70
Time (sec.)

d) A disturbance of 0.2 rad/s with the compensator [illegible] designed in part (c) yields a max plant input of about 2 rad - much to big

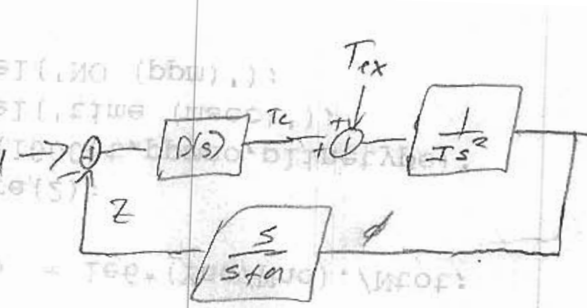
Easiest way to lower this is to reduce the gain - a gain of 0.13 gets the input to $\pi/6$ but also lowers bandwidth / raises settling time considerably to $T_s \approx 100$ s - almost 2 min!

Step response shown in Fig. D1

Figure D1


From: U(1)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Amplitude
To: Y(1)
0
50
100
150
200
Time (sec.)

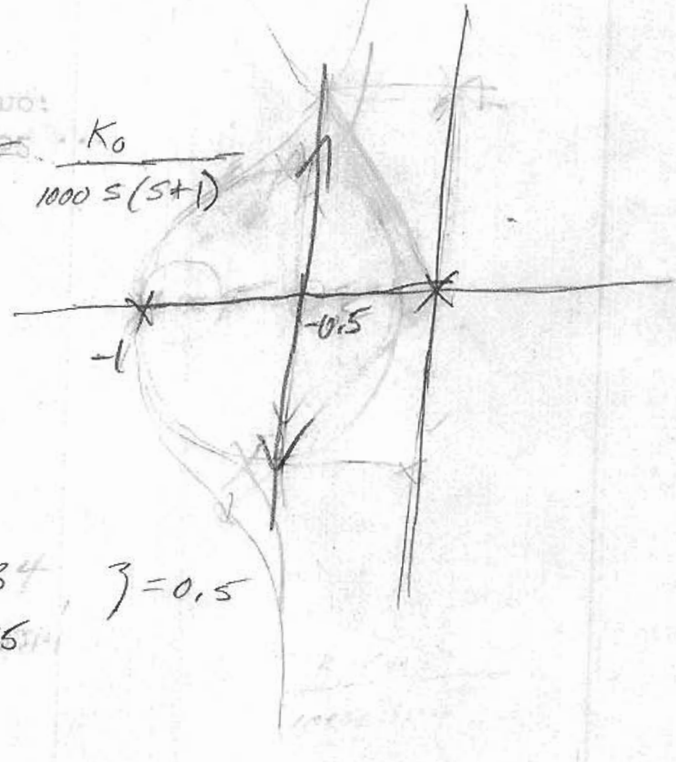
2.)



$Is^2\phi = T_c + T_{ex}$

$sZ = \phi s - aZ \Rightarrow \frac{Z}{\phi} = \frac{s}{s+a}$

a.) If $D(s) = K_0$ $\quad \frac{\phi}{Z_d} = \frac{K_0}{Is^2}\left(\frac{Is}{s+a}\right) = \frac{K_0}{1000\, s(s+1)}$



b.) $K_0 > 0$ stable

c.) closed loop system $\omega_{BW} = 1.3$ $\quad \zeta = 0.5$

$\omega_n \approx 1.15$

$$D(s) = K_1\left(\frac{s+z}{s+1}\right)$$

closed Loop: $\frac{\phi}{Z_d} = \frac{\frac{K_1(s+z)}{(s+1)}\left(\frac{1}{1000 s^2}\right)}{1 + \frac{K_1(s+z)}{(s+1)}\left(\frac{1}{1000 s^2}\right)\left(\frac{s}{s+1}\right)}$

open Loop: $\frac{K_1(s+z)}{1000\, s(s+1)^2}$

$1000 s^3 + 2000 s^2 + (1000 + K_1)s + Kz = 0$

Have from system ① $s^3 + 2s^2 + \left(1 + \frac{K}{1000}\right)s + \frac{Kz}{1000} = 0$

$(s+\alpha)(s^2 + 2(0.5)(0.0314)s + 0.0314^2) = 0$

$s^3 + (2(0.5)(0.0314) + \alpha)s^2 + (2(0.5)(0.0314)\alpha + 0.0314^2)s + 0.0314^2\alpha = 0$

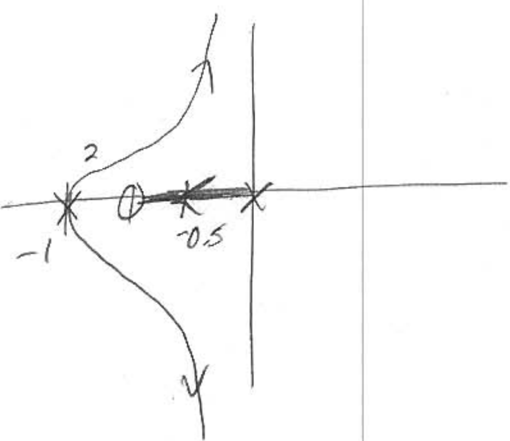
Want: $(s+\alpha)(s^2 + 2\zeta\omega_n s + \omega_n^2) = 0$ where $\zeta = 0.5, \omega_n = 1.3$

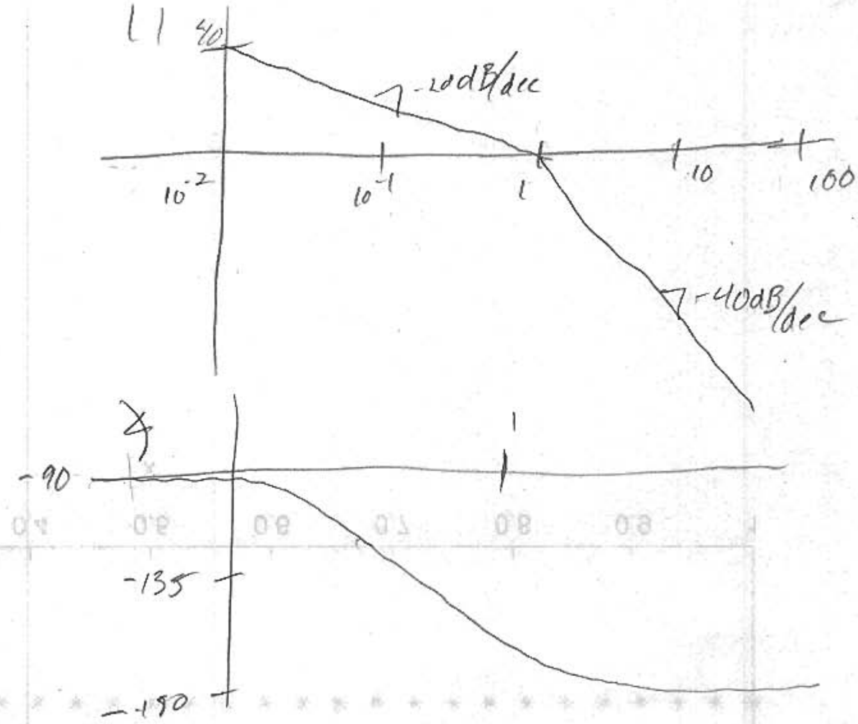
② $s^3 + (2(0.5)(1.13) + \alpha)s^2 + (2(0.5)(1.13)\alpha + 1.13^2)s + 1.13^2\alpha = 0$

Equate coefficients of ① and ② $\boxed{K_1 = 1300}$, $z = 0.864$

$$\boxed{G_{lead}(s) = 1300\frac{(s + 0.864)}{s+1}}$$

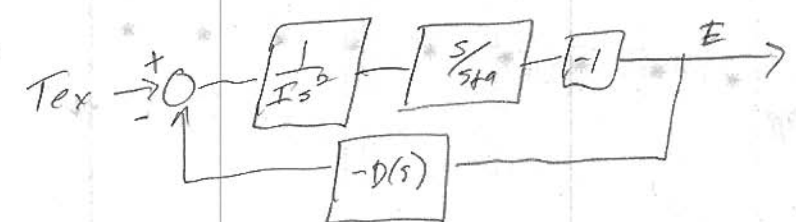
GT





d.) $K > 0$ stable

e.)



$$D(s) = \frac{K(s+z)}{(s+1)}$$

$$\lim_{s \to 0}\left(s\frac{E(s)}{T_{ex}(s)}\right) \text{ to a step } \quad \lim_{s \to 0}\left(\cancel{s}\cdot\frac{1}{\cancel{s}}\cdot\frac{\frac{-s}{s+a}\,\frac{1}{Is^2}}{1 + -D(s)\,\frac{1}{Is^2}\,\frac{s}{s+a}}\right)$$

$$\lim_{s \to 0}\left(\frac{\frac{-1}{Is(s+a)}}{1 + \frac{-D(s)}{Is(s+a)}}\right) \Rightarrow \lim_{s \to 0}\left(\frac{-1}{Is(s+a) - \frac{K(s+z)}{(s+1)}}\right) \Rightarrow \lim_{s \to 0}\left(\frac{-(s+1)}{Is(s+1)^2 - K(s+z)}\right)$$

$$= \frac{-1}{-Kz} = \frac{1}{1300(0.864)} = 8.9\times10^{-4}$$ or something close. Depends on controller.

f.) $\phi_m = 52.3^\circ$ } Computed in MATLAB
g_m is infinite }