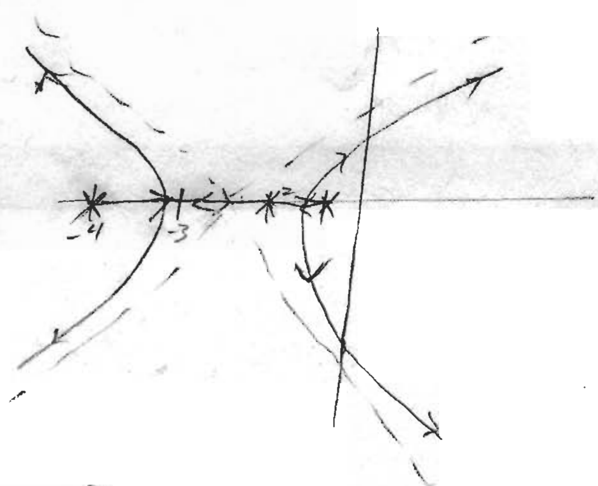
PSET 4

GT

1. a.) $G(s) = \dfrac{1}{(s+2)^2(s+4)(s+1)}$



$\alpha = \dfrac{-2-2-4-1}{4} = -9/4 = -2.25$

$\phi_l = \pm 45, \pm 135$

b.) $-180 = -2\tan^{-1}(\omega/2) - \tan^{-1}(\omega/4) - \tan^{-1}(\omega)$

$\omega = 2$

$1 = \dfrac{K}{(2^2+2^2)(2^2+4^2)^{1/2}(2^2+1)^{1/2}}$ $\boxed{K=80}$

c.)

$1 + \dfrac{K}{(s+2)^2(s+4)(s+1)} = 0$

$s^4 + 9s^3 + 28s^2 + 36s + 16 + K = 0 = (s^2 + 2\alpha s + \beta^2)(s^2 + 2\zeta\omega_n s + \omega_n^2)$

$= s^4 + 2\zeta\omega_n s^3 + \omega_n^2 s^2$
$+ 2\alpha s^3 + 4\alpha\zeta\omega_n s^2 + 2\alpha\omega_n^2 s$
$\beta^2 s^2 + 2\zeta\beta^2\omega_n s + \omega_n^2\beta^2$

$= s^4 + (2\zeta\omega_n + 2\alpha)s^3 + (\omega_n^2 + 4\alpha\zeta\omega_n + \beta^2)s^2$
$+ (2\alpha\omega_n^2 + 2\zeta\beta^2\omega_n)s + \omega_n^2\beta^2$

Equate coefficients, $(2\zeta\omega_n + 2\alpha) = 9$, etc...

$\boxed{K = -34.9\ ,\ 5.93}$

d.) $K = 5.93$, $G_m = 22.601$ dB or 13.5 $\phi_m = \infty$

$K = -34.1$ $G_m = -6.774$ dB or 0.46 $\phi_m = -127.5$

ϕ_m: Since $G(s) = \dfrac{5.93}{16}\left(\dfrac{1}{(s/2+1)^2(s/4+1)(s+1)}\right)$ has a magnitude always <1 i.e. $\dfrac{5.93}{16} < 1$, the $\phi_m = \infty$.

G_m: $-180 = -2\tan^{-1}(\omega/2) - \tan^{-1}(\omega/4) - \tan^{-1}(\omega)$ $\Rightarrow$ $1 = \dfrac{K(5.93)}{(2^2+2^2)(2^2+4^2)^{1/2}(2^2+1)^{1/2}} \Rightarrow \underline{K = 13.5}$

$\omega = 2$

2) $-180 = \tan^{-1}(w) + \tan^{-1}(w/2) - 180 - \tan^{-1}(w/3) - \tan^{-1}\left(\frac{2w}{25-w^2}\right) = \measuredangle G(jw)$

Instability occurs when $\phi(s_{y5}) = 180$, $\boxed{w = 4.96}$

Plug w in $|G(s)|\Big|_{w=4.96} = 1$, Solve. $\boxed{K = 52.35}$

$$|G(s)|\Big|_{w} = 1 = \frac{K(w^2+1)^{1/2}(w^2+2^2)^{1/2}}{w^2(w^2+3^2)^{1/2}\left((2w)^2+(25-w^2)^2\right)^{1/2}}$$

For p.m. of 20°

$-160 = \measuredangle G(jw)$ for some w, Calc. w.

$-160 = \tan^{-1}(w) + \tan^{-1}(w/2) - \tan^{-1}(-w/3) - 180 - \tan^{-1}\left(\frac{2w}{25-w^2}\right) \Rightarrow w = 4.6$

$|G(s)|\Big|_{w=4.6} = 1 \Rightarrow \boxed{K = 33.1\,dB \text{ or } 49.0629}$

At that phase margin $\boxed{Gm = 0.563\,dB \text{ or } 1.0669}$

Find w that $\measuredangle 49.0629\,G(s) = -180$ $w = 4.9618$

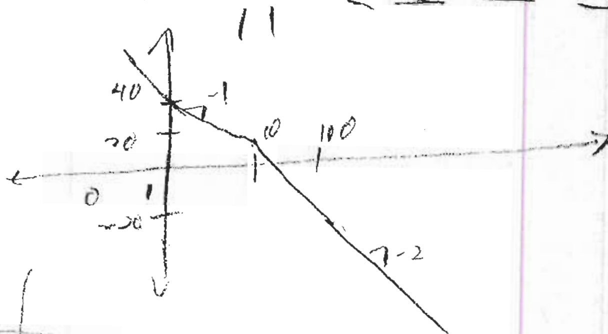
plug into $K|G(s)|\Big|_{w=4.9618} = 1$

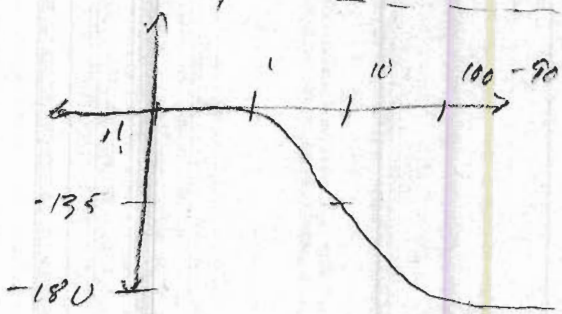
OR You know the additional gain needed is $K_c(49.0629) = 52.35$

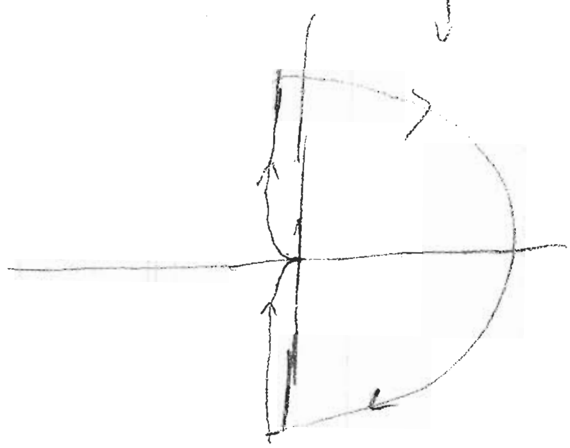
$K_c = 1.0669$

---

3) $\frac{100}{s(s/10+1)}$







	w	P	Z		
$-\infty < -1/K < 0$	0	0	0	$\boxed{0 < K < \infty}$	Stable
$0 < -1/K < \infty$	0	0	1	$-\infty < K < 0$	Unstable

GT

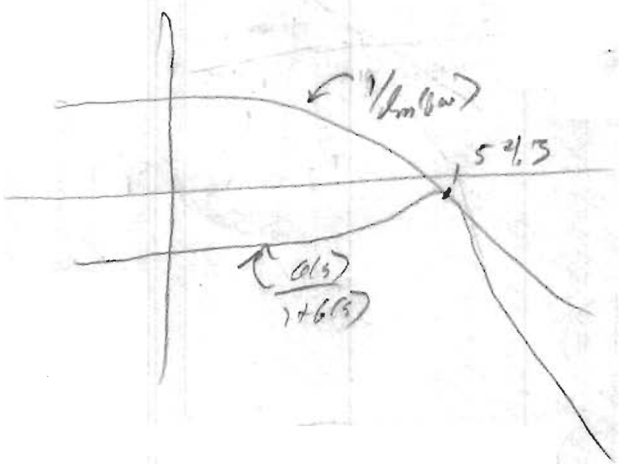
b.) $\frac{1}{l_n(j\omega)} > \left|\frac{G(j\omega)}{1+G(j\omega)}\right|$ $\quad \tilde{G}_p = G_p(1+l(s))$

$$\left|\frac{300}{s+10}\right| > \frac{\frac{100K}{s(s/10+1)}}{1+\frac{100K}{s(s/10+1)}}$$

$$\left|\frac{300}{s+10}\right| > \left|\frac{100K}{s(s/10+1)+100K}\right|$$

$$\left|\frac{300}{s+10}\right| > \frac{100K}{s^2/10+s+100K}$$

$$\left|\frac{300}{s+10}\right| > \frac{1000K}{s^2+10s+1000K} \qquad \omega_n = \sqrt{1000K}$$



$$\frac{300}{(1000K+100)^{1/2}} = \frac{1000K}{\left((10\sqrt{1000K})^2+(1000K-1000K)\right)^{1/2}}$$

$$\frac{300}{(1000K+100)^{1/2}} = \frac{1000K}{10\sqrt{1000K}}$$

$$\frac{300^2}{1000K+100} = \frac{1000^2K^2}{10^2(1000K)}$$

solve for $K = \frac{1}{20}(-1+\sqrt{3601})$

$$\boxed{K = 2.95}$$

c.)

$\omega_n = \sqrt{1000K}$

$\omega_n = 54.3$

$2\zeta\omega_n = 10$

$\zeta = 0.092$

$$\omega_b = \omega_n\left(1-2\zeta^2+\sqrt{2-4\zeta^2+4\zeta^4}\right)^{1/2}$$

$$\omega_b = 54.3\left(1-2(0.092)^2+\sqrt{2-(.092)^2(4)+4(.092)^4}\right)^{1/2}$$

$$\boxed{\omega_b = 83.86}$$